Indian Historical Researches

THE

BAKHSHĀLĪ MANUSCRIPT

A Study in Mediaeval Mathematics

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THE BAKHSHALI MANUSCRIPT

Early Hindu Mathematics
A Study in Mediaeval Mathematics,

KAY G.R.

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PREFACE.

In order to correct an impression that certain passages in this volume might convey unless distinctly qualified, I must here refer to my indebtedness to the late Dr. Hoernle. Indeed, a considerable part of the analysis of the MS. is really his work,* and by his preliminary survey of the manuscript my task was considerably lightened. It was at Dr. Hoernle's special request that I undertook to carry on the work he had started, and he handed over to me most of the material he had himself prepared. Had he lived a little longer I should, no doubt, have had the benefit of further help from him, and this volume might have been issued as our joint work. Dr. Hoernle's lamented death prevented that plan being carried out; and unfortunately my views are so often opposed to those that where held by Dr. Hoernle that it would hardly be proper to make him a participator in them.

I am much indebted to Bodley's Librarian for special facilities that enabled me to examine the original manuscript under the most favourable conditions; to the Oxford University Press for their most excellent work in preparing the photographs of the manuscript and the collotype reproductions of the text; and to the Manager, Government of India Press, Calcutta, for the care and skill with which the transliteration has been printed.

G. R. KAYE.

BANHAM,
Attleborough,
Norfolk.

* Sections B, G, H, K and L are almost wholly the work of Dr. Hoernle, who also transliterated about half of the leaves of the MS. References to his published papers on the MS. are given on page 2.

ERRATA.

On the plate facing page 4 read '51 RECTO B' for '51 VERSO B'.

In table iv, 4 (Part ii) the fifteenth ligature is *sva*, not *sya*.

On p. 41 in the second example x_1, x_2 and x_3 have been wrongly interchanged in the solution and answer.

In the table on p. 14 'folio 55' should come before 'folio 49'. That is M 9 is folio 55 and M 10 is folio 49.

p. 20, § 52. Add "on folio 49 is a phrase *śūkhyair yajamti* &c."

p. 28, foot-note g: for 2+1 read ²1+7.

p. 29, penultimate line: for ⁸7⁶ read ⁸7⁵.

p. 62, § 109. Add 'the terms *prasriti* and *khārī* occur on folio 34, but with doubtful application.'

p. 63. Omit the last line of the capacity table.

p. 127. Fol. 29r(*a*). Second line. Read *guṇabhayāsa*.

(*d*). Second line. Read *dya* for *sya*.

Fol. 29v.(*d*). Second line. Read *tritīya* for *tritīya*.

p. 130. Fol. 33v. Fourth line. Read *pādavyase* for *pāgadhyase*.

p. 131. Fol. 35r. (*b*). Second line. Read *yogosyā* for *yogaṁsvā*.

p. 136. Fol. 40v. (*d*). Second line. Read *prathamaṁ* for *prathamā*.

p. 137. Fol. 42v. Third line. Read *viṁśanāṁ* for *riṁśanaṁ*.

p. 139. Fol. 44v. Second line. Read *bha 8* for *b. ā 8*.

p. 139. Fol. 45v. First line. Read *akṛi* for *asya*.

p. 142. Fol. 49r. Second line. Read *danāstimūdṛi* for *pādanastimūdṛi*.

p. 145. Fol. 54r. (*b*). Read *sūtraṁ* (*c*) *jātaṁ*.

p. 147. Fol. 56r. Penultimate line. Read *yāramarjaye* for *yāvarjaye*.

p. 147. Fol. 57r. Last line. Read *ekorāśi* for *ekonarāśi*.

THE BAKHSHĀLĪ MANUSCRIPT.

PART I.—INTRODUCTION.

CHAPTER I.

1. In 1881 a mathematical work written on birch-bark was found at Bakhshālī near Mardān on the north-west frontier of India. This manuscript was supposed to be of great age and its discovery aroused considerable interest. Part of it was examined by Dr. Hoernle, who published a short account together with a translation of a few of the leaves in 1888. Dr. Hoernle had intended, in due course, to publish a complete edition of the text, but was unable to do so. The present volume gives the complete text.

2. Bakhshālī, or Bakhshalai, as it is written in the official maps, is a village of the Yusufzai sub-division of the Peshawar district of the North-West Frontier Province of India. It is situated on, or near, the river Mukhām, which eventually joins the Kābul river near Nowshera, some twenty miles further south. Six miles W.N.W. of Bakhshālī is Jamalgarhī, twelve miles to the west is Takht-i-Bhai, and twenty-five miles W.S.W. is Chārsada—famous for their Indo-Greek art treasures.

Bakhshālī is about 150 miles from Kābul, 160 from Srinagar, 50 from Peshawar, 350 from Balkh and 70 from Taxila. It is in the trans-Indus country and in ancient times was within the Persian boundaries—in the Arachosian satrapy of the Achaemenid kings. It is within that part of the country to which the name Gandhāra has been given, and was subject to those western influences which are so bountifully illustrated in the so-called Gandhāra art.

3. The only authentic record of the discovery of the manuscript appears to be contained in the following letter, dated the 5th of July 1881,[1] from the Assistant Commissioner at Mardān.

"In reply to your No. 1306, dated 20th ultimo, and its enclosures, I have the honour to inform you that the remains of the papyrus MS. referred to were brought to me by the Inspector of Police, Mian An-Wan-Udin. The finder, a tenant of the latter, said he had found the manuscript while digging in a ruined stone enclosure on one of the mounds near Bakhshāli.[2] These mounds lie on the west side of the Mardān and Bakhshālī roads and are evidently the remains of a former village. Close to the same spot the man found a triangular-shaped 'diwa,' a soap-stone pencil, and a large lota of baked clay with a perforated bottom. I had a further search made but nothing else was found.

"According to the finder's statement the greater part of the manuscript had been destroyed in taking it up from the place where it lay between stones. The remains when brought to me were like dry tinder, and there may be about fifty pages left some of which would be certainly legible to any one who knew the characters. The letters on some of the pages are very clear and look like some kind of *Prakrit*, but it is most difficult to separate the pages without injuring them. I had intended to forward the manuscript to the Lahore Museum in the hope that it might be sent on thence to some scholar, but I was unable to have a proper tin box made for it before I left Mardān. I will see to this on my return from leave. The papyrus will require very tender manipulation. The result will be interesting if it enables us to judge the age of the ruins where the manuscript was found."

---

[1] Apparently the manuscript was found in May 1881.

[2] General Cunningham in a private letter to Dr. Hoernle, dated Simla, 5th June 1882, says: "Bakhshālī is 4 miles north of Shāhbāsgarhī. It is a mound with the village on the top of it. The birch-bark manuscript was found in a field near a well without trace of any building near the spot, which is outside the mound village........."

This account is very unsatisfactory and there are indications that it is not altogether reliable. It was written, apparently from memory, some month or so after the discovery of the manuscript. The "ruins," it appears, were the creation of the writer's imagination, and the statement generally does not give the impression of exactitude.

4. In the meantime notices of the discovery had found their way into the Indian newspapers. Professor Bühler, who had read of the discovery in the "Bombay Gazette"[1], communicated the announcement to Professor Weber, who brought it to the notice of the fifth International Congress of Orientalists then assembled in Berlin.[2] In Bühler's letter to Weber it was stated that the manuscript had been found "carefully enclosed in a stone chamber," and it was thought that the newly discovered manuscript might prove to be "one of the Tripitakas which Kanishka ordered to be deposited in Stūpas."

There is nothing whatever in the record of the find to justify Bühler's statement, which seems to have originated in a rather strange interpretation of the words "while digging in a stone enclosure" that occur in the letter quoted above, and which are themselves of doubtful reliability. And Bühler's views would hardly have been worth recording here had it not been that their effect was altogether disproportionate to their value. Perhaps the exaggeration of the value of the find, however, served a useful purpose at the time; but now it has become embarrassing—for refuting it makes the present editor of the manuscript appear to be decrying his wares.

5. The manuscript was subsequently sent to the Lieutenant-Governor of the Punjab, who, on the advice of General Cunningham, directed it to be transmitted to Dr. Hoernle, then head of the Calcutta Madrasa, for examination and publication. In 1882 Dr. Hoernle gave a short description of the manuscript before the Asiatic Society of Bengal, and this description was published in the Indian Antiquary of 1883.[3] At the seventh oriental conference held at Vienna in 1886 he gave a fuller account which was published in the proceedings of the conference,[4] and also, with some additions, in the Indian Antiquary of 1888.[5] In 1902 Dr. Hoernle presented the manuscript to the Bodleian Library.

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[1] This account appears in the BOMBAY GAZETTE of Wednesday, August 13th, 1881, and is as follows:—

"The remains of a very ancient papyrus manuscript have been found near Baskhāll, in the Mardān tahsil, Peshawar District. On the west side of the Mardān and Baskhāll road are some mounds, believed to be the remains of a former village, though nothing is known with any certainty regarding them, and it was while digging in a ruined stone enclosure on one of these mounds the discovery was made. A triangular-shaped stone 'diwa', and a soap stone pencil, and a large lotah of baked clay, with a perforated bottom, were found at the same place. Much of the manuscript was destroyed by the ignorant finder in taking it up from the spot where it lay between the stones; and the remains are described as being like dry tinder, in some of the pages. However, the character, which somewhat resembles *Prakrit*, is clear, and it is hoped it may be deciphered when it reaches Lahore, whither we understand it is shortly to be sent."

[2] The official record is given in the report of the Congress (Part I, p. 79) as follows:—

Der Präsident (Weber) verlas darauf aus einem Briefe von Prof. Bühler folgende hochwichtige Mittheilung:

"Ein Penjābi Landmann soll beim Steinegraben einen alten Papyros gefunden haben, der sorfältig in einen Steinkammer eingeschlossen war. Derselbe soll sehr umfangreich gewesen, doch Vieles vom Finder durch Unvorsichtigkeit zerstort sein. Die sehr bedeutenden Reste sind nach Lahore gebracht. Ganz Seiten sollen lesbar sein und die Schrift 'wie Prâkrit' aussehen. Es konnte dies wohl eines der Tripiṭaka's sein, die Kanishka in Stûpa's niederlegen liess. Ich habe gleich an Cunningham geschreiben und um ein Stuck wenigstens in Photographie gebeten."

[3] Vol. xii, pp. 89-90.

[4] Verhandlungen des vii Internationalen Orientalisten-Congresses, Arische Section, 127 *seq.*

[5] Vol. xvii, pp. 33-48 and 275-279.

CHAPTER II.

6. The manuscript consists of some 70 leaves of birch-bark, but some of these are mere scraps. The largest leaf measures about 5·75 by 3·5 inches or 14·5 by 8·9 centimetres. The leaves, which are numbered according to the Bodleian Library arrangement from 1 to 70, may be classified according to their size and condition as follows :—

In fair condition but broken at the edges—size, not less than 5 by 3 inches (13 by 8 cm.)

1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 32, 33, 34, 36, 37, 43, 44, 47, 49, 59, 60, 61, 62, 63—total 35.

Rather more damaged but otherwise in fair condition—not less than 4¾ by 2 inches (12 by 5 cm.)

2, 25, 26, 42, 45, 46, 48, 50, 51, 52, 55, 56, 57, 58, 64, 69—total 16.

Much damaged.

21, 31, 35, 41, 53, 66, 68—total 7.

Scraps.

27, 28, 29, 30, 35, 38, 39, 40, 54, 68, 70—total 11.

One folio (19) is entirely blank.

Certain folios consist of two leaves stuck together, namely 7, 31, and 65, and possibly others. It would not be difficult to separate these double leaves without damaging the manuscript.

7. The eleven folios classified as " scraps " at first appeared completely refractory but the exercise of considerable patience has led the following to submit to some arrangement. Here the letters a, b, c, etc., refer to the scraps in the order in which they appear on the right sides of the leaves as illustrated and the revised arrangement is indicated below :—

Revised order.	References to the plates.
A1	$40^{a}+39^{a}+39^{b1}+39^{c}+38^{h}$.
A2	$39^{b}+40^{e}+38^{e}+40^{b}+39^{d}$.
A3	$40^{d}+39^{e}+38^{a}$.
A6	$51^{b}+35^{a}$.
A9	$29^{d}+29^{b}+29^{c}$.
A10	$27+29^{a}$.

Of these rearrangements one side of each of A1, A6 and A9 are shown in the illustration facing p. 4. These rearrangements were made from the reproductions —a much more difficult task than working with the originals.

8. Dr. Hoernle's first estimate of the original size of the leaves was 7 by 8¼ inches, and this estimate was based upon " the well-known fact that the old birch-bark manuscripts were always written on leaves of a squarish size ", and upon the

obvious incompleteness of folio 17 (xliii),[1] but this estimate he himself corrected some twelve years later.[2]

Although no leaf of the manuscript is now complete there are 35 leaves in comparatively fair condition, and these vary in size[3] from about 5¼ by 3¼ inches to 6 by 3¾ inches. There are some lines of writing quite complete, though not many—and these complete lines measure from 5½ to 6 inches in length. There is also evidence of a small margin on either side, and the actual space occupied is just over 6 inches or 15½ centimetres. I estimate that the original length was from about 6½ to 7 inches.

With regard to the depth I estimate from 4 to 4½ inches. In a few cases we have the complete set of lines (generally from 10 to 11 to the page) and in some other cases the amount missing can be roughly estimated, while the actual maximum depth that occurs is 3¾ inches. My estimate of the original size of the leaves is therefore 6½ to 7 inches long by 4 to 4½ inches deep or 16·6 to 17·8 centimetres by 10·2 to 11·5 centimetres. Dr. Hoernle's final estimate was 7 by 4 inches.

8 (*a*). In ordinary Sanskrit manuscripts it is the custom to number the leaves (not the pages) in order, generally in the left hand margin of the reverse.[4] No single leaf of our manuscript has this part intact and there is no evidence of such numbering.

9. The leaves are now mounted between sheets of mica and placed within an album. The mica sheets are about 7·4 by 4·6 inches and are fixed together by strips of gummed paper at the edges leaving a clear area of 6½ by 3¾ inches. The general arrangement is shown in Plate I (Part II). Some other method of mounting (*e.g.*, between glass plates) might be safer. It should be possible to separate the leaves now stuck together, and the possibility of thereby discovering new material would justify considerable trouble being taken in this matter.

10. Birch-bark is an outer bark of the Silver Birch (*Betula utilis, Betula bhojpatra*, or the *Bhurja tree*, as it is variously called) which flourishes in the Himalayas from Kashmir to Sikkim. It grows on all the higher ranges of the Kashmir hills from a height of about 6,000 feet to 12,000 feet. The forests in the Gurais district supply most of the *bhojpatra* that is sold in Srinagar. The bark is used chiefly for the roofing of houses, for wrapping up things, for lining baskets, etc., and the villagers still use it as a writing material.

To obtain the bark from the tree a deep cut is made vertically down a clean piece of bole, and the bark is then peeled off by the hand. The operation is very much the same as that employed in "girdling" pine trees except that the upper and lower cuts are not made. The paper bark appears to be thrown off by an under red bark and apparently one layer is produced each year. If all the laminæ are stripped off from a tree, it either dies, or, if it survives, it does not give good bark a second time. The most suitable size of tree is from 2 feet to 4½ feet in girth. In

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[1] *Indian Antiquary*, xvii, 1888, p. 33. The 'well-known fact' is not a fact at all.

[2] JASB, LIX, 1900, p. 126.

[3] The reproductions are of the same size as actual leaves.

[4] This is the general practice in Northern India. In the South the number is usually given on the obverse.

larger trees the paper bark of the bole is rough and lignified and is of no use as a writing material.[1]

11. Each layer of bark is white or pinky-white on the outer side, but is a reddish or yellowish buff on the inner side. The number of layers varies and I have counted 47 in a strip taken from an old tree. A marked feature of the bark is the existence of numerous lenticels (glands) from 1 to about 6 cm. in length and from about 1 to 3 mm. in depth. These lenticels are reddish-brown in colour[2] and of darker shade than the natural reverse of the lamina, and each of them is continued throughout the several laminæ. On the natural obverse they appear much more distinct by contrast with the lighter back-ground, and in the reproductions of manuscripts this contrast sometimes appears to be emphasised. On the bole of the tree the lenticels are *horizontal* (*i.e.*, they are always at right angles to the axis of the bole or branch). On the older specimens they are slightly convex on the obverse and concave on the reverse. Traces of these lenticels can be seen in almost all the reproductions of our manuscript (see Fig. 1 and Plate XLI; but they show much clearer in some of the Bower manuscript plates). The lenticels are of importance from the point of view of the scribe because they are of different structure from the rest of the bark, and they sometimes break away (see folio 16). There is a sort of grain running parallel with the lenticels and the bark tears easily in that direction.

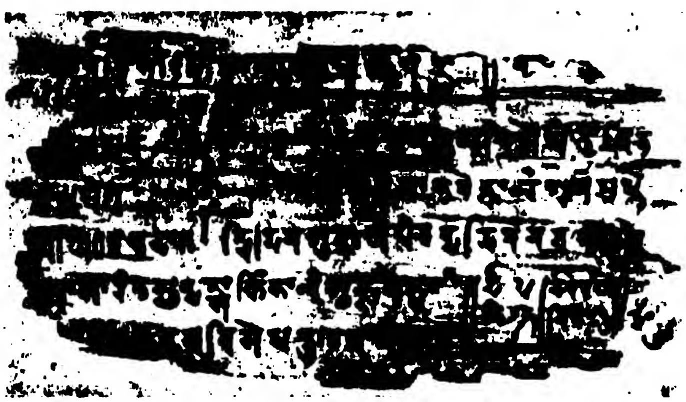


Fig. 1.

[1] *See* notes by Mr. E. Radcliffe in the *Indian Forester* (xxviii, 1902, pp. 25-27). I am much indebted to Lt.-Col. A. Gage, Director of the Botanical Survey of India, Mr. P. H. Clutterbuck, Inspector-General of Forests, and Mr. B. Coventry, Conservator of Forests, Kashmir, for information very kindly supplied by them.

The following particulars of specimen strips of birch-bark taken from trees of various ages, and kindly sent to me by Mr. B. Coventry, are perhaps worthy of record.

Girth of tree.		Size of Strip. Length.*		Size of Strip. Breadth.*		Number of laminæ.
Ft.	Inches.	Ft.	Inches.	Ft.	Inches.	
..	6	2	...	...	6	20
1	...	2	10	...	10	19
2	...	3	4	1	6	29
3	...	4	7	2	...	Shows previous stripping.
3	...	3	7	2	2	19
4	...	2	10	2	6	27 Bottom of trunk woody.
5	...	3	6	4	...	33 ,, ,, ,, ,,
6	...	3	7	1	10	47 Taken from an upper branch.

* Here length means vertical length measured at right angles to the lenticels and breadth means the measurement *parallel* to the lenticels.

[2] On the outside of the tree they are sometimes black, and on very old trees they form woody excrescences.

12. The art of preparing the bark for writing upon appears to be lost, but Albīrūnī tells us that the strips were rubbed with (?) oil and polished. The manuscripts preserve no evidence of either of these processes. All that they tell us is something of the process of sub-division, and arrangement. Each leaf of the Bakhshālī manuscript appears to consist of half of the original thickness of the strip, *i.e.*, the original strip was divided into layers each of which consisted of some six laminæ. In the Bower manuscript the number of laminæ to a leaf varies from two to six while one leaf consists of at least twelve liminæ. The Kashmirian Arthava Veda exhibits rather more elaboration: the process of sub-division is carried to the extreme limits and each leaf consists of two single laminæ pasted together.

13. Possibly the original strip of birch-bark from which the leaves of the Bakhshālī manuscript were taken was roughly of the shape of the annexed diagram and was cut up into the oblongs indicated. If A, B, C, etc., represent the upper layer, and A^1, B^1, C^1, etc., the lower layer, then, according to the evidence of the leaves themselves, they were arranged for purposes of writing upon in the order A, A^1; B, B^1; C, C^1; etc., or A, A^1; D, D^1; etc.

A	B	C
D	E	F
G	H	I
J	K	L
M	N	O
P	Q	R

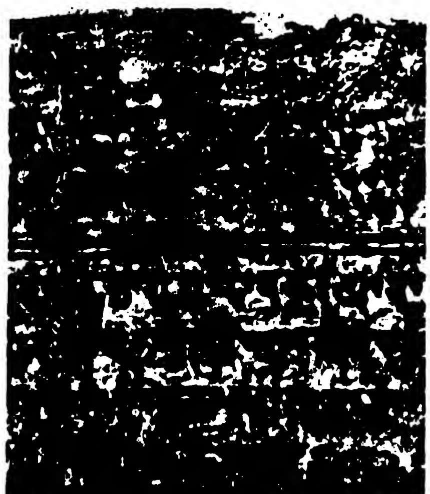


Fig. 2.

14. Birch-bark, even after preparation for writing upon, retains the natural marks of the wood showing the lenticels and occasionally knots. Such knots are like thumb marks in their individuality, so that, if a knot recurs, it can generally be identified. Returning to our diagram—it is obvious that if a knot occurs on, say, D its doublet will occur on D^1. In the Bower manuscript three knots that occur on folio II, 7 recur on II, 8; the knot on III, 8 recurs on III, 9; several knots on II, 23 recur on II, 24; one on II, 31 recurs on II, 32; and so on; and in no case does any particular knot occur on more than two leaves. In the Bakhshālī manuscript the order of the leaves was uncertain but it was noticed that folios 12 and 13 had a common knot, and so had folios 32 and 36, folios 44 and 49, folios 51 and 52, folios

53 and 66. If our scheme of the arrangement of leaves also holds good here, then each of these pairs of leaves should consist of consecutive leaves, and the final order, based on other considerations, places them thus—

Bodleian order.	Final order.
12	G3
13	G4
32	M3
36	M4
44	M10
49	M9
51	A8, 9
52	A10
53	E2
66	E1

15. It should be noted also that where the reproductions are of double leaves stuck together the two sides should have no common natural markings (see, for example, folio 65); and, conversely, when the two sides have no natural marks in common there is ground for suspicion (*e.g.*, folios 46 and 3). In this matter too much reliance should not be placed on the reproductions.

Format.

16. Of the very few early birch-bark manuscripts the following are perhaps the best known :—

(*a*) The Kharoshṭhī Dhammapada from Khotan. Size 8 by 36[1] inches, (Ratio 0·2)[1]. Period 2nd century A.D.

(*b*) The Bower manuscript from Khotgarh. Size 9 by 2 to 11½ by 2½ inches, (Ratio 4·5). Period 5th century A.D.

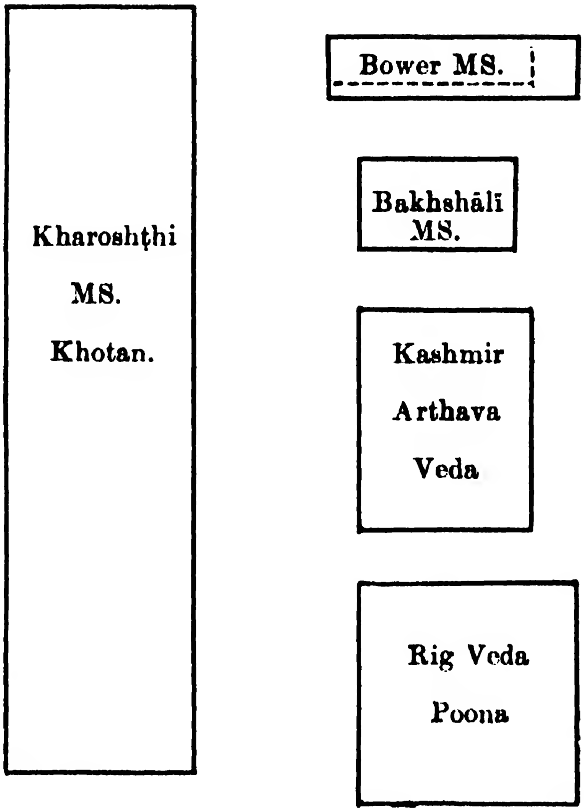
(*c*) The Bakhshālī manuscript (Oxford). Size 7 by 4 inches, (Ratio 1·7). Period ?

(*d*) The Kashmirian Arthava Veda. Size 9·8 by 7·8 inches, (Ratio 1·25). Period 15th century A.D.

(*e*) The Deccan College Rig Veda (Poona). Size 10¼ by 9½ inches, (Ratio 1·08). Period ?

[1] The sizes here given are, of course, only approximations to the average sizes. See (*a*) *Journal Asiatique*, 1898, pp. 192 ff. (*b*) *The Bower Manuscript*, by A. F. R. Hoernle. (*d*) *The Kashmirian Arthava Veda*, Bloomfield & Garbe. (*e*) *Cat. MSS. of the Deccan College, Poona*, Vol. I, p. 1.

The shapes and relative sizes of these manuscripts are represented in the accompanying diagram—



17. The format of a book is generally determined by several semi-independent factors, *e.g.*, material, economy, fashion, convenience, contents. Our first example of a birch-bark book, the Khotan manuscript, seems to have been shaped in imitation of the western papyrus roll[1]; the second example, the Bower manuscript, is probably in imitation of the palm leaf manuscripts of India; while the later examples are in imitation of paper books. The Bakhshālī manuscript appears to stand in an intermediate position and there is, of course, a possibility that its format may indicate its age within certain limits.

18. In all birch-bark manuscripts the writing is parallel to the lenticels, which on the bole of the tree are horizontal; and the reason for this direction of the writing is that the bark tends to split in the same direction. In the Bower manuscript there are one or two exceptional examples of writing inclined to the lenticels (Plates I and II) but the angle of inclination is small. It should also be borne in mind that the length of the strip of bark (parallel to the writing) is limited to the circumference of the tree, and the depth of the strip is also practically limited,[2] and that the format was, at least, partly determined by the necessity for dividing these strips economically.

19. But fashion was probably a more powerful factor than material in the determination of the format of birch-bark books, and it is possible that the method of preparing papyrus for writing upon to some extent influenced those who prepared birch-bark for the same purpose. The stem of the papyrus plant was cut into

[1] Numbers of such rolls were discovered by Sir M. A. Stein in Central Asia.

[2] See footnote on page 5.

longitudinal strips which were laid on a board side by side to the required width, and across this layer another of shorter strips was laid at right angles. After soaking, the combined sheet was hammered and dried and polished with ivory or a smooth shell. Some twenty such sheets were pasted together to form a roll. Later it was the practice to back one sheet with a second in order to obtain a more suitable paper. The depth of the rolls varied from 4 to 12 inches. The later limit for the use of papyrus as a writing material was about the 9th century A.D.

20. The palm leaf manuscripts of India are made from the leaves of *Corypha umbraculifera* or *Borassus flabellifer*. The former is indigenous in south India but the latter was probably introduced from Africa. The leaves of both of these trees are long and tapering, with central ribs. All the earlier palm leaf manuscripts are made from the leaves of Corypha. The following are examples :—

Reference.	Locality.	Length.	Depth.	Ratio.	Date.
Horiuzi	W. India . .	11 in.	2 in.	5·5	? 520 A.D.
Bendall Add. 1049 . .	„ „ . .	16 „	2 „	8·0	859 „
„ „ 1683 . .	Nepal . . .	21 „	2 „	10·5	1039 „
„ „ 1688 . .	Bihar . . .	22 „	2 „	11·0	1054 „
Kielhorn No. 42 . . .	W. India . .	18 „	2½ „	5·8	1123 „
Peterson No. 220 . . .		33½ „	2¼ „	15·0	1205 „

From measurements of a large number of Corypha manuscripts it appears that the most usual depth was about two inches and that the lengths varied from about 10 to 34 inches. The chief mode was from 10—16 inches but there was a second mode from 30—34 inches. It appears that the whole length of the prepared leaf was occasionally used, but that more often it was cut into thirds, or sometimes into halves.

About the middle of the fifteenth century the use of palm leaf as a writing material entirely ceased in western India, but in eastern India it continued to be used side by side with paper until much later.

21. The art of paper making appears to have been practised by the Chinese at a very early date. From the Chinese the Muslims learnt the process in the eighth century and they introduced it into Europe and also India about the twelfth century. Among the earliest Muslim paper manuscripts now preserved, one was written in A.D. 866, others in 974, 980, 990, etc. The earliest Indian paper manuscripts known were written in A.D. 1231 and 1343. The earlier one measures 6 by 4 inches (ratio 1·5) and the other 13½ by 5 inches (ratio 2·7).

22. Another very common writing material in early India was copper. Bühler states that the size of the copper plate used was largely determined by the common writing material of the locality, *e.g.*, the palm leaf, birch-bark, etc. This statement, however, is not fully supported by actual evidence, for the more usual ratios of lengths to depths of copper plates vary from about 1·4 to 2·6 as against 5 to 15 for palm leaf manuscripts.[1]

[1] There are rare examples of copper plates of ratio over 4 (*e.g.*, two Taxila plates of ratio 4·7).

23. Before making definite deductions from the formats of our birch-bark manuscripts, a great deal more investigation is obviously required, and in particular the formats of the western tablets and codices, the early Chinese and Muslim paper books should be studied. Until the introduction of paper into India there does not seem to have been much consistency in the format of birch-bark manuscripts. The Khotan manuscript may have been in imitation of the papyrus rolls of the west. The width is about 8 inches (20 cm.), the length is unknown, but the total length is about 4 feet (1 m. 23) and the longest piece of the Paris portion is about 24 inches (61 cm.). The scapus was made by joining pieces of unknown depth together[1] and at each side a fine cord was threaded through the bark about one cm. from each edge. Most probably it was never intended that the manuscript should be rolled up : possibly it was to be hung on a wall. The writing is parallel to the short side, and in this matter differs entirely from the papyri, on which the writing is generally parallel to the length of the roll, and in columns of from two to three and a half inches wide. The writing is on one side only of most of the fragments of the Khotan manuscript, but on one long piece (C) both sides are written upon. This piece (C) consists of several layers of bark (it is, of course, impossible to count these layers without examining the original). The lenticels are identical on the two sides and the *verso* side is the lighter side of the bark. The manuscript appears to have been folded up to the size of 20 by 5 cm., but obviously this was not the original intention, for the folds are independent of the position of the lines of writing, and folding birch-bark manuscripts was never a wise procedure.

That the Bower manuscript format was determined by the palm leaf *pothī* is probably true, although it should be noted that 9×2 inches is a very uncommon size for an early palm leaf manuscript. (See Dr. Hoernle's lists in the JASB, 1900, pp. 99ff.)

24. Regarding the format of the Bakhshālī manuscript (ratio 1·6) as a criterion of age, I can come to no positive conclusion from the evidence before me. Further investigation is required and might be profitably undertaken if time allowed. Dr. Hoernle, however, writes as follows :—

> "It is noteworthy that the two oldest (Indian paper) manuscripts known to us point to their having been made in imitation of such a birch-bark prototype as the Bakhshālī manuscript."

It is not easy to accept this argument, for it would be quite as reasonable to conclude that the Bakhshālī format was determined by the paper manuscript formats, and that it is of later date than the introduction into India of paper as a writing material; and this would place the Bakhshālī manuscript about the twelfth century of our era at the earliest. However I only give this hypothesis as a set-off to Dr. Hoernle's unjustifiable deduction.

The script.

25. The Bakhshālī text is written in the Śāradā script, which flourished on the north-west borders of India from about the ninth century until within recent times. Its distribution in space is fairly definitely limited to a comparatively small area lying between longitudes 72 and 78 east of Greenwich and latitudes 32 and 36 north. Dr. Vogel distinguishes between Śāradā proper, of which the latest examples are of the early thirteenth century, and modern Śāradā.

[1] But I can see no evidence of this in the illustrations given in the *Journal Asiatique* (1898, p. 308).

The writing of the Bakhshālī manuscript is of the earlier period and is generally very good writing indeed. It was written by at least two scribes. In the table on p. 97 the styles of writing are indicated by the letters α and β of which β pertains wholly to the "M" section. Style α is divided into four sub-sections which possibly belong to the work of four separate scribes, although it is not easy to point out any fundamental differences between these styles. Folio 65 possibly exhibits the writings of two separate scribes on the two sides, which do not belong to the same original leaf. Compare also folios 29 and 17.

Possible points of differentiation between the sub-divisions α_1, α_2, α_3 and α_4 are the length of medial ī, the length of the final up-stroke of initial *u*, the length of the *virāma* mark, the occurrence of the sickle shaped ī (folios 1, 35, 52, 60), of the "clubbed" *ai* and *e* (folios 16 and 60), of the "curved" medial *e* (folios 3, 18, 5), and the different methods of forming medial *e*, *ai* and *o*.

The style (β) of section "M" is distinguished by its boldness (contrast plates II and XXV), by "tails" or flourishes (including very long *virāma* marks), the methods of writing medial *ai* and *o*, the looped six, etc., etc.

In Part II the script is examined in detail and further peculiarities of the different sections are given, *e.g.*, section "M" has no examples of *ṅ* and contains practically all the examples of *ŏ* Sections L (α_4) and G (α_3) show marked differences in the methods of writing medial *e*, while section F is, in this matter, very much like section L. Section C (α_2) has no example of either the *jihvāmūlīya* or *upadhmānīya*, and so on; but it must be borne in mind that these statistics are only of value in the mass.

The language.

26. The language of the text may be described as an irregular Sanskrit. Nearly all the words used are Sanskrit, and the rules of Sanskrit grammar and prosody are followed with some laxity. The peculiarities of spelling, sandhi, grammar, etc., that occur in the text are exceedingly common in the inscriptions of the eleventh and twelfth centuries found in the north-west of India. Dr. Hoernle, however, implies that the language is much older. He states that the text "is written in the so-called Gāthā dialect, or in that literary form of the North Western Prākṛit which preceded the employment, in secular composition, of the classical Sanskrit." He also states that this dialect "appears to have been in general use, in North Western India, for literary purposes, till about the end of the 3rd century A.D."[1]

The subject of language is discussed to some extent in Chapter viii and will be more fully dealt with in Part III of this work.

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[1] *Ind. Ant.* xvii, 1888, pp. 37 & 38.

CHAPTER III.

ORDER.

27. The Bodleian Library order of the folios, which was definitely fixed by circumstances not within the control of the editor, is necessarily followed in this volume in the arrangement of the facsimiles and the first transliteration of the text. But a more satisfactory arrangement was considered desirable, and a very considerable amount of labour has been involved in the attempt to achieve an approximately correct order.

The leaves were disarranged to some extent before they reached Dr. Hoernle, but unfortunately he did not leave any proper record of the order in which the leaves reached him.[1]

28. A detailed analysis of all the leaves showed that Dr. Hoernle's rearrangement was faulty,[2] that the "find order", as far as known, was better, but not altogether reliable. Further rearrangement was necessary and had to be attempted; but leaves were missing, some were in fragments placed anyhow, not a single leaf was complete, and the connecting portions of the text were all wanting.

29. Dr. Hoernle attempted to arrange the leaves on the basis of the numbered *sūtras;* but the numbered *sūtras* were too few and too unevenly distributed to serve this purpose, and in some ways they were even misleading. The chief criterion of order is, of course, the nature of the contents of the leaves, but I have seized upon any means available that has offered any help towards the solution of the problem of order. Had all the leaves been extant, even in fragments, the problem could have been completely solved; but some leaves are completely missing and many fragments have disappeared altogether, so that the problem is only partially soluble. I have already explained how the knots in the birch-bark were of assistance, and besides this natural aid there was the accidental one of the effects of the method of storage. Possibly for some hundreds of years the bundle of leaves was subject to a certain amount of pressure, and was exposed, particularly at the edges, to chemical and other disintegrating actions. Some of the leaves stuck together, the edges of all became frayed and certain leaves became so frail as to break up into scraps when handled. On the principle that contiguous leaves would be affected approximately to the same extent, we might, if no disintegrating effects had taken place since the find, rebuild in layers the original bundle. But we know that further disintegration has taken place, and is still not altogether eliminated; nevertheless, similarity in size and shape and mechanical makings were distinctly helpful in rearranging the leaves.

30. The script was helpful in a general way: indeed it led me to differentiate the "M" section from the rest of the text. The language employed was also helpful, particularly with reference to the use of special technical terms; while such examples as the following were noted after some order had been restored:—*ghna* "multiplied by" occurs only on folios 7, 45, 46, 57 and 65, all of which belong to the same section (C); *pratyaya, yuta* and *rūpa* occur very often but never in the "M" section, etc., etc.

31. The chief criterion—that of the subject matter—could not always be utilised, owing to the absence of all the actual connecting portions and to the

[1] In the following notes by 'find order' I mean the order in which the leaves reached Dr. Hoernle. This order is not completely known but Dr. Hoernle marked the order of certain of the leaves.

[2] It would perhaps be more correct to say that it was incomplete. He placed leaves 10 to 18 and 60 to 63 in proper sequence.

numerous larger gaps, but occasionally it was definitely decisive : *e.g.*, one example goes from folio 65 to folio 66 and then to 64, another from 66 to 67, and another from 29 to 27, etc. We also find the same topic spread over a number of leaves, and here it was often possible to reconstruct a logical sequence that must be in some correspondence with the original.

Finally the numbered *sūtras* were useful as a check and also as indicating to some extent the relative positions of certain sections.

32. The problem of rearrangement, however, had not only to do with the order of the leaves but with the order of the pages and also the order of the fragments. In a number of cases the Bodleian arrangement places the proper first side of a leaf *in verso*, and the fragments are very often placed with wrong leaves. The illustration facing page 4 shows some of these fragments rearranged, and the table on page 14 enumerates some 13 leaves placed with the obverse *in verso*.

33. The order now given does not pretend to be the exact original order : it is merely a compromise. It brings together those portions of the remains of the text that deal with the same topics. It is not necessarily a logical order according to modern views : indeed, few mediæval mathematical works exhibit such a logical order. Neither is the order a final one, and probably it is not the best that could have been obtained ; but a working order had to be achieved even before the detailed examination of the manuscript could be completed, and now if the labour were not too great I should be tempted to revise the order once more. Further, if an attempt is made, as I trust it will be, to separate those leaves that are stuck together, then the question of order will possibly once more have to be considered.

34. The following tables show the various orders-

BODLEIAN ORDER.

(The facsimiles shown in plates ii to xlvii are in this order.)

Folio.	Revised order.	Folio.	Revised order.	Folio.	Revised order.	Folio.	Revised order.
1	A 11	21	E 4	41	J 3	61	L 2
2	A 12	22	E 5—F 1	42	M 13	62	L 3
3	A 13	23	F 2	43	M 12	63	L 4
4	B 4	24	F 3	44	M 11	64	C 6
5	C 1	25	F 4	45	C 8	65	C 4+J 2
6	C 2	26	F 5+?	46	C 9+D 1	66	E 1
7	B 3+C 3	27	A 10	47	M 7	67	D 6
8	B 1	28	D 7	48	M 8	68	D 4
9	B 2	29	A 6—7	49	M 9	69	D 3
10	G 1	30	J 1	50	M 14	70	D 2
11	G 2	31	D 5	51	A 6 - 7		
12	G 3	32	M 3	52	A 8		
13	G 4	33	M 2	53	E 2		
14	G 5	34	M 5	54	A 4		
15	G 6	35	A 5—6	55	M 10		
16	G 7+H 7	36	M 4	56	C 5		
17	H 2	37	M 6	57	C 7		
18	H 3	38	A 1—3	58	E 3		
19	Blank	39	A 1—3	59	K		
20	M 1	40	A 1—3	60	L 1		

REVISED ORDER, &c.

Revised order.	Folios.	Numbered Sûtras.	Find order.
A 1	$40^a+39^a+{}^{b'}+{}^{c}+38^b$		32
A 2	$39^b+40^c+38^c+40^b+39^d$		32
A 3	$40^d+39^c+38^b$		32
A 4	54		?
A 5	35^b		?
A 6	51^b+35^a		37
A 7	51^a		37
A 8	52		37
A 9	29		32
A 10	27 + 29		32
A 11	1	ix, x	33
A 12	2		34
A 13	3	xiii, xiv	49
B 1	8	?, ?	48
B 2	9 v.R.		43
B 3	7 v.		45
B 4	4	xv, xvi	44
C 1	5	xviii	47
C 2	6		46
C 3	7 R.		45
C 4	65 v.		?
C 5	56 v.R.		16
C 6	64		?
C 7	57 v.R.		?
C 8	45		10
C 9	46 R.		9

Revised order.	Folio.	Numbered Sûtras.	Find order.
D 1	46 v.		9
D 2	70		65
D 3	69		64
D 4	68		64
D 5	31		63
D 6	67 v.R.		62
D 7	28		60
E 1	66		58
E 2	53		?
E 3	58		?
E 4	21		55
E 5	22 R.		
F 1	22 v.	?, ?	54
F 2	23		52
F 3	24		?
F 4	25		51
F 5	26		50
G 1	10	xxiv	42
G 2	11		41
G 3	12		40
G 4	13		39
G 5	14		?
G 6	15	xxv	29
G 7	16 R.		30

Revised order.	Folio.	Numbered Sûtras.	Find order.
H 1	16 v.	?	30
H 2	17		28
H 3	18	? xxviii	27
J 1	30		32
J 2	65 R.	?	?
J 3	41		?
K	59	l	?
L 1	60	li, lii	?
L 2	61	liii	?
L 3	62	liv, lv	?
L 4	63	lvi, lvii	?
M 1	20		26
M 2	33		25
M 3	32		24
M 4	36		23
M 5	34		22
M 6	37		21
M 7	47 v.R.		20
M 8	48		19
M 9	49 v.R.		11
M 10	55		14
M 11	44 v.R.		12
M 12	43		13
M 13	42		18
M 14	50 v.R.		17

NOTE.—R. stands for *Recto*, v. for *Verso* and v.R. indicates that the facsimile should be reversed.

CHAPTER IV.

THE CONTENTS OF THE MANUSCRIPT.

35. The portions of the manuscript that have been preserved are wholly concerned with mathematics. Dr. Hoernle described the work in 1888 in the following words[1] :—

"The beginning and end of the manuscript being lost, both the name of the work and its author are unknown. The subject of the work, however, is arithmetic. It contains a great variety of problems relating to daily life. 'The following are examples:—' In a carriage, instead of 10 horses, there are yoked 5, the distance traversed by the former was one hundred, how much will the other horses be able to accomplish?' The following is more complicated:—'A certain person travels 5 *yojanas* on the first day, and 3 more on each succeeding day; another who travels 7 *yojanas* on each day, has a start of 5 days; in what time will they meet?' The following is still more complicated:—'Of 3 merchants, the first possesses 7 horses, the second 9 ponies, the third 10 camels; each of them gives away 3 animals to be equally distributed amongst themselves. The result is that the value of their respective properties becomes equal; how much was the value of each merchant's original property, and what was the value of each animal?' The method prescribed in the rules for the solution of these problems is extremely mechanical and reduces the labour of thinking to a minimum."

36. It is necessary to emphasise the fact that this early estimate of the value of the work is inadequate and misleading. One reason for this is, that, when the estimate was made, only a comparatively small portion of the contents of the manuscript had been understood, and that was by no means the most interesting portion of the work.

37. The following is a summary list of the contents of the work as far as its present state allows of such analysis :—

	Section.
Problems involving systems of linear equations	A
Indeterminate equations of the second degree	A & K
Arithmetical progressions	B & C
Quadratic equations	C
Approximate evaluations of square-roots	C
Complex series	F
Problems of the type $x(1-a_1)(1-a_2)\ldots(1-a_n)=p$	G
The computation of the fineness of gold	H
Problems on income and expenditure, and profit and loss	L, D & E
Miscellaneous problems	M
Mensuration.	

Such is a very rough outline of the work as it now stands. Perhaps the most interesting sections are C, A and M; and of these C is the most complete and was evidently treated as of considerable importance. Section A is also of special interest as it contains examples which may be described as of the *epanthem* type. Section M is of interest principally on account of the methods of expressing the numerous measures involved and also because of its literary and social references.

38. Although the work is arithmetical in form it would be misleading to describe it as a simple arithmetical text-book. No algebraical symbolism is employed, but the solutions are often given in such a general form as to imply the complete general solution, *i.e.*, the solutions, though arithmetical in form, are really generalised arithmetic, or algebra. First of all a particular rule is given, which is

[1] *Ind. Ant.* XVII, 1888, p. 33.

intended to apply to the particular set of examples that follows. These rules are often expressed in language that would be impossible to interpret without the light thrown upon them by the solutions. The examples are themselves sometimes trivial, but the solutions, often expressed with what at first glance appears to be meticulous care, often redeem the examples from their apparent triviality. Proofs or verifications are often given with some elaboration and on occasions are multiplied.

39. The work may be divided roughly into algebraic, arithmetical and geometrical sections; but the boundaries of these sections are not clear, and perhaps it would be more correct to classify the problems as (*a*) academic, (*b*) commercial, (*c*) miscellaneous.

Judging by the manuscript as it now stands, the problems involving geometrical notions were comparatively very few, and we can only guess at the meanings of the remaining fragments dealing with this branch of mathematics.

One pleasing feature is the small space occupied by commercial problems. There is only one problem on interest, and a rather unobtrusive section containing problems on profit and loss.

Those problems classed as academic are concerned with particular mathematical notions that in early mediæval times had a traditional value and interest, such as the *epanthema*, the *regula virginum*, certain indeterminate equations of the second degree, and certain sets of linear equations.

The miscellaneous problems include examples where the chief interest is rather in the illustrative material than in the mathematical notions involved; *e.g.*, there are problems concerned with the abduction of Sītā by Rāvana, the prowess of Haihaya, the constitution of an army, the Sun's chariot, the daily journey of the planet Saturn, gifts to Śiva, etc., etc.

40. Such, or similar features, are, however, common to many mediæval mathematical works, and we now turn to the consideration of the distinguishing features. Characteristics that completely differentiate one such mathematical work from another of the same period are generally hard to find and generally in order to so differentiate we have to sum up a number of minor characteristics. The Bakhshālī manuscript is, however, almost unique in at least two respects of some mathematical importance. The first of these is the employment of a special sign in the form of a cross—exactly like our *plus* sign but placed after the quantity it affects—to indicate a negative quantity. The possible connexion of this with the Diophantine sign and its value as a chronological test will be dealt with in due course. The second special characteristic consists of the set of methods for indicating the change-ratios of certain measures. This characteristic also will be dealt with in some detail in due course; and it must suffice for the present to point out that neither of these peculiarities is very helpful in placing the work.

41. If the work were actually of the period to which Dr. Hoernle assigned it, then by far the most remarkable feature would be the employment of the modern place-value arithmetical notation; but Dr. Hoernle's estimate of the age of the work was wrong and the occurrence of this notation is a common-place matter. It draws our attention, however, to the skill of our author in the manipulation of numbers. Large quantities are dealt with and one particular number contains 23 digits. These large numbers do not appear to be given for mere effect for they occur quite naturally as the result of a rigorous logic. Indeed the author seems generally to prefer simple numbers.

These large quantities again lead us to note other rather special characteristics of the work, namely the apparently over-elaborated exposition of the "workings" of the solutions (to which further reference will be made), to the preservation of the generality of the solutions throughout such workings, and to the consequent necessity of preserving the quantities that occur so that the penultimate statement shall involve the whole formula. That is, although every operation is arithmetical in form, the quantities involved are not "simplified" or "cancelled" without some special justification until the final result is achieved. The (unwritten) rule followed by the author was something like this: The integrity of the method of solution must be preserved, and so long as it is preserved the calculation may be simplified—otherwise not. Indeed the numerical quantities in these problems are treated almost like algebraic symbols.[1]

42. There is no actual algebraic notation employed but the unknown quantity appears to be indicated in certain examples by the usual symbol for "nought," which symbol, however, is never the subject of operation. Where it occurs the *regula falsi* is employed—probably because of the lack of an efficient symbolism. Neither is there any symbol of operation. The negative sign (+), already alluded to, is never used as such. Operation is generally indicated by some definite *ad hoc* term but sometimes by relative position. Fractions, for example, are indicated in the modern way but without any horizontal bar between the numbers, and sometimes division proper is so indicated.

43. If we considered the Bakhshālī text to be a work of pure Indian origin, then it would be also unique in another respect, namely in the rather extensive employment of the square root rule that may be expressed by $\sqrt{A^2+b} \sim A+b/2A$. For this rule, the early history of which is well known, was never used in the early Indian works to any extent, whereas the Bakhshālī text employs it for a comparatively large number of examples and applies the rule to second approximations in a very thorough manner.

On the same assumption as to an Indian origin the use of the sexagesimal notation in connexion with an approximation to a square-root of a non-astronomical quantity would also be unique, but it may be noted that there is only one such example preserved and this is given in a rather timid way, and the notion involved is not pursued.

44. Whether of a purely Indian origin or not, the work is Indian in form. It is written in a sort of Sanskrit and generally conforms to the Indian text-book fashion, but there are certain apparent omissions. Perhaps the most noteworthy feature of the classical Hindu texts is their treatment of indeterminate equations of the first degree, while their greatest achievement is the full solution of the so-called Pellian equation. A great part of the texts of Brahmagupta, Mahāvīra and Bhāskara are devoted to one or both of these topics, but there is no evidence of either in what remains of the Bakhshālī text; and this apparent omission is the more noticeable, inasmuch as there is evidence of considerable skill in the treatment of systems of linear equations and certain indeterminates of the second degree. Another omission to note is of a different character altogether. Every early Hindu work of this kind has a section relating to the "shadow of a gnomon," but in our text there is no evidence of such a section. We must not, however, pay too much attention to these apparent omissions. The possibility of entire sections of the manuscirpt being destroyed is not great, but negative evidence and a mutilated manuscript do not carry us very far.

[1] These remarks do not apply to the 'M' section.

Non-mathematical elements.

45. Many of the mathematical examples contain references to the affairs of gods and men and some of these references are of rather exceptional interest. The names of certain deities and semi-historical beings occur, and there are references to legends which may give some clue as to the origin of the work. Of interest also are the animals, crops, metals, etc., mentioned. Before indulging in conjectures as to the significance of these references it will be convenient to exhibit them in some sort of order.

46. Śiva. The name of Śiva occurs on a fragment (folio 50) that appears to exhibit a sort of colophon, in which the gift of calculation to the human race is attributed to the god. The actual name Śiva occurs nowhere else, but another name of the god—Śūlin—is given on folio 34, where the example refers to certain offerings made to him. On folio 44, in a similar example, part of the expenditure is in offerings for Śū°, and this, it is conjectured, stands for Śūlin. The term Devī occurs (folio 49) and probably indicates Śiva's consort.

The significance of these references (which all occur in section "M") is emphasised by Dr. Hoernle's remarkable claim that the work is Buddhist or Jain.[1] This appears to have been mere conjecture, for we find nothing whatever in the text that conflicts with the Śaivism indicated. Neither is the authorship of the work by a follower of Śiva in any way surprising, for, it may be noted, Kashmir Śaivism began to re-flourish about the tenth century A.D.,[2] and Kalhaṇa was a Śaivite.

Vāsudeva is a name applied to Kṛishṇa. The phrase that occurs (folio 44) is *vāsudevasya chārchanet.*

Suras and Asuras. Suras are generally classed as minor deities. In our text (folio 33) they are said to dwell on Sumeru. In the same context Asuras, demons or enemies of the gods, are contrasted with the Suras.

Rākshakas are classed as evil spirits but are not generally very clearly defined. The term occurs on folio 65, recto (part of a double leaf, and perhaps not correctly placed in section J). The problem, which is concerned with *jīva-loka,* is not understood.

Siddhas and Vidyādharas. These two terms occur together on folio 37, where the chariot of the Sun is said to be guided by the god Mahoraga among the Siddhas and Vidyādharas. The Siddhas are semi-divine beings of great holiness, who dwell in the region of the sky between the earth and the sun, while the Vidyādharas are inferior deities inhabiting the same region.

Mahoraga is Śesha or any other great serpent. The connexion between serpents and the Sun is supposed to be somewhat intimate in certain mythologies but I have not found the source of the present reference.

47. Sītā. The name Sītā is not actually preserved in our text, but on folio 32 is an example based upon her abduction by Rāvaṇa. The tale is that Sītā, the wife of Rāma, was carried off through the air by Rāvaṇa, and that, in order to attract the attention of her helpers, she tied up some jewellery in a garment and

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[1] *Ind. Ant.* XVII, 1888, p. 38.

[2] *See* R. G. Bhandarkar, Vaishnavism, Saivism, etc., pp. 129-131.

dropped it to earth, and this circumstance forms the substance of the problem given in our text.

Rāvana is mentioned on folio 32. See Sītā.

Pārtha, irritated in a fight, shot a quiver of arrows to slay Karṇa. With exact relationship between the two terms is not certain, but the problem in which they occur relates to the destruction of an army, and the question is how many arrows were used? A very similar problem is given twice over by Bhāskara (*Līl.* 67; *Vīj. Gaṇ.* 133) as follows :—

> Pārtha, irritated in a fight, shot a quiver of arrows to slay Karṇa. With half of his arrows he parried those of his antagonist; with four times the square-root of the quiver full he killed his horses; with six arrows he slew Śalya, with three he demolished the umbrella, and with one he cut off the head of the foe. How many were the arrows which Arjuna let fly?

In Bhāskara's example Pārtha = Arjuna, the Pāṇḍava prince, but there is no connexion between him and the Haihayas. The Haihaya Arjuna was a great king but an altogether different individual. There is, however, little doubt that the examples in the two texts were connected, either directly or by some common source.[1]

Arjuna is mentioned on folio 34 in an isolated phrase......*ārjunena griddhra* ......Arjuna was connected with the Nāgas and some contest with a form of Garuḍa would not be out of place.

48. **Yudhishṭhira.** On folio 37 is the isolated fragment *rāja yudhisṭhira nāma Pāṇḍu vaṁśa*...........which implies some familiarity with the great epic of India.

Satrudama. A similar fragment occurs on folio 47 *kaschid rāja-kumāra Satrudama*.........If this prince could be identified the reference to him might prove a valuable clue. Compare *Śatrughna* which has the same meaning.

Sundarī. On folio 34 Sundarī, "the beautiful one," is asked to solve the problem, that is, she is addressed in exactly the same way as Līlāvatī is in Bhāskara's well-known work. Here is either imitation or a common tradition.

Chhajaka. The text (*i.e.*, section "M") is written by a Brahman, the son of Chhajaka (folio 50). Possibly this name is the same as Sajjaka, which occurs several times in the *Rājataraṅgiṇī*. Śajjaka was superintendent of the Seda office in Kalhaṇa's time (XIIth century), but there is no real justification for connecting this individual with the author of our text.

49. Besides these names of individuals there are references to various classes of men—kings, princes, priests, learned men, bankers, servants, soldiers, tax-collectors. Merchants not only appear in money transactions but, in one case, with Brahmans and others, as recipients of propitiatory gifts. Pandits and learned men are spoken of as earning wages, and two "Rājputs" are described as servants of a king. The problems are often addressed to an individual or individuals variously described as "the best of calculators" (*gaṇakottama*), "wise" or "learned" man (*budha, prājña, paṇḍita, dharmajñaya*, etc.), or "friend."

[1] The mathematical prototype is common in the Greek Anthology.

50. The collection of animals is limited to elephants, horses, camels, cows, buffaloes, snakes, a worm and a vulture. There is little of significance here, but the occurrence of camels points to the north-west of India. There are, however, two classes of horses mentioned (*aśva*, *haya*); for example, one problem deals with certain numbers of *aśva*, *haya*, and *ushtra* (camel). Dr. Hoernle translated *haya* by "yak," which would be interesting if it were justifiable; but on one occasion the two terms seem to be used synonymously (fol. 8) and the "horse" of an army are designated *haya* and *turaga* (fol. 47).

Of food stuffs and other commodities we have wheat, barley, rice and saffron (this combination being of some significance[1]); gold, iron, salt, molasses, and (?) lapis-lazuli (*ambhaloha*).

51. The mention of a boat sailing against adverse winds is not necessarily "local colouring" but we must assume that it was, at least, comprehensible. Chariots (*ratha*) are mentioned three times twice it is the chariot of the Sun, and on the third occasion the chariots form one section of an army, which is said to consist of chariots (*ratha*), elephants (*gaja*), foot soldiers (*nara*), and "horse" (*haya*) in the ratios 1 : 1 : 5 : 3. Certain "divisions" of an army are also mentioned, namely *chamū*, *pritanā*, *anīkinī* and *akshauhiṇī* (fol. 47).

52. The references to religious matters are few but are significant. We have already mentioned offerings to Śiva, Devī and Vāsudeva; Brahmans also appear to have been fed; other gifts are "for the sake of reverence" (*pūjārtha*), and for hopes for "the future world" (*paraloka*), etc. The "supreme spirit" (*paramātmana*) and (?) "creation" (*srishti*) are also referred to.

Is the work homogeneous?

53. In my first examination of the manuscript I noticed that the writing was not uniform, and that, in particular, certain leaves differentiated themselves from the rest by a bolder and, on the whole, a better style of writing; and I distinguished this set of leaves as the "M" section. This early differentiation was a most useful one for it marked not only a difference in style of writing but also one of matter. Indeed this "M" section proved to have so many peculiarities that the idea that it was possibly a separate work could not be ignored. But the rest of the manuscript is by no means uniform in style of writing or anything else, and I am not convinced that the "M" section is the work of a separate author, although I rather suspect that it is. It is, however, pretty certain that it was the work of a separate scribe; but, as there are slight indications that the other portions of the manuscript were *dictated*, this does not affect the question of authorship conclusively. I cannot point to any definite *evidence* of dictation that would bear examination: it is rather an impression received.

The peculiarities of the "M" section, although they may not prove heterogeneity of workmanship, call for some special mention and are here summarised.

I. The script.

(*a*) The writing is bolder and on the whole, more uniform than that of the rest of the manuscript.

[1] "Saffron (*kuṅkuma*) has to the present day remained a famous product of Kashmir." STEIN, *Rājataraṅgiṇī* II. 428. We ought also to mention birch-bark, although there is no reference to it in the text.

(*b*) Flourishes or extensions of the bottom end-strokes are common. These flourishes occur particularly at the end of ligatures, but also in the cases of the numeral figures "5," "7" and "9," and even in the case of the stop bars, and occasionally they even occur at the ends of the frame-works of the "cells" (*e.g.*, see fol. 47, etc.).

(*c*) The numerical symbols of section "M" are shown in Table IV (7) line 1, where the looped "6" should be noted. This is a useful but not an infallible criterion.

(*d*) The following table relating to the formation of the medial vowels *a*, *i* and *o* is taken from Part II.

	Medial ai		Medial o[1]		
	ài	*·ài*	*ŏ*	*ò·*	*·o·*
'M' section	0%	100%	32%	32%	36%
Whole manuscript	24%	76%	75%	15%	10%

Here the indication appears to be very definite indeed, but it must be borne in mind that such criteria only apply in the mass, that the total number of *·ài* examples is only 19, and so on.[1] For a fuller discussion of these interesting statistics see Part II.

II. It is curious that all the mythological and semi-historical references (see Sections 46—48) occur in the "M" section. Indeed this section is peculiarly Hindu in contrast with the remainder of the manuscript.

III. The mathematical contents of the "M" section may be described as miscellaneous problems, which are generally solved by simple "rule of three"; but a special feature is the occurrence of numerous "measures" and a special method of exhibiting their change-ratios. But, of course, these points in no way indicate a separate work—rather otherwise—for if section "M" were an entirely different work we might expect some duplication, and there is none here.

IV. The method of exposition is somewhat different. The example is followed by a statement and the answer is then given, generally, without any detailed working, and generally there is no "proof" or verification. There are, however, exceptions.

V. There are differences in language. Certain technical terms that are extremely common in the rest of the manuscript do not occur, *e.g.*, *pratyaya*, *yuta*.

[1] The symbols used here are merely mnemonic. *See* page 95.

CHAPTER V.

Exposition and Method.

54. The text consists of rules (*sūtras*) and examples. There is no explanation whatever of the processes by which the rules were obtained, that is, there is no mathematical theory at all. In this the work follows the usual Indian fashion, as exhibited in all early texts. But there is a good deal of mathematical theory implied, and the rules and examples are often set forth in such a way as to convey the principles followed quite clearly to the student.

55. The rules (*sūtras*) are written in verse and are generally numbered; and often in the solutions of the examples phrases from the *sūtras* are quoted. When Dr. Hoernle tried to re-arrange the leaves of the manuscript, he took the numbered *sūtras* as the basis of his order, but they were too few in number to lead to a satisfactory result. These *sūtras* do not represent, as might be expected, the most valuable part of the text. They are usually of particular application rather than general and are often very obscurely expressed.

56. The examples given are generally formally stated in full—without the use of notation or abbreviation of any kind; and in most cases they are stated in verse. They are introduced by the term *udā*, an abbreviation for *udāharanaṁ* "an example." After the question sometimes comes a formal statement with numerical symbols and abbreviations, often arranged in cells. Then comes the solution or working (*karaṇa*), and here, sometimes, fragments of the *sūtras* are quoted. Finally come demonstrations—often more of the nature of verification than proof. Generally these demonstrations, by the aid of the answer found to the question, rediscover one of the original elements of the problem; and sometimes several such demonstrations are attached to an individual problem, but sometimes the variation is merely a matter of the form of statement.

The full scheme of exposition is therefore—

Sūtram or rule.
Udāharanaṁ or "example": indicated by *udā°*.
Sthāpanam or "statement."
Karaṇam or "solution."
Pratyayam or "verification."

The end of each *sūtra* is marked after the last example by the device and the number of the *sūtra* is also given at the end.

57. The method of grouping sets of figures is of interest, and shows features in common with mediæval Sanskrit mathematical manuscripts, where also it is the practice to place groups of numbers in cells. In our manuscript, however, this fashion is rather more elaborated than in any Sanskrit manuscript I have examined. The mathematical possibilities of this scheme do not appear to have been realised and the student must always be careful to interpret any group of figures from the context, and not from any similarity with other groupings. However there is a certain amount of consistency in the arrangements, as the examples exhibited below will show. The real purpose of the arrangements appears to be to prevent confusion by demarcating the numerical figures from the text itself. The text is often written almost independently of the figure groups, and a word may be arbitrarily divided by the cell arrangement, which may also cut into several lines of the text not necessarily connected with it. The economic necessity of utilising

the whole of the writing surface of the birch-bark available seems to have been the determining factor. The text itself should be consulted but the following examples may be helpful :—

(*a*) Integral numbers occasionally occur without any marking off by lines or cells, but often

(*b*) each integral number has a cell to itself, *e.g.*

| 1 | | 42 | | 39 |

(*c*) Sometimes an integer is marked off by two vertical bars : thus | 14 | and invariably a series of integers is thus demarcated, *e.g.*

| 20 | 40 | 60 | 80 | evaṁ 200 ||

(*d*) Fractions and groups of fractions are placed in cells or groups of cells, *e.g.*

(i) | $\begin{matrix}132\\33\end{matrix}$ | (ii) | $\begin{matrix}6055040625\\3227520000\end{matrix}$ | (iii) | $\begin{matrix}1\\4\end{matrix}$ $\begin{matrix}1\\3\end{matrix}$ | $\begin{matrix}1\\6\end{matrix}$ $\begin{matrix}1\\12\end{matrix}$ |

(iv) | 1
1
3
1
4
1
5 | (v) 40
1
$\begin{matrix}1\\3\end{matrix}+$
$\begin{matrix}1\\4\end{matrix}+$
$\begin{matrix}1\\5\end{matrix}+$ | (vi) | 21
20
21 | (vii) | 7
12
12 |

(*e*) Complete sets of operations are sometimes marked off in a similar manner. For example, (*d*) (iii) means $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}$; (*d*) (iv) means $(1+\frac{1}{3})$ $(1+\frac{1}{4})$ $(1+\frac{1}{6})$; (*d*) (v) 40 $(1-\frac{1}{3})$ $(1-\frac{1}{4})$ $(1-\frac{1}{5})$.

(*f*) Series of operations may be connected together by cell arrangements, for example

2 dī° 1 2	1 di° 1 2	100000 dī° 947	phalaṁ di° 60000 947
3 dī° 1 2	1 di° 1 3	157500 dī° 947	phalaṁ di° 60000 947
4 dī° 1 2	1 di° 1 4	216000 dī° 947	phalaṁ di° 60000 947

which means

$2\frac{1}{2}$ dīnāras : $1\frac{1}{2}$ days : : $\frac{100,000}{947}$ dīnāras : $\frac{60,000}{947}$ days

$3\frac{1}{2}$,, : $1\frac{1}{3}$,, : : $\frac{157,500}{947}$,, : $\frac{60,000}{947}$,,

$4\frac{1}{2}$,, : $1\frac{1}{4}$,, : : $\frac{216,000}{947}$,, : $\frac{60,000}{947}$,,

(*g*) The data of the problem and the solution may be indicated in one combined statement, *e.g.*

si°	1	16	4	a°	chhe°	2187*		phalaṁ	śarā	2624400
	1	1	1			1				
		1								
		4								
		1								
		3								

This is a statement of proportion where the second term means 16 $(1+\frac{1}{4})$ $(1+\frac{1}{2})$ and the number marked with an asterisk is a change-ratio.

(*h*)	0	2	1	3	3	12	4	dṛi°	300
	1	1	1	1	1	1	1		1

may be roughly expressed **by means of $x + 2x + 3 \times 3x + 12 \times 4x = 300$**

58. The use in our text of the sexagesimal notation in the form in which it occurs is of rather special interest, for there is, as far as I know, no other example of the kind in any of the classical Sanskrit works. The Hindus, from Āryabhaṭa onwards, were well aware of the advantages of the sexagesimal notation for astronomical purposes, but they never used it for arithmetical purposes.

Apparently there is only one purely arithmetical example of the use in the text and this example occurs, in connexion with a problem in arithmetical progression, on folio 6, verso, and 7, recto, where the fraction 178/29 is expressed as $6 + 8' + 16'' + 33''' + 6''''\,\frac{6}{29}$. This sexagesimal fraction is actually written thus—

6	
8	
60	
16	cha°
60	
33	li°
60	
6	vi°
60	
śe°	6
	29

The upper three figures are missing in the manuscript but the restoration is certain. Of the abbreviations *li°* stands for *liptā* (Gk. *lepté*) which in Sanskrit works ordinarily means a minute of arc, or the sixtieth part of a degree; *vi°* stands for *viliptā*, ordinarily a second of arc[1]: while *śe°* stands for *śesham* or "remainder."

The purely arithmetical (and perfectly legitimate) use of the notation here points to extra-India influence, for, although such a use is unknown in Sanskrit works, it was extremely common in mediæval Muslim works.

[1] On folio 37, verso: *liptā* and *viliptā* are used 'astronomically.'

It will be noticed that the term *liptā* here applies to "third parts" instead of "first parts," and *viliptā* to "fourth parts." The abbreviation *cha*° has not yet been traced to its origin.

59. The modern place-value arithmetical notation is employed throughout the text and there is not the slightest indication that, to the author, it was a new or strange invention. There is nothing in the slightest degree remarkable in the employment of this notation in the text, neither is there in the text the slightest indication of any other evidence that makes the employment of this notation in any way particularly noteworthy; but its occurrence in the text has been given an artificial importance by the arguments of Dr. Hoernle and Dr. Bühler about the age of the work. This topic will be discussed in some detail in the chapter dealing with the age of the manuscript and the age of the work.

60. As already indicated algebraic symbols are not generally employed, but a symbol for the unknown quantity is. This symbol is the arithmetical symbol • for "nought" or "zero," and on several occasions it is referred to by the term *śūnya* "empty," "a cypher," and in some places by *śūnya sthāna* or "empty place."[1]

The symbol occurs some twenty times but only in sections B, C, F, G, H, K. Its employment is illustrated in the following examples :—

(i)

ā°	1	u°	1	pa°	•	labdhaṁ	10
	1		1		1		1

This is a "statement" of an arithmetical progression where the first term is 1, the common difference is 1, the number of terms is unknown, and the "quotient" is 10 (where $10x$ = the sum of the series). Here the symbol • simply indicates that the number of terms (*pada*) is unknown, *i.e.*, that the place in the statement is empty. The symbol does not enter into any operation here or elsewhere. The giving it a denominator of unity is curious and really indicates that it is an integral number.

(ii)

ā°	5	u°	6	pa°	•	dha°	•
	1		1		1		1
ā°	10	u°	3	pa°	•	dha°	•
	1		1		1		1

Here are two equivalent arithmetical progressions in which the numbers of terms are equal and the sums also are the same in both cases, but both are unknown.

(iii)

•	1	1	1	1	bhā°	śe°	16
1	1	1	1	1			1
	3	3	3	3			

which means $x = 16/(1-\frac{1}{3})\ (1-\frac{1}{3})\ (1-\frac{1}{3})\ (1-\frac{1}{3})$ and since this is a deduction from a problem it is a distinct step towards a proper algebraic symbolism.

(iv)

•	2	3	4	dṛishya	200
1	1	1	1		1

which may be represented by $x+2x+3x+4x=200$. But the method of solution is by the *regula falsi*. Any number is put in the place of the • and the sum of the

[1] In Hindu works proper the dot is the symbol for negative quantity, and the zero symbol is a small circle.

series is obtained. Here the given sum multiplied by the assumed value and divided by the (false) sum is the correct solution. The connexion between this method and that of operating with unknown quantities is discussed later on.

$$\text{(v)}\quad \left|\begin{array}{cccccccccc} \bullet & 5 & \text{yu} & \text{mū} & \bullet & \text{sā} & \bullet & 7 & \text{mū} & \bullet \\ 1 & 1 & & & 1 & & 1 & 1 & & 1 \end{array}\right|$$

which means $x+5=s^2$, $x-7=t^2$. Here the symbol $\overset{\bullet}{1}$ stands for three different unknown quantities. It simply indicates in each case an unknown number.

Negative sign.

61. The only distinct mathematical symbol employed is the sign for a negative quantity, which takes the form of a cross + placed after the number affected. This is peculiar and has given rise to discussion. In Sanskrit manuscripts a dot before the quantity affected is the usual method of indicating a negative quantity. In the transliteration of the text the original negative sign (+) is, perhaps illogically, preserved; but it is a rather special feature of the manuscript, easily printed, and leading to no ambiguity.

On this negative sign Dr. Hoernle writes[1]:

"Here, therefore, there appears to be a mark of great antiquity. As to its origin I am unable to suggest any satisfactory explanation. I have been informed by Dr. Thibaut of Benares that Diophantos in his Greek arithmetic uses the letter ψ (short for λεῖψις) reversed (thus ↑) to indicate the negative quantity. There is undoubtedly a slight resemblance between the two signs; but considering that the Hindus did not get their elements of the arithmetical science from the Greeks,[2] a native Indian origin of the negative sign seems more probable. It is not uncommon in Indian arithmetic to indicate a particular factum by the initial syllable of a word of that import subjoined to the terms which compose it . . . . . The only plausible suggestion I can make is, that it is the abbreviation (*ka*) of the word *kanita*, diminished." (He also points out that the letter *k* in its ancient shape as used in the Asoka inscriptions is a cross and goes on to say) "Another suggestion is, that the sign represents the syllable *nū*, an abbreviation of ***nyūna***, 'diminished.' The *akshara* for ***nū*** (or *nu*) in the Asoka characters would very closely resemble a cross (+) . . . . . . However for the present, the question must be left an open one."

With this last remark I agree and I would at the same time point out the danger of attempting to trace an isolated symbolic form back through the ages. The suggested connexion with Aśoka times is only part of the special pleading for the extreme antiquity of our manuscript. The striking resemblance with the Diophantine symbol, so lightly discarded by Dr. Hoernle, is of interest, but lacks support. From the time of Diophantus to the time of the Bakhshālī manuscript is too great[3] to allow us to ignore the lack of other examples.

Abbreviations.

62. Abbreviations are employed to such an extent as to become, at times, embarrassing, and they embrace almost every type of term. None of these abbreviations is used in an algebraic sense, although, at first sight, when we find *ā*°, *u*°, *pa*° used consistently for the elements of an arithmetical progression, an algebraic symbolism does not seem very far off. But *a*°, *ha*°, *ū*°, *go*° are the abbreviations of names of animals, *ya*°, *go*° and *sa*° of plants, etc., etc. The names

[1] *Indian Antiquary*, xvii (1888), p. 34.

[2] They got a good deal from them, and practically all their later astronomy came from the Greeks.

[3] But Dr. Hoernle assumes that the Bakhshālī text was practically contemporaneous with Diophantus!

of measures, which are numerous, are nearly always abbreviated. Certain common terms of operation are often abbreviated, and of these the following occurs most often—

bhā° for *bhāga*, placed after a term to indicate that it is a divisor.
śe° for *śeshaṁ*, a remainder.
mū° for *mūlaṁ*, a root, a quantity that has a root, capital.
pha° for *phalaṁ*, an answer.
etc., etc.

Indeed the writers seem to have used abbreviations whenever there was no ambiguity incurred in their own minds. We are not so fortunately placed in this matter and occasionally it has been impossible to rediscover the term implied.

Fundamental operations.

63. In the classical Sanskrit works there is generally very little formal information about mathematical principles and method. Axioms or postulates seldom or never occur and strict definition is seldom attempted. Rules are of particular application rather than general; order appears to have been a matter of convenience rather than logic; and the fundamental operations receive scanty attention. In a work that is essentially Indian in form we must not, therefore, expect much formal attention to such matters, and, indeed, in the Bakhshālī text there is little We cannot even tell how the detailed processes of such operations as multiplication and division were actually performed. All that remains are the formal statements of terms and results, and all I can now present are examples of such statements.

64. Examples of addition

(*a*) 960 64 yutaṁ jātaṁ 1024

(*b*) $\begin{array}{|c|}10\\3\end{array}$ sa rūpaṁ $\begin{array}{|c|}13\\3\end{array}$

(*c*) 2 rūpa saṁyutaṁ $\begin{array}{|c|}3\\1\end{array}$

(*d*) 840 49 datvā jātaṁ 889

(*e*) 924 | 836 | 798 | eshāṁ yutiṁ kṛiyate jātā 2558

(*f*) 120 | 90 | 80 | 75 | 72 | eshāṁ yoga kṛite jātā 437

(*g*) $\begin{array}{|c|}120\\90\\80\\75\\12\end{array}$ evaṁ 377

(*h*) | 10 | 30 | 90 | ekatram $\begin{array}{|c|}130\\1\end{array}$

(*a*) 960 and 64 added: 1024 is produced. (*b*) $\frac{10}{3}$ plus unity $=\frac{13}{3}$. (*c*) 2 and unity added together $=3$. (*d*) 49 having been given to 840, 889 is produced. (*e*) 924, 838, 798: the sum of these may be determined: they produce 2·558. (*f*) 120, 90, 80, 75, 72: the sum of these is made: they produce 437. (*g*) 120, 90, 80, 75, 12: thus 377. (*h*) 10, 30, 90: altogether 130.

(*i*) | 29 48 58 | 7 | anena yutaṁ | 36 48 58 |

(*k*) | 5 śe 1 1 16 | | 10 śe 15 16 | evaṁ 16

(*l*) 45 2 sārdha traya yutaṁ 52 2

65. Examples of subtraction

(*a*) | 5 | 9 | viśeshaṁ | 4 |

(*b*) | 3 2 2 1 | viśeshaṁ | 1 2 |

(*c*) | 5 | | 3 | rahitaṁ jātaṁ | 2 |

(*d*) 6 | 3 śuddhi | 3 |

(*e*) | 42 | tryūnam | 39 |

(*f*) | 3 | | 7 | viśoddhya | 4 |

(*g*) | 77 11 | 294 11 | pātya śeshaṁ | 217 11 |

66. Examples of multiplication.

(*a*) | 2 | dvigunaṁ | 4 |

(*b*) | 30 | ashṭa gunaṁ | 240 |

(*c*) | 2 40 5 1 | guṇita jātāṁ 16

(*d*) | 4 1 1 1 2 4+ | guṇita jātā | 27 8 |

(*i*) $29\frac{48}{58}$ by this (7) increased $= 36\frac{48}{58}$. (*k*) $5\frac{1}{16} + 10\frac{15}{16}$ thus 16. (*l*) $\frac{45}{2}$ with three and a half added $= \frac{52}{2}$.

(*a*) 5, 9 : the difference is 4. (*b*) $\frac{3}{2}$, 2 : the difference is $\frac{1}{2}$. (*c*) 5, 3 : subtracted, 2 is produced. (*d*) 6, 3 : the difference is 3. (*e*) 42 less three is 39. (*f*) 3, 7 : having subtracted, 4. (*g*) $\frac{77}{11}$, $\frac{294}{11}$: having subtracted the difference is 2[illegible].

(*a*) 2 multiplied by two $= 4$. (*b*) 30 multiplied by eight $= 240$. (*c*) $\frac{2}{5}$ and 40 multiplied : 16 is produced. (*d*) $1\frac{1}{4}$ and $4\frac{1}{2}$ multiplied $\frac{27}{8}$ is produced.

(*e*) $\left|\begin{array}{cc} 6 & 1 \\ 1 & 1 \\ & 4+ \end{array}\right|$ anena guṇitaṁ jātaṁ $\left|\begin{array}{c} 4 \\ 1 \\ 2 \end{array}\right|$

(*f*) $\left|\begin{array}{c|c|c} 2 & 3 & 4 \\ 3 & 4 & 5 \end{array}\right|$ guṇita jātaṁ $\begin{array}{c} 2 \\ 5 \end{array}\Big|$

(*g*) $\left|\begin{array}{c} 40 \\ 1 \\ 1 \\ 3+ \\ 1 \\ 4+ \\ 1 \\ 5+ \end{array}\right|$ phalaṁ 16

(*h*) $\left|\begin{array}{c|c} 3 & 4 \end{array}\right|$ abhyāsaṁ 12

(*i*) $\left|\begin{array}{c} 8 \end{array}\right|$ ātma gunaṁ $\left|\begin{array}{c} 64 \end{array}\right|$

(*j*) $\left|\begin{array}{c|c} 880 & 964 \\ 84 & 168 \end{array}\right|$ guṇita jātaṁ $\left|\begin{array}{c} 848320 \\ 14112 \end{array}\right|$

(*k*) $\left|\begin{array}{c} 737 \\ 58 \end{array}\right|$----$\left|\begin{array}{c} 178 \\ 29 \end{array}\right|$ anena guṇitaṁ jātaṁ $\left|\begin{array}{c} 65593 \\ 841 \end{array}\right|$

(*l*) $\left|\begin{array}{c} 108625 \\ 65600 \end{array}\right|$ pada-guṇa $\begin{array}{c} 6455949625 \\ 3227520000 \end{array}\Big|$

(*m*) $\left|\begin{array}{c|c} 405280 & 44004 \\ 38724 & 77448 \end{array}\right|$ saṁguṇya jātaṁ $\left|\begin{array}{c} 179945781120 \\ 2999096352 \end{array}\right|$

67. *Division.* Fractional quantities are expressed in the usual Indian way with the numerator written above the denominator without any dividing line, and they are usually placed in cells. This indication of the operation of division often does away with the necessity for an explanatory word or phrase.

Examples of division.

(*a*) $\left|\begin{array}{c|c|c} 168 & 168 & 168 \\ 4 & 6 & 7 \end{array}\right|$ labdhaṁ 42 | 28 | 24

(*b*) | 4 | vibhaktaṁ $\left|\begin{array}{c} 1 \\ 4 \end{array}\right|$ 35 guṇitaṁ $\left|\begin{array}{c} 36 \\ 4 \end{array}\right|$

(*c*) | 10 | | 3 | vibhaktvaṁ $\left|\begin{array}{c} 10 \\ 3 \end{array}\right|$ |

(*e*) 6 and $1-\frac{1}{4}$: by this ($\frac{3}{4}$) multiplied, $4\frac{1}{2}$ is produced. (*f*) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ multiplied, $\frac{2}{5}$ is produced. (*g*) 40 $(1-\frac{1}{3})$ $(1-\frac{1}{4})$ $(1-\frac{1}{5})$ $=16$. (*h*) 3, 4 : the product is 12. (*i*) 8 multiplied by itself $=64$. (*j*) $\frac{880}{84}$, $\frac{964}{168}$ multiplied ; $\frac{848320}{14112}$ is produced. (*k*) $\frac{737}{58}$, $\frac{178}{29}$ by this ($\frac{178}{29}$) multiplied, $\frac{65593}{841}$ is produced. (*l*) $\frac{108,625}{65600}$ multiplied by the number of terms ([illegible]) $=\frac{6,455,040,625}{3,227,520,000}$. (*m*) $\frac{405280}{38724}$: $\frac{44004}{77448}$: having multiplied these together $\frac{179,945,781,120}{2,999,096,352}$ is produced.

(*a*) *Labdham* = quotient. (*b*) This periphrastic exposition is common—the divisor is turned into a multiplier by being inverted. The example here means : 4 divided $=\frac{1}{4}$, which multiplied into $35=\frac{35}{4}$. (*c*) 10, 3 : having divided $=\frac{10}{3}$.

(*d*) $\frac{447}{29}$ dalita $\frac{447}{58}$

(*e*) $\boxed{\frac{132}{33}}$ vartyaṁ jātaṁ $\boxed{\frac{4}{1}}$

(*f*) $\frac{798}{1463}$ projjhya 798

(*g*) 2558 chchheda projjhyaṁ 1095

(*h*) $\boxed{60}$ anena dṛishyaṁ bhājitaṁ $\boxed{\frac{1}{60} \quad \frac{300}{1}}$ jātā $\boxed{5}$

Square-root.

68. The method of extracting square-roots exhibited in the text is of special interest. The rule is much mutilated but as it is given on three separate occasions the fragments pieced together enable us to give the complete *sūtra* :

> akṛite slishṭha kṛityūnā śesha chchhedo dvisaṁgunaḥ tad vargaḥ dala saṁslishṭha hṛiti śuddhi kṛiti kshayaḥ

This means somewhat as follows :—

> The mixed surd is lessened by the square portion and the difference divided by twice that. The difference is divided by the quantity and half that squared is the loss.

Then follows a note to the effect that " by means of this rule an approximation (*anaya*) to the proper root of a mixed quantity is found . . . ."

The rule as it stands is cryptic and hardly translatable, but fortunately there are examples given in some detail, and these show that the rule was extended to " second approximations."

The rule means that the first approximation to $\sqrt{Q} = \sqrt{A^2+b}$ is $A+b/2A$ or q_1; but $q_1^2 - Q = (b/2A)^2 = e_1$.

No rule for 'second approximations' is preserved but there are several examples; and, of course, no fresh rule is really required,[1] for $\sqrt{Q} = \sqrt{q_1^2 - e_1} \backsim q_1 - e_1/2q_1 = A + \frac{4A^2b+b^2}{8A^3+4Ab}$, and the 'second error' may be indicated by $e_2 = (e_1/2q_1)^2 = \left(\frac{b}{2A}\right)^4 / 4\left(A+\frac{b}{2A}\right)^2$.

(*d*) $\frac{447}{29}$ halved $= \frac{447}{58}$. (*e*) $\frac{132}{33}$ reduced gives 4. (*f*) Having discarded the denominator $\frac{798}{1463}$ becomes 798. (*g*) 2558—1463 = 1095 where 1463 is the denominator of the fraction. (*h*) By this (60) the known quantity (300) is divided and $\frac{1}{60}$ of 300 = 5.

[1] Note that if $\sqrt{A^2+b} = r$ then $r = A + \cfrac{b}{2A + \cfrac{b}{2A + \cfrac{b}{2A + \cdots}}}$ .... The text uses $r' = A + \frac{b}{2A}$, $r''' = A + \cfrac{b}{2A + \cfrac{b}{2A + \cfrac{b}{2A}}} = A + \frac{4A^2b+b^2}{8A^3+4Ab}$, but not $r'' = A + \cfrac{b}{2A + \cfrac{b}{2A}} = A + \frac{2Ab}{4A^2+b}$.

All the examples preserved belong to section C and appear to be merely subsidiary to the solution of certain quadratic equations arising out of problems in arithmetical progressions. These problems will be found fully worked out in § 86 and I give here merely a summary of the square-root evaluations.

(i) $\sqrt{41}=\sqrt{36+5}$ and $q_1=6\frac{5}{12}$ while $e_1=\frac{45}{144}$ and $q_2=6\frac{745}{1848}$.

(ii) $\sqrt{105}=\sqrt{100+5}$ and $q_1=10\frac{1}{4}$; $e_1=\frac{1}{16}$; $q_2=10\frac{1}{4}-\dfrac{\frac{1}{16}}{2\times 10\frac{1}{4}}$

$=10\frac{81}{328}$; $e_2=\left(\dfrac{\frac{1}{16}}{2\times 10\frac{1}{4}}\right)^2=\frac{1^*}{107584}$.

(iii) $\sqrt{481}=\sqrt{21^2+40}$†; $q_1=21\frac{40}{42}$; $\frac{e_1}{8}=\frac{1}{8}\left(\frac{40}{42}\right)^2=\frac{1600}{14112}$;

$q_2=21\frac{20}{21}-\left(\frac{20}{21}\right)^2/2\times 21\frac{20}{21}=21\frac{9020}{9681}$; $\frac{e_2}{8}=\frac{1}{8}\left(\frac{40}{42}\right)^4/\left(2\times 21\frac{40}{42}\right)^2=\frac{160,000}{2,999,096,352}$.

(iv) $\sqrt{889}=\sqrt{29^2+48}$; $q_1=29\frac{48}{58}$; $\frac{e_1}{24}=\frac{1}{24}\left(\frac{48}{58}\right)^2=\frac{24}{841}$.

(v) $\sqrt{336009}=\sqrt{579^2+768}$; $q_1=579\frac{384}{579}$; $q_2=579\frac{515,225,088}{777,307,500}$;

$\frac{e_2}{8d}=\frac{21,743,271,936}{7,250,483,394,675,000,000}$.

69. There is not much doubt about the exegesis of the rule. It was neither connected with continued fractions nor with the so-called Pellian equation.[1] Brahmagupta gave the converse of the rule, namely $(A+x)^2\sim A^2+2Ax$ from which the square-root rule given in our text is immediately deducible. But, as already pointed out, the square-root rule itself was not used by the Hindus and was not even noticed by them until the sixteenth century.[2] Indeed the Hindus had a very good practical rule of their own, which was given by Srīdhara (*Triśatika*, 46) and Bhaskara (*Līlāvatī*, 138), namely—"Multiply the quantity whose square-root cannot be found by any large square number, take the square-root of the product—leaving out of account the remainder—and divide by the square-root of the multiplier." For example $\sqrt{41}=\sqrt{41\times 1000000}\div 1000\sim 6{\cdot}403+$

This rule was used in conjunction with the usual modern scholastic square-root rule, which dates at least from the time of Heron.

The examples in the text may be further conveniently summarised thus :—

Q	q_1	q_2	q (approximately correct).
41	6·41667	6·40313	6·403124
481	21·9524	21·9307 . .	21·9317122
889	29·828 . .		29·8161030
336009	579·664	579·6615	579·65 . . . .

* In the text we find $\frac{e}{8d}$ and not e by itself, for reasons explained in §85.

† The form $\sqrt{481}=\sqrt{22^2-3}\sim 22-\frac{3}{44}=21\frac{41}{44}$ suggests itself; but the negative sign does not appear to have been used in first approximations.

Note.—The Appendix to Chapter vi (page 53) may be helpful to those who are interested in the details of the calculations.

[1] For a full discussion of the topic see P. TANNERY *Mémoires scientifiques*, I, pp. 189ff. ; and II, pp. 157ff.

[2] A short historical note is given on page 45.

"Rule of three."

70. The term *trairāśika* "relating to three quantities" occurs about a dozen times—always employed in the sense of arithmetical proportion. Most often it occurs in the phrase *pratyaya trai-rāśikena* or "proof by the rule of three" and the proportion is generally set out in the following manner—

1	1	4	phalaṁ	18
3	1	1		1
	2			

which means $\frac{1}{3} : 1\frac{1}{2} :: 4 : 18.$

The term *trairāśika* is orthodox but the term *phalaṁ* is used throughout our text quite appropriately as equivalent to "answer", and is not applied to the second term of a proportion as in the *Līlāvatī*.

There are very many examples of the "rule of three" used either as the direct method of solution or as a proof. There are also some very much damaged examples of what appear to be problems in so-called compound proportion. There is nothing of the nature of a theory of proportion discussed but several examples of the following principle occur—

$$\text{If } \frac{a_1}{b_1} = \frac{a_2}{b_2} \ldots\ldots = \ldots = \frac{a_n}{b_n} = \frac{A}{B}, \text{ then } \frac{\Sigma a}{\Sigma b} = \frac{A}{B}.$$

Regula falsi.

71. The rule of "false position" or "supposition" is used in two ways in our text[1] in solving linear equations.

(i) $f(x) = p$ is solved by assuming a value e for x. This gives $f(e) = p'$ and the correct value of x is ep/p'

An example is

> The amount received by the first is not known. The second receives twice as much as the first, the third thrice as much as the first two and the fourth four times as much as all the others. Altogether they receive 300. How much did the first receive?

Suppose the first receives one, then the second receives 2, the third 9, and the fourth 48; or altogether they receive 60. Actually, therefore, the first received $300/60 = 5$.

(ii) If $f(x) = p$ reduces to $bx + c = p$ and $f(e) = p'$ reduces to $be + c = p'$ then $x = (p - p')/b + e$, which appears to have been considered useful when it was desired to keep b and c unchanged.

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[1] No rule is preserved in the text but Bhāskara gives the following (*Līlāvatī*, 50).

"Any number assumed at pleasure is treated as specified in the particular question, being multiplied and divided, raised and diminished by fractions; then the given quantity, being multiplied by the assumed number and divided by the result yields the number sought."

For example if $x_1 + x_2 = a_1$, $x_2 + x_3 = a_2$, $x_3 + x_1 = a_3$ we have $2x_1 + (a_2 - a_1) = a_3$; and if in place of x we put e then $2e + (a_2 - a_1) = a'_3$ and the correct value of x_1 is $(a_3 - a'_3)/2 + e$

72. This rule of "false position" is interesting as being, in a way, a precursor of algebraic symbolism. It connotes the idea of an unknown quantity and even of a symbol for that quantity (*e.g.*, as in our text) but it does not embrace the notion of such a quantity being subject to operation or being isolated. As soon as we introduce algebraic symbols the rule, as it were, disappears. Algebraically the rule solves $bx = p$ by $x = ep/p'$ where $p' = be$, which shows that the transformation from $bx = p$ to $x = p/b$ was not conceived.

There is no algebraic symbolism in our text but it may be noted that Bhāskara gives both the *regula falsi* and also an early form of algebraic symbolism. Neither Āryabhaṭa, Brahmagupta nor Śrīdhara gives this rule or makes use of the principle. Its occurrence in the *Līlāvatī* therefore seems to indicate that it was introduced into northern India after the time of Śrīdhara (XIth cent.). Mahāvīra (IXth cent.), however, uses the method in rather a special way in connexion with a geometrical problem.[1]

The rūpoṇa method.

73. There are several references to the *rūpoṇa* method, and the phrases *rūpoṇā karaneṇa* or *pratyaya rūpoṇā karaneṇa* occur. In all the cases the application is to the summation of a series of terms in arithmetical progression according to the rule

$$s = [\,(t-1)\,d/2 + a\,]\;t$$

The recurrence of the phrase *rūpoṇā karaneṇa* seems to imply that the rule in question began with the term *rūpoṇā* which corresponds to the $(t-1)$ of the formula.[2] The rule is not preserved in our text but we find the following in the *Ganita Sāra-saṅgraha* of Mahāvīra (ii, 63)

> rūpenono gachchho dalī kṛitaḥ pra chayatādi to miśraḥ |
> prabhavena padābhyas tas saṅkalitaṁ bhavati sarveshām ||

"The number of terms is diminished by one, halved and multiplied by the increment. This when combined with the first term of the series and multiplied by the number of terms becomes the sum of all."

The rule is exemplified in two ways in our text of which the following are particular cases :—

(i)	ā° 1 1	u° 1 1	pa° 19 1	rūpoṇā karaneṇa phalaṁ	190 1

which means : " $a = 1$, $d = 1$, $t = 19$; by the rūpona method the answer is $s = 190$."

Similar bare summaries of the "rūpona" method are found in section "B" mainly.

[1] *Gaṇita Sāra-saṅgraha* vii, 112.

[2] Is this a faint echo of Pythagorean style ? Anatolius writes : "The Pythagoreans state that their master, in connexion with numbers that form a right-angled triangle, showed how such could be composed by means of unity." *i.e.*, $[\frac{1}{2}(a^2-1)]^2 + a^2 = \frac{1}{2}(a^2+1)^2$. See P. TANNERY *Mémoires scientifiques*, iii, 14.

(ii) In section "C" the application of the *rūpoṇa* method is always worked out step by step. For example when $a=5$, $d=3$, $t=178/29$; then $s=((t-1)\frac{3}{2}+5)\,t$; and the *rūpoṇa* method is applied as follows :—

$$t-1=\frac{149}{29};\ 3\times\frac{149}{29}=\frac{447}{29};\ \frac{447}{29}\times\frac{1}{2}=\frac{447}{58};\ \frac{447}{58}+5=\frac{737}{58};\ \frac{737}{58}\times\frac{178}{29}=\frac{65593}{841}=77\frac{836}{841}.$$

Proofs.

74. To many solutions are attached proofs, which are generally introduced by the term *pratyayaṁ* "proof" or "verification." This is sometimes amplified into *pratyaya-trai-rāśikena* "proof by the rule of three," or *pratyaya-rūponā-karanena,* "proof by the *rūpona* method."

Sometimes the "proof" seems to be merely a matter of rewriting a statement in another form, but generally the answer is utilised and one of the original terms of the problem is rediscovered. Occasionally the "proof" consists of a solution of the original problem in another way, *e.g.*, by steps; but sometimes it deals with a subsidiary aspect of the original problem. In certan problems approximations are employed and then the "proof" may be described as a process of reconciliation. These "reconciliations" entail a comparatively high degree of mathematical skill. Occasionally several different proofs are attached to a problem. The following are specimens.

(1) *Problem.* $\frac{5}{3}t-7=\frac{6}{5}t+7$

Solution. $t=2\times 7/\left(\frac{5}{3}-\frac{6}{5}\right)=30$

Proof. $3:5::30:50$, $\quad 5:6::30:36$ and $50-7=36+7$

(2) *Problem.* A gives $2\frac{1}{2}$ in $1\frac{1}{2}$ days, B gives $3\frac{1}{2}$ in $1\frac{1}{3}$ days, C gives $4\frac{1}{2}$ in $1\frac{1}{4}$ days. In what time will they have given 500 dināras altogether?

Solution. $t=\dfrac{500}{\frac{2\frac{1}{2}}{1\frac{1}{2}}+\frac{3\frac{1}{2}}{1\frac{1}{3}}+\frac{4\frac{1}{2}}{1\frac{1}{4}}}=\dfrac{60{,}000}{947}$ days;

and A gives $100{,}000/947$, B gives $157{,}000/947$, C gives $216{,}000/947$ dīnāras.

Proof. $2\frac{1}{2}:1\frac{1}{2}::100{,}000/947:60{,}000/947$

$3\frac{1}{2}:1\frac{1}{3}::157{,}000/947:60{,}000/947$

$4\frac{1}{2}:1\frac{1}{4}::216{,}000/947:60{,}000/947$

(3) *Problem.* One earns e and spends f daily. How long will a capital of C last?

Solution. $t=C/(f-e)$

Proof. $1:f::t:F$ (the total expenditure)

$1:e::t:E$ (the total earnings) and $F-E=C$.

(4) *Problem.* If 7 are bought for 2 and 6 sold for 3, and the capital is 24, what will be the profit?

Solution. $p = C\,(c/s - 1) = 24\,(\frac{7}{2} \div \frac{6}{3} - 1) = 18.$

Proof. $2 : 7 :: 24 : 84$ (the number of articles)

and $6 : 3 :: 84 : 42$ (the total proceeds), and $42 - 24 = 18.$

or—$1 : c :: C : n,\quad s : 1 :: n : C + p$ and $C + p - C = p.$

(5) *Problem.* $x\,(1 - \frac{1}{2})\,(1 - \frac{1}{4})\,(1 - \frac{1}{5}) = x - 280.$

Solution. $x = 280/(1 - \frac{3}{10}) = 400.$

Proof. $400/2 = 200,\qquad 400 - 200 = 200,$

$200/4 = 50,\qquad 200 - 50 = 150,$

$150/5 = 30,\qquad 150 - 30 = 120,$ and $400 - 120 = 280.$

(6) *Problem.* $1 + \frac{1}{2}\,(2 + \frac{1}{2}\,(3 + \frac{2}{3}(4 + \frac{3}{2}\,(5 + \frac{1}{2})))).$

Solution. $= \frac{93}{16}.$

Proof. $(((((\frac{93}{16} - 1)\,2 - 2)\,2 - 3) - 4)\,\frac{3}{2} - 5 - \frac{1}{2}) = 0.$

(7) *Problem.* Solve $x + 5 = s^2,\qquad x - 7 = t^2.$

Solution. $x = (\frac{1}{2}(\frac{5+7}{2} - 2))^2 + 7 = 11.$

Proof. $11 + 5 = 4^2\qquad 11 - 7 = 2^2$

(8) *Problem.* $F = \dfrac{\frac{1}{2}(1) + \frac{1}{3}(2) + \frac{1}{4}(3) + \frac{1}{5}(4)}{1 + 2 + 3 + 4}.$

Solution. $F = \frac{163 \div 60}{10} = \frac{163}{600}.$

Proof. $10 : 163/60 :: 1 : 163/600$

$10 : 163/60 :: 2 : 163/300$

$10 : 163/60 :: 3 : 163/200$

$10 : 163/60 :: 4 : 163/150,$

or, since $F = \frac{\Sigma f \times w}{\Sigma w},\ \Sigma w : \Sigma f \times w :: w_r : w_r F.$

(9) *Problem.* $Dt = ((t - 1)\,d/2 + a)\,t.$

Solution. $t = 2\,(D - a)/d + 1.$

Proof by the *rūpona* method: $s = ((t - 1)\,\frac{d}{2} + a)t$ and $Dt = s$

(10) *Problem.* $s = ((t-1)d/2 + a)\ t.$

Solution. $t = \{\sqrt{(2a-d)^2 + 8s} - (2a-d)\ \} \div 2d.$

Generally the solution is an approximation t_1 or t_2 depending on the method of evaluation of the surd quantity; and the *rūpona* method gives s_1 or s_2, neither of which is the same as the original s. A process of reconciliation is therefore called for, and this is given by $\acute{s} - s = e/8d$ where $\acute{s}$ is the approximation to s given by the proof and e may be termed the square-root error.

Problem. $a = 1,\ d = 1,\ s = 60.$

First solution. $t_1 = \{\sqrt{(2 \cdot 1 - 1)^2 + 480} - 1\} \div 2 \cdot 1 = (21\frac{40}{42} - 1) \div 2 = \frac{880}{84}.$

First proof. $s_1 = t_1\ (t_1 + 1) \div 2 = \frac{880}{84} \times \frac{964}{168} = \frac{848,320}{14112}$

$$e_1 = \left(\frac{40}{42}\right)^2 \text{ and } s = s_1 - \frac{e_1}{8d} = \frac{848,320 - 1600}{14112} = 60.$$

Second solution. $t_2 = \left(\frac{424,642}{19360} - 1\right) \div 2 = \frac{405,280}{38724}$

Second proof. $s_2 = \frac{405,280}{38724} \times \frac{444,004}{77448} = \frac{179,945,941,120}{2,999,096,352}.$ Now $e_2 = \left(\frac{40}{42}\right)^4 + 4\ (21 + \frac{40}{42})^2$, and $\frac{e_2}{8d} = \frac{160,000}{2,999,096,352}$;

$$\text{and } s = s_2 - e_2/8d = \frac{179,945,781,120}{2,999,096,352} = 60.$$

(11) The problems in section "G" are characterised by numerous proofs—as many as five separate proofs being attached to one example. The following is fairly typical:—

Problem. $x = 500(1 - \frac{1}{4})(1 - \frac{1}{4})(1 - \frac{1}{4})(1 - \frac{1}{4}).$

Solution. $x = 158\frac{13}{64}$

Proof (i). $x'(1 - \frac{1}{4})\ (1 - \frac{1}{4})\ (1 - \frac{1}{4})\ (1 - \frac{1}{4}) = 158\frac{13}{64}$, whence $x' = 500$. This is written horizontally.

Proof (ii). $x'(1 - \frac{1}{4})(1 - \frac{1}{4})(1 - \frac{1}{4})(1 - \frac{1}{4}) = 158\frac{13}{64}$, whence $x' = 500$. This is written vertically.

Proof (iii). $x' = \frac{158\frac{13}{64}}{(1 - \frac{1}{4})\ (1 - \frac{1}{4})\ (1 - \frac{1}{4})\ (1 - \frac{1}{4})} = 500.$

Proof (iv). By steps.

$$500 \div 4 = 125,\ 500 - 125 = 375;$$

$$375 \div 4 = 93\tfrac{3}{4},\ 375 - 93\tfrac{3}{4} = 281\tfrac{1}{4};$$

$$281\tfrac{1}{4} \div 4 = 70\tfrac{5}{16},\ 281\tfrac{1}{4} - 70\tfrac{5}{16} = 210\tfrac{15}{16};$$

$$210\tfrac{15}{16} \div 4 = 52\tfrac{47}{64},\ 210\tfrac{15}{16} - 52\tfrac{47}{64} = 158\tfrac{13}{64}.$$

Solutions.

75. The solutions are sometimes very detailed, proceeding most carefully step by step, so that they become expositions in general terms of the processes involved. Also they often give actual quotations from the rules—to such an extent sometimes that the original wording of the rule can be reconstructed. Unfortunately, however, such helpful suggestions are more often missing than not: in many cases the

particular portions of the manuscript are lacking or damaged beyond repair. In some sections, *e.g.*, section " M ", nothing but the bare answer is generally given, and in others the outline only of the working is given.

The following examples illustrate these remarks : 1. The problem is $DT + Dt = ((t-1)+a)\,t$ where *a* and *d* are respectively the first term and common difference of an arithmetical progression, and *t*, the number of terms, to be found. The rule is

$$t = \left\{ 2(D-a)+d+\sqrt{(2(D-a)+d)^2+8dDT} \right\} \div 2d$$

and the particular example gives $D=5$, $T=6$, $a=3$, $d=4$. The actual working of the solution, translated as literally as possible, is as follows :—

> *The daily rate diminished by the first term ;* the daily rate is five yojanas, 5 ; the first term is 3 ; their difference is 2 ; this doubled is 4 : *this increased by the common difference* is 8 ; and squared is 64, which is *known as the *kshepa* quantity.* *Multiplied by eight* ; the fixed term (30) multiplied by eight is 240 ; and multiplied by the common difference—multiplied 960. *The quantity known as *kshepa* is added.* Now the quantity known as *kshepa* is 64, which added gives 1024. The root of this is 32 ; the quantity set aside is 8 and this added gives 40. *Divided by twice the common difference :* twice the common difference is 8—divided 5.

The phrases placed between asterisks are quotations from the rule. A perfectly literal rendering would not be very intelligible to the ordinary reader and one or two gaps have been filled in ; but the translation is a perfectly fair representation of the original.

ii. The problem may be represented by $x\,(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5}) = x - 24$

and the solution is given as follows :—

> *Having calculated the loss on unity* the terms become $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and these multiplied together give $\frac{2}{5}$. This subtracted from unity gives $\frac{3}{5}$, which, inverted and multiplied† by the given amount 24, is $\frac{5}{3}$ of $24 = 40$.

iii. Something travels 3 *yavas* a day. How long will it take to go 5 yojanas ?

Here the solution is indicated by the following proportion only :

3 *ya*° : $\frac{1}{360}$ years : : $5 \times 4{,}608{,}000$ *ya*° : 21,333 years 4 months.

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* The phrases marked off by asterisks are quotations from the *sūtra* or rule.

† Literally " divided and multiplied " into. This is generally used to indicate division by a fraction.

CHAPTER VI.

AN ANALYSIS OF THE CONTENTS.

76. The present chapter contains an analysis of the mathematical contents of the text. To those who are chiefly concerned with the history of mathematics this chapter will possibly be of rather special interest. Indeed, taken with chapter I of Part III (which will appear in the second volume) it forms the real corpus of the present study, for the latter gives the re-arranged text, while the present chapter is an attempt at a fairly complete explanation of the intelligible parts of that text. The analyses of the different sections are not exhibited on a uniform plan, some are given in great detail, while others are mere summaries; some are arithmetical in form and some algebraical; but these differences correspond to some extent to the spirit of the text, and it is hoped that the analysis as a whole gives a proper representation of the original.

77. In order to avoid innumerable cross references the following table is given to show the connexion between the paragraphs of the present chapter and the original and re-arranged texts.

Chapter VI.	Illustrations (Bodleian order).	New order.
§78	Folio(s) 29, 27	$A_{11, 12}$.
§79	,, 1, 2, 3	$A_{13, 15}$.
§80	,, 38ᵃ, 39ᶜ, 40ᶜ·ᵈ 58	A_7.
§81	,, 59	K.
§82	,, 27 recto	A_{12}.
§83 (i)	,, 4 recto	B_1.
(ii)	,, 4 verso	B_1.
(iii)	,, 8 recto, 9 recto, 7 verso	$B_{2, 3, 4}$.
§84 (i)	,, 8 verso	B_2.
(ii)	,, 9 verso	B_3.
§85	Section 'C' generally	....
§86 (i)	Folio(s) 5	C_1.
(ii)	,, 6, 7 recto	$C_{2, 3}$.
(iii)	,, 65 verso, 56 verso, 56 recto, 64 recto	$C_{4, 5, 6}$.
(iv)	,, 64 verso, 57	$C_{6, 7}$.
(v)	,, 45 recto	C_8.
(vi)	,, 45 recto, et verso, 46 recto	$C_{8, 9}$.
§87	,, 23, 24, 25, 26	$F_{2, 3, 4, 5}$
§88	,, 51	A_9
§89	,, 10, 11, 12, 13, 14, 15, 16 recto	G_{1-6}.

Chapter VI.	Illustrations (Bodleian order).	New order.
§90	Folio(s) 16 verso, 17, 18	$H_{1, 2, 3}$
§91 (i)	,, 60 verso	L_1.
(ii)	,, 60 recto, 61	$L_{1, 2}$.
(iii)	,, 21 verso, 22 recto	E_4, F_1.
§92 (i)	,, 61	L_2
(ii)	,, 62	L_3
(iii)	,, 62 verso	L_3.
(iv)	,, 63	L_4.
§93	Section 'M'	M.
§94	Folio(s) 47 recto	M_7.
§95	,, 44 recto, 43 verso	M_{12}
§96	,, 44 verso	M_{11}.
§97	,, 36 verso	M_4.
§98	,, 20 verso	M_1.
§99	,, 32	M_3.
§100 (i)	,, 37 recto	M_6.
(ii)	,, 37 recto	M_6.
(iii)	,, 37 verso	M_6.
§101 (i)	,, 55 recto	M_{10}
(ii)	,, 55 verso	M_{10}.
(iii)	,, 49 recto	M_9.
(iv)	,, 49	M_9.

Systems of linear equations.

78. Examples of the following type occur

$x_1 + x_2 = a_1$, $\quad x_2 + x_3 = a_2$, ..., $x_n + x_1 = a_n$ where n is always odd.

We have $a_n = (a_2 - a_1) + ... + (a_{n-1} - a_{n-2}) + 2x_1$; and if we assume $x_1 = p$

then $a'_n = (a_2 - a_1) + ... + (a_{n-1} - a_{n-2}) + 2p$.

Subtracting we get $x_1 = p + (a_n - a'_n)/2$, which is the solution employed in the text.

The following examples occur

(i) $x_1+x_2=13, \quad x_2+x_3=14, \quad x_3+x_1=15.$

The value of x_1 is assumed to be 5, then by 'subtraction in order' $x_2=8$, $x'_3=6$ and $x'_3+x_1=11$. The correct values are therefore $x_1=5+(15-11)\div 2=7$, $x_2=6$ and $x_3=8$.

(ii) $x_1+x_2=16$, $x_2+x_3=17$, $x_3+x_4=18$, $x_4+x_5=19$, $x_5+x_1=20$.

Here the value of x_1 is assumed to be 7 and $x'_5+x'_1=16$, therefore the correct values are $x_1=9$, $x_2=7$, $x_3=10$, $x_4=8$, $x_5=11$.[1]

The following are implied

(iii) $x_1+x_2=9$, $x_2+x_3=5$, $x_3+x_1=8$.

(iv) $x_1+x_2=70$, $x_2+x_3=52$, $x_3+x_1=66$.

(v) $x_1+x_2=1860$, $x_2+x_3=1634$, $x_3+x_1=1722$.

(vi)[2] $x_2+x_3+x_4+x_5=317$

$x_1+x_3+x_4+x_5=347$

$x_1+x_2+x_4+x_5=357$

$x_1+x_2+x_3+x_4=365$

and the following, which, however, is too mutilated to be sure of

(vii) $x_1+x_2=36$, $x_2+x_3=42$, $x_3+x_4=48$, $x_4+x_5=54$, $x_r+x_1=60$.

The directions seem to indicate that we should cancel by six. We then get

$a'_1=6$, $a'_2=7$, $a'_3=8$, $a'_4=9$, $a'_5=10$ of which the solution is $x'_1=4$, $x'_2=2$, etc.; whence $x_1=24$, $x_2=12$, $x_3=30$, etc.

79. The next set of examples can be represented in the form $\Sigma x - x_1 = c - d_1x_1$, $\Sigma x - x_2 = c - d_2x_2 \ldots\ldots \Sigma x - x_n = c - d_nx_n$.

If we set $a=1-d$ these become

$\Sigma x - a_1x_1 = \Sigma x - a_2x_2 = \ldots\ldots = \Sigma x - a_nx_n = c$, and $a_1x_1 = a_2x_2 = \ldots\ldots = a_nx_n = \Sigma x - c = k$.

---

[1] Since in these two particular examples the values of a_1, a_2, etc., are in arithmetical progression a simpler solution would be $x_1=\frac{m}{2}$ where m is the mean of the series a_1, a_2, etc.

[2] This can be arranged in the form $y_1+y_2=365$, $y_2+y_3=347$, $y_3+y_4=362$, $y_4+y_5=317$, $y_5+y_1=357$. Solving this we get $x_1+x_2=210$, $x_2+x_3=170$, $x_3+x_4=155$, $x_4+x_5=147$, $x_5+x_1=192$, which solved gives $x_1=120$, $x_2=90$, $x_3=80$, $x_4=75$, $x_5=72$. (See folio 1.)

Also note that most of these six examples can be expressed in the form

$$\Sigma x - x_1 = a_1,\ \Sigma x - x_2 = a_2,\ \ldots\ldots\ldots\ \Sigma x - x_n = a_n.$$

The examples given in the text are undoubtedly akin to the 'Ephanthema', usually attributed to Thymaridas, which may be expressed by $x_0=\frac{\Sigma a - \Sigma x}{n-1}$, where $x_0+x_1=a_1$, $x_0+a_2=x_2$, ... $x_0+x_n=a_n$. In Aryabhaṭa's *Gaṇita* is a similar rule $\Sigma x=\frac{\Sigma d}{n-1}$ where $\Sigma x - x_1 = d_1$, $\Sigma x - x_2 = d_2$...... $\Sigma x - x_n = d_n$, which Cantor (*Vorlesungen über der Geschichte der Mathematik* i, 624) considers to be a modification of the rule given by Thymaridas. See also P. TANNERY *Mémoires scientifiques*, Tome ii, pp. 192-195.

Therefore $\Sigma x = (\frac{1}{a_1} + \frac{1}{a_2} + \dots \frac{1}{a_n})k = \frac{p}{q}k$.

A solution is $x_1 = \frac{q}{a_1}$, $x_2 = \frac{q}{a_2} \dots\dots x_n = \frac{q}{a_n}$ and $c-p=q$[1],

There are four examples illustrating this process which may be tabulated thus—

	d_1	d_2	d_3	d_4	d_5	$p \div q$	c	x_1	x_2	x_3	x_4	x_5
(i)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{437}{60}$	377	120	90	80	75	72
(ii)	$-\frac{7}{12}$	$-\frac{3}{4}$	$-\frac{11}{6}$			$\frac{2558}{1463}$	1095	924	836	798		
(iii)	5	7	8				262	42	28	24		
(iv)	2	3	4				17	6	3	2		

Of these only fragments remain. For example, of the first we have :[2]

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{120}{60}+\frac{90}{60}+\frac{80}{60}+\frac{75}{60}+\frac{72}{60}=\frac{437}{60}$$

$$90+80+75+72=317, \quad 317+\frac{120}{2}=377.$$

$$120+80+75+72=347, \quad 347+\frac{90}{3}=377.$$

$$120+90+75+72=357, \quad 357+\frac{80}{4}=377.$$

$$120+90+80+72=362, \quad 362+\frac{75}{5}=377.$$

$$120+90+80+75=365 \quad 365+\frac{72}{6}=377.$$

The second of these examples may be expressed in the form

$$x_1+x_2-(\tfrac{1}{3}+\tfrac{1}{4})\,x_3 = x_3+x_1-(\tfrac{1}{4}+\tfrac{1}{2})\,x_2 = x_2+x_3-(\tfrac{1}{2}+\tfrac{1}{3})\,x_1.$$

From this $\frac{19}{12}x_1 = \frac{7}{4}x_2 = \frac{11}{6}x_3$, and $\Sigma x/(\Sigma x - c) = \frac{12}{19}+\frac{4}{7}+\frac{6}{11} = \frac{2558}{1463}$.

The solution given is $\Sigma x = 2558$ and $c = 2558 - 1463 = 1095$, whence $x_1 = 924$, $x_2 = 836$, $x_3 = 798$.

Of the third there is sufficient of the formal question preserved to enable it to be restored.

One possesses seven horses, another nine mules (?), and a third ten camels. Each gives one of his animals to each of the others and then their possessions become of equal value.

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[1] Compare with this the treatment of the *epanthem* by Jamblichus (Heath, *Greek Mathematics*, i, 93-94).

[2] The nearest approach to this that I have come across in an Indian work is given by Mahāvira in his *Ganita-Sāra-Sangraha* (vi, 239-240).

Five merchants saw a purse of money. They said one after another, by obtaining $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$ or $\frac{1}{10}$ of the contents of the purse I shall become three times as rich as all of you.

Let p be the value of the purse and x_1, x_2, etc., the original capitals; then $3(\Sigma x - x_1) = \frac{p}{6} + x_1$, $3(\Sigma x - x_2) = \frac{p}{7} + x_2$, etc.; whence $11\Sigma x = (\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10})\,p = \frac{6508p}{10080}$, or $\Sigma x = \frac{6508p}{110880}$. If $p = 110880$ then $x_1 = 261$.

But al-Karkhī (xith cent.) gave (iii, 6) the following :—A certain sum is divided among three people, one-half being given to the first, one-third to the second and one-sixth to the third. But then one-half the share of the first, one-third the share of the second and one-sixth that of the third is pooled and shared equally by the three.

In A.D. 1225 this problem became famous as it was one of those propounded to Leonardo of Pisa and solved by him.

If x_1, x_2, x_3 be the shares and $3c$ be the amount pooled, then $x_1 - \frac{x_1}{2} + c = \frac{\Sigma x}{2}$, $x_2 - \frac{x_2}{3} + c = \frac{\Sigma x}{3}$, $x_3 - \frac{x_3}{6} + c = \frac{\Sigma x}{6}$ and $\Sigma x - x_1 = 2c$, $\Sigma x - 2x_2 = 3c$, $\Sigma x - 5x_3 = 6c$, whence $\Sigma x = \frac{47c}{7}$. Make $c = 7$ and then $x_1 = 33$, $x_2 = 13$, $x_3 = 1$.

The fourth example is of similar form.[1]

80. The following occurs on some very mutilated scraps that have been pieced together. It is now impossible to say how it originated or what its context was.

$$x(x+y+z)=60$$
$$y(x+y+z)=75$$
$$z(x+y+z)=90$$

We have $(x+y+z)^2=225$ whence $x+y+z=15$, and $x=4$, $y=5$, $z=6$.

80. (*a*). There is one nearly complete statement and solution of the following pair of equations :—

$$x+y+z=20$$
$$3x+\tfrac{3}{2}y+\tfrac{1}{2}z=20$$

of which the only solution in positive integers is $x=2$, $y=5$, $z=13$.

This type of problem was a favourite in Europe and Asia in early mediæval times. Later it was known as the 'Regula Virginum,' 'Regula Potatorum,' etc. It was given by Chang-ch'iu-chien (sixth century A.D.),[2] by Alcuin the Englishman (eighth century) and others. About 900 A.D. it was pretty fully treated by Abū Kāmil al-Misrī who gives some six problems, varying from three to five terms and attempts to find *all* the positive integral solutions.[3]

In the earlier Indian works problems of this type do not occur, but *exactly* the same example is given by both Mahāvīra[4] (? ninth century) and **Bhāskara**[5] (twelfth century).

Quadratic indeterminate equations.

81. There are two types of quadratic indeterminates preserved.

$$\text{(i)}\ x+a=s^2, \qquad x-b=t^2$$

The solution given may be represented by

$$x=\left(\tfrac{1}{2}\left(\frac{a+b}{c}-c\right)\right)^2+b$$

which makes both $x+a$ and $x-b$ perfect squares. In the actual example preserved $a=5$ and $b=7$ and the solution is $x=11$, which is the only possible integral (positive) solution obtainable from the formula. No general rule is preserved but the solution itself indicates the rule. It proceeds by steps thus : $5+7=12$, $12\div 2=6$, $6-2=4$, $4\div 2=2$, $2^2=4$, $4+7=11$. The value of this type of detailed exposition is here

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[1] This set of examples is introduced by a rule which simply means : change the fraction $\frac{A}{B}$ into $\frac{B}{B-A}$. This illustrates the fact that the *sūtras* or rules often make no pretence of indicating in any way the general theory : they are merely intended to be helpful.

[2] Yoshio Mikami.

[3] H. SUTER, *Das Buch der Seltenheit*, etc. Bib. Math. xi (1910-11), pp. 100-120. See also SUTER's *Die Mathematiker und Astronomen der Araber und Ihrer Werke*, p. 43.

E.g. of $x+y+z+w=100$, $2x+\frac{y}{2}+\frac{z}{3}+w=100$ there are said to be 304 solutions and twenty are given. See also L. E. DICKSON's *History of the Theory of Numbers*, vol. ii, pp. 77, *et seq.*

[4] *Gaṇita Sāra-Saṅgraha* vi, 152.

[5] *Vīja Gaṇita*, §158.

self-evident. It seems to have been almost necessitated by the absence of a suitable algebraic symbolism.[1]

82. The second type is

$$\text{(ii)}\quad xy-ax-by-c=0$$

The solutions which appear to be followed in the text are $x=(ab+c)\div m+b$, $y=a+m$; or $y=(ab+c)\div m+a$, $x=b+m$

where m is any assumed number. The only example that is preserved[2] is $xy-3x-4y\pm 1=0$ of which the solutions given are 15 and 4, and 16 and 5, *i.e.*,

$$(3\cdot4-1)\div1+4=15,\ 3+1=4;\ (3\cdot4-1)\div1+3=16,\ 4+1=5.$$

Simple 'motion' problems.

83. From the mathematical point of view this section is not of particular interest but it was possibly intended as a sort of introduction to the section (C) that follows directly after. The examples can be classified into three types :

(i) *A* travels at a certain rate r_1 for a number of days *T*, and then *B* starts at a daily rate r_2. When will *A* and *B* have travelled equal distances ?

Since $r_1T+r_1t=r_2t$, $t=r_1T/(r_2-r_1)$ which is expressed by a *sūtra* and illustrated by two examples.

(ii) *A* travels a distance a_1 the first day, a_1+d_1 the second day and so on; *B* travels a_2 the first day, a_2+d_2 the second day and so on. When will they have travelled equal distances ?

Since $((t-1)d_1\div2+a_1)t=((t-1)d_2\div2+a_2)t$, $t=2(a_1-a_2)/(d_2-d_1)+1$

(iii) One goes at a fixed rate *A* and another goes *a* the first day, $a+d$ the second day and so on. In what time will they have travelled equal distances ?

Since $tA=t((t-1)d\div2+a)$, we have $A=(t-1)d\div2+a$, and $t=(2A-2a)\div d+1$

[1] This type of equation occurs in many mediæval works from the time of Diophantus onwards (*e.g.*, see DIOPHANTUS ii, 11ff. ; BRAHMAGUPTA, xviii, 84 ; AL-KARKHĪ, p. 63 ; etc.) and has here no very special interest beyond the indication it gives that the Bakhshālī text followed the fashion. Dr. Hoernle, however, thought that it indicated a 'peculiar' connexion between the Bakhshālī MS. and Brahmagupta's work ; and from this deduced that our text 'may have been one of the sources from whence the later astronomers took their arithmetical information.' (*Ind. Ant.*, xvii, 1888, p. 37.)

[2] Unfortunately the text (folio 27, recto) is so mutilated that the correctness of the interpretation here given cannot be guaranteed. But the equation was well known to mediæval mathematicians and has historical interest. Brahmagupta gave a general solution (xviii, 61) which, however, he appears to have thought unnecessary. Bhāskara also gives a general solution (*Vīja Ganita*, 212-214) together with demonstrations in both algebraical and geometrical forms ; but he also on another occasion (ib. 208-209) gives an arbitrary solution. The general solution is also given by al-Karkhī. Mahāvīra gives (vi, 284) the equation in a different form, namely $(a+\frac{1}{x})(b+\frac{1}{y})=ab+A$ which reduces to $xy-\frac{a}{A}x-\frac{b}{A}y-\frac{1}{A}=0$ of which the solution is $x=\frac{b}{A}+\frac{1}{m}\left(\frac{ab+A}{A^2}\right)$, $y=\frac{a}{A}+m$; or $y=\frac{a}{A}+\frac{1}{m}\left(\frac{ab+A}{A^2}\right)$, $x=\frac{b}{A}+m$. If we make $m=\frac{ab}{A}+1$ then $x=\frac{b+1}{A}$, $y=\frac{a+ab+A}{A}$ which is the solution given by Mahāvīra.

His examples are : The product of 3 and 5 is 15, and the required product is 18 or 14. What are the quantities to be added or subtracted ? *i.e.* (i) $(3+\frac{1}{x})(5+\frac{1}{y})=15+3$, (ii) $(3-\frac{1}{x_1})(5-\frac{1}{y_1})=15-1$. His answers are (i) 2 and 7, (ii) 6 and 17.

84. There are two examples in this section of different types—

(i) A chariot is drawn by A horses a at a time. How many stages should there be in a distance l, and how many stages should each horse do?

The argument seems to be : the 'mileage' done is la therefore each horse does la/A miles ; the number of stages is A consisting of l/A miles each and each horse does a stages.[1]

(ii) A and B start on a journey together. A goes at the rate of r_1 and meets B, whose rate is r_2, on the return journey. If the distance between the two places is l, when will they meet?

Let x be the distance gone by the slower traveller then the other goes $2l-x$ and $tr_1=2l-x$, $tr_2=x$ whence $t=2l/(r_1+r_2)$. A proof is given in the form $l : r_1 : : t : tr_1$ and $l : r_2 : : t : tr_2$

Quadratic equations and square-root approximations.[2]

85. Ostensibly this section deals with certain problems in arithmetical progression that were common property in comparatively early times; but it is of special interest because it elaborates a square-root rule and gives a number of examples, some of which involve large numbers. It also employs, although to a very limited extent, the sexagesimal notation. Of particular interest are the methods of reconciliation necessitated by employing approximations. There are two types of arithmetical progression exhibited—

(i) $DT+Dt=((t-1)d\div 2+a)t$, where T is a given period and t is to be determined. The solution is $t=\{2(D-a)+d\pm\sqrt{(2(D-a)+d)^2+8DT}\}\div 2d$.

The second type is the ordinary one of finding the number of terms of an arithmetical progression when the other elements are given, the solution being

(ii) $t=\{-(2a-d)\pm\sqrt{(2a-d)^2+8ds}\}\div 2d$.

These two types may conveniently be reduced to one by the formula $t=(p\pm\sqrt{q})\div 2d$ or $(p\pm q)\div 2d$, where $q=\sqrt{Q}$ and $p=2D-(2a-d)$ or $-(2a-d)$. The negative sign for the root quantity is not used so $t=(p+q)\div 2d$ always. Unless Q is a square number the solutions given depend upon approximations, and the most interesting results exhibited in the text are those connected with the proofs or verifications of the approximate answers obtained.

In type (i) let $s'=DT+Dt$ and $s''=((t-1)d\div 2+a)t$. If approximate values of t are used such that $s_1'=DT+Dt_1$, and $s_1''=((t_1-1)d\div 2+a)t_1$, it is evident that s_1' does not equal s_1, and the problem of reconciliation arises.

Now $s_1'=DT+Dt_1=DT+(Dp+Dq_1)\div 2d=(8dDT+4Dp+4Dq_1)\div 8d$, $s_1''=((t_1-1)d/2+a)t_1=(4Dp+4Dq_1-p^2+q_1^2)\div 8d$, and $s_1''-s_1'=q_1^2-(p^2+8dDT)\div 8d$; but $p^2+8DdT=Q=q^2$, therefore $s_1''-s_1'=(q_1^2-q^2)\div 8d=e/8d$ (see below).

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[1] This example is evidently traditional and we find it in Mahâvîra (vi, 157-158) in the following form :—

"It is well known that the horses belonging to the Sun's chariot are seven. Four horses being harnessed to the yoke to draw it. They have to do a journey of seventy yojanas. How many times are they yoked and unyoked in four?"

His rule is the same as that given above.

[2] Approximations are here indicated by subscripts : thus s_1 represents a first approximation to the value of s, and s_2 represents an approximation of the second order, etc.

This form of reconciliation occurs only once, but that about to be explained occurs a number of times. In type (ii) $s=((t-1)d/2+a)t=t^2d/2+(2a-d)t/2=(t^2d-pt)/2$; and since $2dt-p=q$, we have $4d^2t^2-4dtp=q^2-p^2$, and $8ds=4d^2t^2-4dtp=q^2-p^2$. Therefore $s_1-s=(q_1^2-p^2)/8d-(q^2-p^2)/8d=(q_1^2-q^2)/8d=e/8d$ as before.

The square-root rule comes in incidentally when a surd quantity occurs. In this way it is actually given three times (see §68) and this seems to indicate that it was not treated separately and was considered as subsidiary to the arithmetical progression problem. To the student of the history of mathematics, however, it will be probably considered the most interesting part of the work. The rule may be expressed by $\sqrt{A^2+b}=A+b/2A$ approximately.[1] But since $(A+b/2A)^2=A^2+b+(b/2A)^2$, the error may be shown by $e_1=(b/2A)^2$. Again $\sqrt{A^2+b}=\sqrt{r_1^2-e_1}=r_1-e_1/2r_1$ approximately, where $r_1=A+b/2A$ and the second error may be represented by $e_2=(e_1/2r_1)^2=b^4/64A^4(A+b/2A)^2$. Thus for a first approximation $s_1-s=e_1/8d$, and for a second approximation $s_2-s=e_2/8d$.

86. The examples of this section are fragmentary but in most cases they are quite unambiguous and can be interpreted with certainty. I now give an accurate representation of these examples; but it should be noted that those portions that I have enclosed in angular crotchets do not occur in the text as it now stands.

Rule. $t=\{2(D-a)+d+\sqrt{(2(D-a)+d)^2+8DTd}\}\div 2d$

(i) *Example.* $D=5$, $T=6$, $a=3$, $d=4$

. . . . . . $DT=30$. $D-a=5-3=2$, $2(D-a)=4$,

$2(D-a)+d=8$, $(2(D-a)+d)^2=64$. $DT=30$.

$8DT=240$, $8DTd=960$, $\{2(D-a)+d\}^2+8DTd=960+64$

$=1024$, $\sqrt{1024}=32$, $2(D-a)+d=8$, $8+32=40$,

$<$ $2d=8$ and $40\div 8=5=t$ $>$.

Proof by the rule of three; $1:5::5:25$ and $25+30=55$.

(ii) *Example.* $D=7$, $T=5$, $a=5$, $d=3$.

$DT=35$, $D=7$, $a=5$, $<$ $D-a=2$, $(2(D-a)+d)^2=49$ $>$

$8DTd=840$, $840+49=889$. $<$ $\sqrt{889}=\sqrt{29^2+48}$ $>$ $=29\frac{48}{58}$.

$2(D-a)+d=7$, $29\frac{48}{58}+7=36\frac{48}{58}=\frac{2136}{58}$. $<$ $2d=6$ and $\frac{2136}{58}\div 6=\frac{178}{29}=t_1$. $>$ $\frac{178}{29}=6+8'+16''+3'''+6\frac{6}{29}^{iv}$.

$t_1-1=\frac{178}{29}-1=\frac{149}{29}$, $(t_1-1)d=\frac{149}{29}\times 3=\frac{447}{29}$, $(t_1-1)d/2=\frac{447}{58}$, $(t_1-1)d/2+a=\frac{447}{58}+5=\frac{737}{58}$.

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[1] The history of this rule is of considerable interest. It was given by Heron (HEATH, Greek Math. ii, 324) who indicates the second approximation; and it was known to Planudes (xiiith century), Barlaam and Nicholas Rhabdas (xivth cent.) and to al-Qalasādī (xvth cent. See WOEPCKE, *Jour. Asiatique*, 1854, p. 384). It does not appear in any early Indian work but it is given by Sūryadāsa (xvith cent.), who attributes it to his father, Jnyāna Rāja. Colebrooke's translation (*Algebra, etc., from the Sanscrit*, p. 155) of this Indian version is as follows:—"The root of a near square, with the quotient of the proposed square divided by that approximate root, being halved, the moiety is an approximate root; and repeating the operation as often as necessary, the nearly exact root is found. Example 5. This divided by two which is first put for the root gives $\frac{5}{2}$ for the quotient, which added to the assumed root 2, makes $\frac{9}{2}$; and this divided by 2, yields $\frac{9}{4}$ for the approximate root." Sūryadāsa's approximation may thus be represented by $\dfrac{A+\frac{A^2+b}{A}}{2}$. Jnyāna Rāja's work on astronomy is entitled *Siddhānta Sundara*. See the reference to SUNDARI on folio 34 of our text

$s_1'' = ((t_1 - 1)d/2 + a)t_1 = \frac{737}{58} \times \frac{178}{29} = \frac{65,593}{841}$.

$s_1' = DT + Dt_1 = 7\ (5 + \frac{178}{29}) = \frac{65569}{841}$; and $s_1'' - s_1' = \frac{24}{841} = \langle \frac{48^2}{4 \times 841 \times 24} = \frac{b^2}{4A^2.8d}$ $\rangle$

Proof by the 'rule of three', $1 : 7 :: \frac{178}{29} : 42\frac{28}{29}$ $\langle$ and $42\frac{28}{29} + 35 = 77\frac{812}{841}$. $\rangle$

(iii) *Rule*. $t = \{-(2a - d) + \sqrt{(2a - d)^2 + 8ds}\} \div 2d$.

Example. $a = 1,\ d = 1,\ s = 60$.

Solution. $8ds = 480$, $2a - d = 1$, $480 + 1 = 481$, $\sqrt{481} \sim 21\frac{40}{42} = \frac{882 + 40}{42} = \frac{922}{42}$; $\langle$ and $t_1 = \frac{1}{2}(\frac{922}{42} - 1)$ $\rangle = \frac{880}{84}$.

$\langle$ Now $s_1 = t_1(t_1 + 1)/2$ $\rangle = \frac{880}{84} \times \frac{964}{168} = \frac{848,320}{14112}$.

$\langle$ But $s_1 - s = e_1/8d = \frac{1}{8}(\frac{40}{2.21})^2$ $\rangle = \frac{1600}{14112}$, and $\frac{848,320}{14,112} - \frac{1600}{14112} = \frac{846,720}{14112} = 60$.

$\langle$ Again $\sqrt{481} = \sqrt{(21\frac{20}{21})^2 - (\frac{20}{21})^2}$ $\rangle \sim 21\frac{20}{21} - \frac{(20/21)^2}{2 \times 21\frac{20}{21}} = \frac{425,042 - 400}{19,362} = \frac{424,642}{19,362}$, $\langle$ and $t_2 = (\frac{424,642}{19,362} - 1) \div 2$

$= \frac{405,280}{38,724}$; $\langle$ and $s_2 = ((t_2 - 1)\ d \div 2 + a)t_2 = t_2(t_2 + 1)/2$ $\rangle = \frac{405,280}{38,724} \times \frac{444,004}{77,448} = \frac{179,945,941,120}{2,999,096,352}$.

$\langle$ But $s_2 - s = e_2/8d = \frac{10^4}{8^2 \times 21^4 \times (21\frac{20}{21})^2}$ $\rangle = \frac{160,000}{2,999,096,352}$ and $\frac{179,945,941,120 - 160,000}{2,999,096,352} = \langle \frac{179,945,781,120}{2,999,096,352} = 60.$ $\rangle$

(iv) *Example*. $\langle$ $a = 1,\ d = 1,\ s = 5$; therefore $t = (-1 + \sqrt{41})/2$; $\rangle$ $\sqrt{41} \sim 6\frac{5}{12} = \frac{77}{12}$, and $t_1 = \frac{65}{24}$.

But $s_1 = ((\frac{65}{24} - 1)\frac{1}{2} + 1)\frac{65}{24} = (\frac{1}{2} \cdot \frac{41}{24} + 1)\frac{65}{24} = (\frac{41}{48} + 1)\frac{65}{24} = \frac{89}{48} \times \frac{65}{24} = \langle \frac{5785}{1152} = 5\frac{25}{1152} = 5 + \frac{e_1}{8d} \rangle$.

Again a second approximation to $\sqrt{41}$ is given by $6\frac{5}{12} - \frac{1}{2}(\frac{5}{12})^2/6\frac{5}{12} = \frac{11858}{1848} - \frac{25}{1848} = \frac{11833}{1848}$; and t_2

$= \frac{1}{2}(\frac{11833}{1848} - 1) = \frac{1}{2}(\frac{9985}{1848}) = \frac{9985}{3696}$......

(v) *Example*. $\langle$ $a = 1\frac{1}{2},\ d = 1\frac{1}{2},\ s = 2$; therefore $t = \{-\frac{3}{2} + \sqrt{(\frac{3}{2})^2 + 8.\frac{3}{2}.2}\} \div 3 = (-3 + \sqrt{105}) \div 6$.

Now $\sqrt{105} = 10\frac{1}{4}$ approximately, and a second approximation is given by $10\frac{1}{4} - \frac{1}{16}/2(10\frac{1}{4}) = 10\frac{81}{328}$.

With this value $t_2 = (-3 + 10\frac{81}{328})/6 = \frac{59425}{49200}$, and $s_2 = ((t_2 - 1)\frac{d}{2} + a)t_2 = ((\frac{59425}{49200} - 1)\frac{3}{4} + \frac{3}{2})t_2$ $\rangle$

$= (\frac{10225}{32800} \times \frac{1}{2} + \frac{3}{2})t_2 = (\frac{10225}{65600} + \frac{3}{2})t_2 = \frac{108625}{65600} \times \frac{59425}{49200} = \frac{6,455,040,625}{3,227,520,000}$. Now $s_2 - s = e_2/8d^* = \frac{625}{3,227,520,000}$

and $s = \frac{6,455,040,625 - 625}{3,227,520,000} = \frac{6,455,040,000}{3,227,520,000} = 2$.

(vi) *Example*. $a = 1\frac{1}{2}$, $d = 1\frac{1}{2}$, $s = 7000$; therefore $\langle t = (-3 + \sqrt{33609})/6$, and since $579^2 = 335,241 = 33,609 - 768$, we have as a first approximation to the root quantity $579\frac{384}{579}$, while the second approximation is $579\frac{384}{579} - (\frac{384}{579})^2 \div 2(579\frac{384}{579})$ $\rangle = 579\frac{384}{579} - (\frac{384}{579})^2 \times \frac{1158}{671,250} = 579\frac{768}{1158} - \frac{294,912}{777,307,500} = 579\frac{515,520,000 - 294,912}{777,307,500}$

$= 579\frac{515,225,088}{777,307,500} = \frac{450,576,267,588}{777,307,500}$. $\langle$ Therefore $t_2 = (\frac{450,576,267,588}{777,307,500} - 3) \div 6$ $\rangle = \frac{448,244,345,088}{4,663,845,000}$.

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[1] The Appendix to this Chapter (page 53) may be useful to those who are interested in the details of the calculations.

* Here instead of $d = 1\frac{1}{2}$ we must use $4.d = 6$, for we have really given a and d new values by cancelling, namely $a = 3$, $d = 3$ and e is four times greater than it should have been (since $e = q_2^2 - q^2$); otherwise since $q^2 = 26\frac{1}{4}$, $q_2 = \frac{3361}{16.41}$, $e = q_2^2 - q^2 = (\frac{3361}{16.41})^2 - 26\frac{1}{4} = \frac{1}{16^2.41^2}$; and $\frac{e}{8.d} = \frac{1}{3.4.16^2.41^2} = \frac{25^4}{3.4.41^2.(4.25)^4} = \frac{625}{3,227,520,000}$.

Now $s_3 = ((t_3 - 1)\frac{d}{2} + a)t_3$ and $t_3 - 1 = \frac{448,580,500,088}{4,063,845,000}$; $(t_3 - 1)d = \frac{221,790,250,044}{1,554,615,000}$; $(t_3 - 1)\frac{d}{2} = \frac{110,895,125,022}{1,554,615,000}$

$(t_3 - 1)d/2 + a = \frac{113,227,047,522}{1,554,615,000}$; and finally $s_3 = \frac{50,753,383,762,746,743,271,936}{7,250,483,394,675,000,000}$.

But $s_3 - s = e_3/8d = \frac{768^4}{6.8^3.579^4.(579\frac{4}{11})^3} = \frac{21,743,271,936}{7,250,483,394,675,000,000}$, which given $s = \frac{50,753,388,762,725,000,000,000}{7,250,483,394,675,000,000} = 7,000$.

Series.

87. There is a set of series that may be represented as follows, where T_1, T_2, T_3, etc., represent the successive terms of any of the series irrespective of their form.

	T_1	+ T_2	+ T_3	+ T_4	S
	x	$+ 2T_1$	$+ 3T_1$	$+ 4T_1$	$= 200$
	$x(1+\frac{3}{2})$	$+ 2T_1 + \frac{5}{2}x$	$+ 3T_1 + \frac{7}{2}x$	$+ 4T_1 + \frac{9}{2}x$	$= \frac{71}{2}$
	$x(1+\frac{3}{2})$	$+ 2T_1 - \frac{5}{2}x$	$+ 3T_1 - \frac{7}{2}x$	$+ 4T_1 - \frac{9}{2}x$	$\frac{29}{2}$
	x	$+ 2T_1$	$+ 3T_2$	$+ 4T_2$	$= 132$
	$x(1+\frac{3}{2})$	$+ 2T_1 + \frac{5}{2}x$	$+ 3T_2 + \frac{7}{2}x$	$+ 4T_2 + \frac{9}{2}x$	$144\frac{1}{2}$
Missing	$< x(1+\frac{3}{2})$	$+ 2T_1 - \frac{5}{2}x$	$+ 3T_2 - \frac{7}{2}x$	$+ 4T_3 - \frac{9}{2}x$	$= \frac{41}{2} >$
	x	$+ 2T_1$	$+ 3(T_1 + T_2)$	$+ 4(T_1 + T_2 + T_3)$	$= 300$
	$x(1+\frac{3}{2})$	$+ 2T_1 + \frac{5}{2}x$	$+ 3(T_1 + T_2) + \frac{7}{2}x$	$+ 4(T_1 + T_2 + T_3) + \frac{9}{2}x$	$= 222$
	$x(1+\frac{3}{2})$	$+ 2T_1 - \frac{5}{2}x$	$+ 3(T_1 + T_2) - \frac{7}{2}x$	$+ 4(T_1 + T_2 + T_3) - \frac{9}{2}x$	$= 78$

These are obviously built up on a definite plan. There are three fundamental series, namely

$$a_1 + 2a_1 + 3a_1 + \ldots + na_1$$

$$a_1 + 2a_1 + 3a_2 + \ldots + na_n ,$$

$$a_1 + 2a_1 + 3(a_1 + a_2) + \ldots + n(a_1 + a_2 + \ldots a_{n-1})$$

and a subsidiary series which may be represented by

$$0 + e + e(1 + d) + e(1 + 2d) + \ldots \text{etc.}$$

There is no attempt at general summation, which, indeed, is not generally possible, and I am afraid we can credit the author with little of mathematical value here. It may be noted that the sums of the series may be given as $10T_1 + 3(e + d)$, $33T_1 \pm (22e \pm 7d)$, $60T_1 \pm (26e + 7d)$, and that in all cases $e = T_1 = \frac{5}{2}$, and $d = \frac{2}{5}T_1$. The answers are therefore $25 \pm \frac{21}{2} = \frac{71}{2}$ or $\frac{29}{2}$, $\frac{156}{2} \pm \frac{124}{2} = \frac{280}{2}$ or $\frac{41}{2}$, and $\frac{800}{2} \pm \frac{144}{3} = 222$ or 78. Of these $\frac{41}{2}$ is missing.

88. There are other series of which the most notable is also a double series. It may be represented by

$$\left\{\begin{array}{l} a_1 + 3a_1 + 3^2a_1 \quad + 3^3a_1 \qquad\qquad + 3^4a_1 \\ 0 + \frac{1}{4}a_1 + \frac{2}{4}(a_1 + a_2) + \frac{3}{4}(a_1 + a_2 + a_3) + \frac{4}{4}(a_1 + a_2 + a_3 + a_4) \end{array}\right\} = 329,$$

whence the value of a_1 is 2.

There are other simple series on certain fragments, that, as they now stand, are not of particular interest.

89. There is a set of some 17 examples that may be represented by

$$C\ (1-a_1)(1-a_2)\ .\ .\ .\ .\ .\ .\ .\ (1-a_n)=R=C-L$$

where R is the 'remainder' and L the 'loss' after successive deductions from C. The values of *a* are generally simple fractions with unity as numerator, and if $(1-a_1)\ .\ .\ .\ .\ (1-a_n)=p/q$ then $Cp/q=R=C-L$ whence $C=\frac{Rq}{p}$ and $C=L/(1-\frac{p}{q})$. The following are specimens

$$4(1-\tfrac{1}{4})(1-\tfrac{1}{4})(1-\tfrac{1}{4})(1-\tfrac{1}{4})=x$$

$$8(1-\tfrac{1}{2})(1-\tfrac{1}{3})(1-\tfrac{1}{4})=x$$

$$60(1-\tfrac{1}{2})(1-\tfrac{1}{3})(1-\tfrac{1}{4})(1-\tfrac{1}{5})=x$$

$$x(1-\tfrac{1}{3})(1-\tfrac{1}{4})(1-\tfrac{1}{5})=x-24$$

$$x(1-\tfrac{1}{3})(1-\tfrac{1}{4})(1-\tfrac{1}{5})=x-280$$

$$x(1-\tfrac{1}{3})(1-\tfrac{1}{4})(1-\tfrac{1}{5})=2x-32$$

The following questions are somewhat restored.

(i) A traveller takes on a journey a bottle containing four *prasthas* of wine. At the end of each stage he drinks one *prastha* and then fills up his bottle with water. How much wine and how much water will there be in the bottle after four stages?

The solution is first given by

$4(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})=1\frac{17}{64}$ *prasthas*, the amount of wine, and $2\frac{47}{64}$ *prasthas* as the amount of water. Secondly it is worked out by steps.

(ii) A tax is paid at the rate of one-third, one-fourth and one-fifth on three separate occasions and the total amount paid in taxes is twenty-four. What was the original amount.[1]

The solution given is
$x\ (1-\frac{1}{3})\ (1-\frac{1}{4})\ (1-\frac{1}{5})=x-24$ and $x=40$.

This section is interesting[2] on account of the numerous proofs or verifications of the calculations, which have been referred to in the previous Chapter (§ 74).

Computation of gold.

90. This topic is treated in most mediæval mathematical works and our text follows the usual treatment but with some slight irregularity due, possibly, to a slight misconception of the author, who apparently looked upon the 'quality' or 'touch' factor as a negative quantity.

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[1] See Bhaskara's *Līlāvatī*, 73.

[2] It is also interesting because of the occurrence of a similar section in a papyrus of Akhmim (Heath, *Greek Math.* ii, 544; but Heath's reference is misleading and so is Cantor's: the papyrus of Akhmim was published by J. Baillet, *Mémoires du Caire*, IX, 1892, pp. 1-88).

The fundamental theorem is, of course, the usual one of averages, namely :—

(i) $x = \frac{f_1w_1+f_2w_2+\ldots\ldots f_nw_n}{w_1+w_2+\ldots w_n}$

where w_r is the mass of a piece of gold, and f_r is its quality or 'touch.' That is f is the amount of pure gold per unit in some unknown measures. There is some indication in the text that *f* is expressed in *māshakas* and *w* in *suvarṇas*, where 12 *māshakas* = 1 *suvarṇa* but this is not supported by Bhāskara or Mahāvīra, and indeed the money measures in our text are nowhere well defined.

Modifications of (i) that are indicated are

(ii) $\frac{f_1w_1+f_2w_2+\ldots\ldots x.w_n}{w_1+w_2+\ldots\ldots w_n} = F$

(iii) $\frac{f_1w_1+f_2w_2+\ldots\ldots f_nw_n}{w_1+w_2+\ldots\ldots x} = F$

(iv) $\frac{(f_1w_1+\ldots\ldots f_nw_n)+f'_1w'_1+\ldots\ldots f'_nw'_n}{w_1+w_2+\ldots\ldots w_n} = x$ which is not understood.

Earning and spending.

91. (i) A earns e_1 in d_1 days ; B earns e_2 in d_2 days. If A gives g to B when will they have equal amounts ?

$$t = 2g \div \left(\frac{e_1}{d_1} - \frac{e_2}{d_2}\right)$$

(ii) A earns e in d_1 days and spends *f* in d_2 days. How long will his capital *C* last ?

$$t = c/\left(\frac{f}{d_1} - \frac{e}{d_2}\right)$$

(iii) One gives a_1 *dīnāras* in d_1 days ; a second gives a_2 *dīnāras* in d_2 days ; and so on. In what time will they have given b_4 *dīnāras* altogether ?

In one day the total gift is $\frac{a_1}{d_1} + \frac{a_2}{d_2} + \ldots\ldots + \frac{a_n}{d_n} = \frac{P}{Q}$ and therefore *b dīnāras* are given in $\frac{bQ}{P}$ days and the respective gifts are $\frac{a_1bQ}{d_1P}$, $\frac{a_2bQ}{d_2P}$, $\frac{a_3bQ}{d_3P}$, &c.

The following occurs—

In $1\frac{1}{2}$ days one gives $2\frac{1}{2}$ *dīnāras*, in $1\frac{1}{3}$ days a second gives $3\frac{1}{2}$ *dīnāras*, and in $1\frac{1}{4}$ days a third gives $4\frac{1}{2}$ *dīnāras*. In what time will they have given 500 *dīnāras* altogether ?

Since $\frac{2\frac{1}{2}}{1\frac{1}{2}} + \frac{3\frac{1}{2}}{1\frac{1}{3}} + \frac{4\frac{1}{2}}{1\frac{1}{4}} = \frac{947}{120}$ *dīnāras* are given in 1 day, 500 *dīnāras* will be given in $500/\frac{947}{120} = \frac{60,000}{947} = 63\frac{339}{947}$ days.

Profit and Loss.

92. Let C be the capital, p the profit, M = C + p, n = Cc the number of articles, c the cost rate and s the sale rate [c = a/b where a is the number purchased for b

drammas, say, but the money measure is not generally stated; and $s = \frac{c}{d}$ where c is the number sold for d.]

The following rules are given with illustrative examples.

(i) $C = \frac{p}{c/s - 1}$ (ii) $p = \frac{Cc}{s} - C$

(iii) $C = \frac{M}{c/s} = \frac{C + p}{c/s}$ (iv) $C = \frac{-p}{1 - c/s}$

The 'M' Section.

93. The examples that belong to the so-called 'M section' may be described as miscellaneous, but most of them are solved by 'the rule of three terms.' Their chief interest is hardly mathematical although certain of them exhibit in very interesting forms the change-ratios of certain measures; it lies rather in what may be termed the social nature of the formal questions (see §§ 45-52). The translations of the questions here offered are 'informal' and sometimes very much 'restored'.

94. An army consists of chariots, elephants, horse and foot in the ratios 1 : 1 : 5 : 3 : If a complete army contains 10×3^7 of these how many of each kind are there?

The answers are

$10 \cdot 3^7 = 21{,}870$ chariots.

$10 \cdot 3^7 = 21{,}870$ elephants.

$10 \cdot 3^7 \cdot 5 = 109{,}350$ foot.

$10 \cdot 3^7 \cdot 3 = 65{,}610$ horse.
$(10^2 \cdot 3^7) = 218{,}700$ (1 akshauhiṇī).

This is the traditional subdivision of a Hindu army. There was probably more information given in the text for the terms *akshauhiṇī*, *anīkinī*, *chamū* and *pṛitanā* occur. According to the dictionaries 1 *akshauhiṇī* consists of 10 *anīkinīs* or 218,700 in all; a *chamū* consists of 129 chariots, 129 elephants, 2,187 horse, 3,645 foot, or 6,090 (*i.e.*, $3 \times 43 + 3 \times 43 + 3^7 + 3^6 \times 5 = 10 \times 3 \times 7 \times 29$); while a *pṛitanā* is said to consist of $243 + 243 + 729 + 1215 = 2430$ (or $3^5 + 3^5 + 3^6 + 5 \times 3^5 = 10 \times 3^5$). See also Albīrūnī's *India*, chap. xlviii.

95. One produces ten and a half in two and one-third days. For the sake of religion he gives thirteen and one-third in three and one-eighth days. He offers to Vāsudeva one quarter less than thirteen in eight and a half days. Desiring reward in a future world he gives to Brāhmans for food one and one-third in three and one-fifth days (. . . $2\frac{1}{4}$ in 5 days . . .) and also twelve and a half in thirty-three and one-third days for the best wine for the consumption of the merchants. In the treasure house is stored twelve hundred. Say, O Pandit, how long can this expenditure continue?

The daily income is $\frac{10\frac{1}{2}}{2\frac{1}{3}} = \frac{9}{2}$ and the daily expenditure is $\frac{13\frac{1}{3}}{3\frac{1}{8}} + \frac{12\frac{3}{4}}{8\frac{1}{2}} + \frac{1\frac{1}{3}}{3\frac{1}{5}} + \frac{1}{1\frac{1}{4}} + \frac{1}{3\frac{1}{4}} + \frac{2\frac{1}{4}}{5} + \frac{12\frac{1}{2}}{33\frac{1}{3}} = \frac{1807}{240}$. The daily loss is therefore $\frac{1807}{240} - \frac{9}{2} = \frac{727}{240}$ and $\frac{727}{240} : 1 : : 1200 : x$ gives the period, and $x = \frac{1200}{727/240} \times \frac{1}{360}$ years $= \frac{800}{727}$ years.

A proof is given in the following way $2\frac{1}{3} : 10\frac{1}{2} : : \frac{800}{727} \times 360$ days: x, and $x = 1782\frac{486}{727}$ which is the total amount earned. Also 1 day: $\frac{1807}{240} : : \frac{800}{727} \times 360 : x^1$, and $x^1 = 2982\frac{486}{727}$. which is the total amount spent; and $1782\frac{486}{727} + 1200 = 2982\frac{486}{727}$.

96. **A certain person earns one and a half in one and one-third days and gives away eight in five and one-third days for Bha(vani), one in thirty-two days for pa(raloka), and as an offering to Śolin one quarter of two and a half in thirty-six days. If he already possesses seven hundred when will it all be consumed?**[1]

The daily income is $1\frac{1}{2} \div 1\frac{1}{3} = \frac{9}{8}$, **and the gross daily expenditure is** $\frac{8}{5\frac{1}{3}} + \frac{1}{32} + \frac{2\frac{1}{2}}{4 \times 36} = \frac{228}{144}$. **The net expenditure is therefore** $\frac{228}{144} - \frac{9}{8} = \frac{61}{144}$, **and** $\frac{61}{144} : 1$ **day : :** $700 : x$, **whence** $x = \frac{700 \times 144}{61 \times 360}$ **years** $= \frac{280}{61}$ **years = 4 years 7 months** $2\frac{8}{61}$ **days.**

A proof is given thus:—1 day : $\frac{228}{144} : : \frac{280 \times 360}{61} : 2559\frac{1}{61}$ **and this is the gross expenditure. Again: 1 day :** $\frac{9}{8} : : \frac{280 \times 360}{61} : 1859\frac{1}{61}$**, which is the total income; and** $700 + 1859\frac{1}{61} = 2559\frac{1}{61}$. **Further** $\frac{280}{61}$ **years :** $2559\frac{1}{61} : : \frac{1}{360} : \frac{228}{144}$**, which is the daily expenditure.**

97. **A boat**[2] **goes one-half of a third of a** ***yojana*** **plus one-third less one quarter in one-half of one-third of a day, but then it is driven back by the wind one-half of one-fifth of a** ***yojana*** **in one-eighth of three days. In what time will it travel one hundred and eight** ***yojanas?***

The details of the question and the solution are not clear.

98. **A snake eighteen** ***hastas*** **long enters its hole at the rate of one-half plus one-ninth of that** ***aṅgulas*** **less one twenty-first part daily. In what time will it have completely entered the hole?**

Since 24 ***aṅgulas*** **= 1** ***hasta*** **we have** $\frac{1}{2} + \frac{1}{9}$ **of** $\frac{1}{2} - \frac{1}{21} : \frac{1}{360} : : 18 \times 24 : x$, **and** $x = \frac{189}{80} = 2$ **years 4 months** $10\frac{1}{2}$ **days.**

99. **A snake gets out of its skin as quickly as possible at the rate of half an** ***aṅgula*** **in one day. Its length is 100** ***yojana*****, 6** ***krośa*****, 3** ***hasta*** **plus 5** ***aṅgulas.***[3] **In what time will it be free?**

$\frac{1}{2}$ **aṅgula : 1 day : : 100** ***yo.*** **+6** ***kro.*** **+3** ***ha.*** **+5** ***aṅg.*** **: x or** $\frac{1}{2}$ ***aṅgula*** **: 1 day : : 77,376,077** ***aṅg.*** **: x, and x = 154,752,154 days = 429867 years, 1 month and 4 days.**

100. **On folio 37 are three very simple astronomical problems:**

(i) **Bhanu (The Sun) travels 500,000,000** ***yojanas***[4] **in one day. State with certainty the amount of progress in one** ***ghaṭika?***

Since 60 ***ghaṭika*** **= 1 day (of 24 hours) we have**

$$60 \text{ gha.} : 500{,}000{,}000 \text{ yo.} : : 1 \text{ gha.} : x \text{ and } x = 8{,}333{,}333\tfrac{1}{3} \text{ yojana}.$$

(ii) **The Sun's chariot is guided by the god Mahoraga among the Siddhas and Vidyādharas. The clever scientist says that, according to the general rule, it travels half a hundred** ***koṭi***[5] **in a day and night. Tell me, O best of calculators, what it will go in one** ***muhūrta?***

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[1] The wording of the example is 'restored' on the basis of certain indications given in the solution preserved, *e.g.*, *Sū°* may stand for Śolin. The numerical portions are certain.

[2] Mahāvīra (V, 23-31) gives a small section dealing with problems of 'forward and backward movement', which he illustrates by a boat problem and a snake problem amongst others. His boat problem is:—

In the course of $\frac{3}{5}$ of a day a boat traverses $\frac{1}{8}$ of a *krośa* of the ocean, but owing to adverse winds it loses $\frac{1}{4}$ of a *krośa*. State in what time it will have advanced $99\frac{1}{2}$ *yojana*, thou who hast powerful arms in crossing easily the ocean of numbers. (*Answer*: 5 years 117 days.)

The snake problem is:—

A powerful, unvanquished, excellent, black snake, 32 *hastas* long enters a hole at the rate of $7\frac{1}{2}$ *aṅgulas* in $\frac{5}{14}$ of a day; and in the course of $\frac{1}{4}$ of a day its tail grows by $2\frac{3}{4}$ of an *aṅgula*. O ornament of arithmeticians, tell me by what time it enters fully into the hole. (*Answer*: $76\frac{4}{5}$ days.)

[3] The snake is here nearly 1,000 miles long! This may have been a modest estimate of the size of the serpent Śesha, who supports the world, causes earthquakes whenever he yawns, and occasionally destroys the universe. (See the *Vishṇu Purāṇa*, ii, v, etc.) But Mr. Hargreaves suggests that the example refers to the Nāga Elāpattra, whose head was at Benares and whose tail reached to Taxila—a distance of just about 100 *yojanas*.

[4] An orbit of 5.10^8 *yojanas* or a mean radius of nearly 80 million *yojanas* or, say, 727 million miles. What is the origin of this?

[5] A *koṭi* = 10^7.

Since 30 muhûrta = 1 day (of 24 hours) we have

30 *mu.* : 500,000,000 : : 1 *mu.* : x and x = 16,666,666⅔ *yojana.*

(iii) If Bhanujā (Saturn) traverses one sign in two and a half years, tell me, thou who art thoroughly learned, how far he will go in one solar[1] day.

Since 1 sign = 30° = 108000″ we have

2½ years : 108,000″ : : $\frac{1}{360}$: x and x = 120″ or 2 minutes of arc.[2]

101. The following examples[3] seem to form a set but the text is mutilated and only the bare skeletons of the problems remain:

(i) 1 *tola* : 5⅓ years : : 1 *to* + 1 *dhā* + 1 *aṁ* + 1 *ya* + 1 *si* + 1 *ka* + 1 *mū* : x, or 1 to : 5⅓ years : : $\frac{21681}{19200}$ to : x and x = 6 years $8\frac{1}{10}$ days.

(ii) 1 *to* : 35 *drammas* : : 1½ *to* + 1½ *mā* + 1½ *ani* + 1½ *ya* : x, or 1 : 35 : : $\frac{819\frac{1}{2}}{192}$: x, and x = $58\frac{31}{128}$ *drammas.*

(iii) 1 *to* : 400 *dīnāra* : : 1 *dha* + 1 *aṁ* + 1 *ya* + 1 *ka* + 1 *pā* + 1 *mū* : x, or 12 *dhā* : 400 *dī* : : $\frac{2441}{1600}$ *dhā* : x, and x = $50\frac{11}{48}$ *dīnāras* = 50 *dī* + 10 *dhā* + 1 *ani.*

(iv) 1 day : (3 to + 2 mā + 3 ani + 3 ya + 1 ka + 1 pā + 1 mū) − (4 ra + 4 si) : : 25 years 5 months 20 days : x, or 1 day : : $\frac{62321}{19200}$ tola − $\frac{1440}{19200}$ tola : : 9170 days : x, and x = $\frac{558,278,770}{19200}$ tola = 1 *bhā* + 1634 *pā* + 5 *to* + 0 *mā* + 0 *ani* + 3 *ya* + 4 *ka* + 1 *pā* + 2 *mū.*

---

[1] See my *Hindu Astronomy*, p. 57.

[2] It should be about 122″

[3] The importance of these examples lies in the use of numerous measures and in the methods of expressing change-ratios. The subject of measures is discussed in chapter VII.

APPENDIX TO CHAPTER VI.

(Aids to the calculations.)

FACTORS.

$384=2^7.3$

$441=3^2.7^2$

$481=13.37$

$492=2^2.3.41$

$576=2^6.3^2$

$579=3.193$

$621=3^3.23$

$656=2^4.41$

$737=11.67$

$768=2^8.3$

$841=29^2$

$889=7.127$

$1024=2^{10}$

$1158=2.3.193$

$1681=41^2$

$3696=2^4.3.7.11$

$5785=5.13.89$

$9681=3.7.461$

$9985=5.1997$

$10225=5^2.409$

$11858=2.7^2.11^2$

$14112=2^5.3^2.7^2$

$19362=2.3.7.461$

$38724=2^2.3.7.461$

$40528=2^4.17.149$

$59425=5^2.2377$

$65593=11.67.89$

$67125=3.5^3.179$

$84672=2^6.3^3.7^2$

$84832=2^5.11.241$

$107\ 584=2^6.41^2$

$108\ 625=5^3.11.79$

$112\ 003=31.3613$

$322\ 752=3.2^6.41^2$

$335\ 241=3^2.193^2$

$336\ 009=3.31.3613$

$425\ 042=2.461^2$

$444\ 004=2^2.11.10091$

$645\ 504=2^7.3.41^2$

$1\ 554\ 615=3^2.5.179.193$

$4\ 663\ 845=3^3.5.179.193$

$7\ 773\ 075=3^2.5^2.179.193$

$515\ 225\ 088=2^9.3^2.7.15973$

$2\ 999\ 096\ 352=2^5.3^2.7^2.461^2$

$6\ 455\ 042\ 625=5^5.11.79.2377$

$17\ 994\ 594\ 112=2^6.11.17.149.10091$

$21\ 743\ 271\ 936=2^{28}.3^4=768^4/2^4$

$113\ 227\ 047\ 522=2.3^2\ (6\ 290\ 391\ 529)=?$

$221\ 790\ 250\ 044=2^2.3^2.11.560{,}076{,}389$

$347\ 892\ 350\ 976=2^{32}.3^4=768^4$

$7\ 250\ 483\ 394\ 675=3^5.5^2.179^2.193^2$

$50\ 753\ 383\ 762\ 725=3^5.5^2.7.179^2.193^2$

$448\ 244\ 345\ 088=2^8.3^2\ (194\ 550\ 497)$

$50\ 753\ 383\ 762\ 746\ 743\ 271\ 936=113\ 227\ 047\ 522\times 448\ 244\ 345\ 088$

$=2^9.3^4.\ (6,\ 290,\ 391,\ 529)\ (194,\ 550,\ 497)$

$=2^9.3^4\ (2^{19}+3.5^{11}.7.179^2.193^2)$

Unresolved 6 290 391 529 and 194 550 497

SQUARES.

$21^2=441$

$24^2=576$

$29^2=841$

$41^2=1681$

$77^2=5929$

$179^2=32041$

$193^2=37249$

$461^2=212521$

$579^2=335241$

$768^2=589824$

$3361^2=11296321$

Powers of 2, 3 and 5

$2^4=16$ $2^5=32$ $2^6=64$ $2^7=128$ $2^8=256$ $2^9=512$ $2^{10}=1024$ $2^{28}=268\ 435\ 456$ $2^{30}=1073\ 741\ 824$ $2^{32}=4294\ 967\ 296$

$3^4=81$ $3^5=243$ $3^6=729$ $3^7=2187$; $5^4=625$ $5^5=3125$ $5^6=15625$

Prime numbers greater than 100

127 149 179 193 241 409 461 1997 2377 3361 3613 10091 15973

CHAPTER VII.

MEASURES.

102. The measures exhibited in the manuscript are of rather special interest. As a whole they are Indian and the terminology is Sanskrit ; but there are some Sanskritised western terms such as *liptā*, *dramma*, *dīnāra*, *satera* employed. Most of the terms are well defined but the values of some are doubtful. Money measures, however, are, as in most early Indian works, very ill defined and hardly show any differentiation from measures of weight.

103. *Change ratios.** The change ratios are often given with considerable care and elaboration, and are expressed in several different ways. The change ratio appears to be considered as a divisor for it is most frequently marked by the term *chhedam*† which indicates the operation of division.

Examples are

chhe° 80 *rakti°-su°* i.e. 80 *raktikā* = 1 *suvaana*¶

chhe° 24 *aṁ°-ha°* i.e. 24 *aṅgula* = 1 *hasta*

chhe° 2 *gha°-mu°* i.e. 2 *ghaṭika* = 1 *muhūrta*

chhe° 4608000 *ya°-yo°* i.e. 4,608,000 *yava* = 1 *yojana*

ūrdha‡ *chchhe°* 768000 *a°*-yo°, i.e. 768,000 *aṅgula* = 1 *yojana*, and in this particular example the operation of multiplication is to be performed.

ūrdha chchhedaṁ 108000 *viliptānām rāśi*, i.e. 108000 viliptā = 1 *rāśi* and multiplication is indicated.

adha chchhedaṁ 2000 *pa°-bhā°*, i.e. 2000 pala = 1 *bhāra* and division is indicated.

Another form is illustrated in the following examples

tolenāsti dhāne 12, i.e. 1 *tola* = 12 *dhāna*.

dhāṇenāsti aṁ° 4, i.e. 1 *dhāna* = *aṇḍikā*

dīnāranāsti dhāne 12, i.e. 1 *dīnāra* = 12 *dhāna*

* By a *change ratio* I mean a number by which one denomination is multiplied or divided to change it to another. Thus, since 20 shillings = 1 pound, 20 is the *change ratio* between pounds and shillings. It is a *multiplier* or a *divisor* according to the direction of change.

† Often abbreviated *chhe.*

‡ Dict. *ūrdhva.*

¶ Or 'division by 80 changes *raktikas* to *suvarnas*.'

104. In tabulated examples the methods of expressing the change ratios are modified. In two cases sets of cumulative ratios are given thus

to°	1	dhā°	1	1*	aṁ°	1	1*	ra°	1	1*	ya°	1	1*	si°	1	1*
	1		1	12		1	48		1	60		1	192		1	480

ka°	1	1*	pā°	1	1*
	1	1200		1	4800

which means primarily

1 *to°* + 1 *dhā°* + 1 *aṁ°* + 1 *ra°* + 1 *ya°* + 1 *si°* + 1 *ka°* + 1 *pa°*

while the other numbers are simply explanatory and mean that 1 *dhā°* = $\frac{1}{12}$ *to°*, 1 *aṁ°* = $\frac{1}{48}$ *to°*, ... 1 *pā°* = $\frac{1}{4800}$ *to.*

A similar table is given without naming the terms in the form

1	1	1*	1	1*	1	1*
1	1	12	1	48	1	192
2	2		2		2	

which means

1½ *to°* + 1½ *dhā°* + 1½ *aṁ°* + 1½ *ya°* and that 1 *dhā°* = $\frac{1}{12}$ *to°*, 1 *aṁ°* = $\frac{1}{48}$ *to°*, 1 *ya°* = $\frac{1}{192}$ *to°*.

Both of the sets given above are also tabulated in a different manner, e.g.—

to	1	
dhā	1	
	12*	
aṁ	1	
	4	
ra	1	1* bhā
		1
		4
ya	1	3* bhā
		1
		5
si	1	2* bhā
		1
		2
ka	1	2* bhā
		1
		2
pā	1	

This again primarily means 1 *to°* + 1 *dhā* + 1 *aṁ* + 1 *ra* + 1 *ya* + 1 *si* + 1 *ka* + 1 *pā* and the other numbers are simply change ratios and indicate that 12 dhā = 1 to°, 4 aṁ = 1 dhā, 1¼ ra = 1 aṁ, 3 $\frac{1}{5}$ ya = 1 ra, 2½ si = 1 ya, 2½ ka = 1 si.

Note that where the change ratios are fractional they are written at the side, obviously to prevent confusion.

Examples of the type just given occur on folios 7 recto, 48 recto 48 verso, 49 verso.

105. A variation of the above scheme in which the charge ratios are *halved* occurs thus

1 1 2		to
1 1 2	1* 6	mā
1 1 2	1* 2	aṁ
1 1 2	1* 2	ya

which means $1\frac{1}{2}$ *to*° + $1\frac{1}{2}$ *mā*° + $1\frac{1}{2}$ *aṁ*° + $1\frac{1}{2}$ *ya*°. The notion of halving the change ratios seems to be like this : $\frac{3}{2}$ *to*° multiplied by $\frac{1}{6} = \frac{3}{12}$ *to*° = 3 mā and so on. Another example of this practice occurs on folio 20 recto and gave considerable trouble. It is written thus

1 3			
1 2	1 2	chhe	1* 2
1 4		chhe	1* 4

and means $\frac{1}{3}\ a + \frac{1}{2}\cdot\frac{1}{2}\ b + \frac{1}{4}\ c$. Until the change ratios between a, b and c could be found the problem was insoluble. The real change ratios are given by $\frac{1}{2}\cdot\frac{1}{3}\ a = b,\ \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{2}\ b = c$.

106. Before proceeding to discuss the several kinds of measures that occur a summary table of them is given for reference. The names of the measures are nearly always abbreviated in the manner indicated : the full terms are

Adh	=	Ādhaka	Guṁ	=	Guñja	Pra	=	Prastha
Aṁ	=	Aṁśa	Ha	=	Hasta	Pra	=	Prasṛiti
Aṁ	=	Aṇḍikā	Kā	=	Kākiṇi	Ra	=	Raktikā
Aṁ	=	Aṅgula	Ka	=	Kalā	Rā	=	Rāśi
Bhā	=	Bhāra	Khā	=	Khāri	Sa	=	Satera
Dhā	=	Dhānakā	Kro	=	Krośa	Si	=	Siddhārtha
Dha	=	Dhanus	Ku	=	Kuḍava	Su	=	Suvarṇa
Di	=	Dina	Li	=	Lipta	To	=	Tola
Dī	=	Dināra	Mā	=	Māsa	Va	=	Varsha
Dram	=	Dramma	Mā	=	Māshaka	Vi	=	Viliptā
Dra	=	Draṅkshaṇa	Mū	=	Mūdrikā	Ya	=	Yava
Dro	=	Droṇa	Mu	=	Muhūrta	Yo	=	Yojana
Gav	=	Gavyūti	Pā	=	Pāda			
Gha	=	Ghaṭika	Pa	=	Pala			

The Bakhshālī measures.

Time.

	gha	mu	di	mā	va	
gha	1					=24 min.
mu	2	1				=48 „
di	60	30	1			=24 hr.
mā			30	1		
va			360	12	1	

Arc.

	vi	li	aṃ	rā	
vi	1				=1"
li	60	1			=1'
aṃ	3,600	60	1		=1°
rā	108,000	1,800	30	1	=1 sign

Money.

	ra	dhā*	dra	di	su
ra	1				
dhā	5	1			
dra	30	6	1		
di	60	12	2	1	
su	80	16	$2\frac{2}{3}$	$1\frac{1}{3}$	1

* Also 4 *aṁ*=1 *dhā* (49r.)

Weight.

	mū	pā	ka	si	ya	ra	aṅ	dhā	dra	to	pa	bhā
mū	1											
pā	4	1										
ka	16	4	1									
si	40	10	$2\frac{1}{2}$	1								
ya	100	25	$6\frac{1}{4}$	$2\frac{1}{2}$	1							
ra	320	80	20	8	$3\frac{1}{5}$	1						
aṅ	400	100	25	10	4	$1\frac{1}{4}$	1					
dhā	1,600	400	100	40	16	5	4	1				
dra					96			6	1			
to	19,200	4,800	1,200	480	192	60	48	12	2	1		
pa	153,600	38,400	9,600	3,840	1.536	480	384	96	16	8	1	
bhā											2,000	1

Length.

	ya	aṅ	ha	dha	kro	ga	yo	
ya	1							= ·12 in.
aṅ	6	1						= ·75 in.
ha	144	24	1					=18·00 in.
dha	576	96	4	1				6 feet.
kro	576,000	96,000	4,000	1,000	1			1·14 m.
ga	2,304.000	384,000	16,000	4,000	8	1		4·52 m.
yo	4,608,000	768,000	32,000	8,000	8	2	1	

Capacity.

	pa	pra	ku	pra	āḍ	dro	khā
pa	1						
pra	2	1					
ku	4	2	1				
pra	32	8	4	1			
āḍ	128	32	16	4	1		
dro	256	128	64	16	4	1	
khā	4,096	2,048	1,024	256	64	16	1

Measures of Time.

107. The measures of time used in the text are the usual practical measures employed in India and there is nothing in any way remarkable about them. The measures found in early Hindu texts are given below.[1] All the authorities agree on one point, namely in dividing the day (24 hours) into 30 *muhūrtas*. The half-*muhūrta* was possibly introduced for astronomical purposes, as was the *vinadi* of the *Sūrya Siddhānta*. Also it is noteworthy that the tables give neither hours nor weeks, although the 'hour' was used for astrological purposes and the 'week' came into general use in the early centuries of the present era. Neither of these measures occurs in our manuscript but in this matter the text is quite orthodox.

The measures of time employed therefore call for little comment, but Dr. Hoernle suggested[2] that the year of 360 days might be an indication of the age of the work! He could hardly have been aware that it was the common practice to give this value in Hindu arithmetical text-books.[3] Śrīdhara, indeed, remarks: "Time is calculated according to this rule in all mathematical works."

Measures of Arc.

The measures of arc, which with the early Hindus were lengths rather than angles,[4] are such as occur in all mediæval Hindu astronomies. The term *liptā* is a Sanskritised form of λεπτή.

[1] In the sections that follow I have used with considerable freedom the chapter on 'Weights and Measures' given in L. D. Barnett's *Antiquities of India*, which is the most useful contribution to the subject (of those at my command) since that given in [illegible]'s *India*.

[2] Indian Antiquary, xvii, 1888, p. 37.

[3] See Mahāvīra and Śrīdhara.

[4] See my Hindu Astronomy, p. 54.

Hindu Measures of Time.

Manu.

Nimesha .	1					·2 sec.
Kāshṭhā .	18	1				3·2 ,,
Kalā . .		30	1			1·6 min.
Muhūrta .		900	30	1		48 ,,
Aho-rātra .		27,000	900	30	1	24 hrs.

Vishṇu Purāṇa.

Nimesha . .	1					·21 sec.
Kāshṭhā . .	15	1				3·2 ,,
Kalā . .		30	1			1·6 min.
Muhūrta .		900	30	1		48 ,,
Day . .		27,000	900	30	1	24 hrs.

The Kauṭiliya.

Truṭis . .	1								·06 sec.
Lava . .	2	1							·12 ,,
Nimesha .		2	1						·24 ,,
Kāshṭhā .			5	1					1·2 ,,
Kalā . .				30	1				36 ,,
Nālikā . .					40	1			24 min.
Muhūrta .						2	1		48 ,,
Day . .						60	30	1	24 hrs.

Sūrya Siddhānta.

Gurvakshara .	1					·4 sec.
Prāṇa . .	10	1				4 ,,
Vināḍī . .	60	6	1			24 ,,
Nāḍī . .	3,600	360	60	1		24 min.
Day . .			3,600	60	1	24 hrs.

Mahāvīra.

Uchchhvāsa	1											= ·75 sec.
Stoka	7	1										=5·3 ,,
Lava		7	1									=37½ ,,
Ghaṭī			38½	1								=24 min.
Muhūrta				2	1							=48 ,,
Day					30	1						=24 hrs.
Paksha						15	1					=15 days.
Month						30	2	1				=30 ,,
Ṛitu						60	4	2	1			=60 ,,
Ayana						180	12	6	3	1		=180 ,,
Year						360	24	12	6	2	1	=360 ,,

Śrīdhara.

Ghaṭī . .	1			
Day . .	60	1		
Month . .	1,800	30	1	
Year . .	21,600	360	12	1

Bhāskara.

Truṭis . .	1								·00003 sec.
Tatpara . .	100	1							·003 ,,
Nimesha . .	3,000	30	1						·1 ,,
Kāshṭhā . .	54,000		18	1					1·6 ,,
Kalā . .	$162 \cdot 10^4$			30	1				48 ,,
Ghaṭikā . .	$486 \cdot 10^5$				30	1			24 min.
Kshaṇa . .	$972 \cdot 10^5$					2	1		48 ,,
Day . .	$2{,}916 \cdot 10^6$					60	30	1	24 hrs.

Bakhshālī.

	Gha	Mu	Di	Mā	Va
Gha	1				
Mu	2	1			
Di	60	30	1		
Mā			30	1	
Va			360	12	1

Measures of Length.

108. (*a*) Although individual measures of length occur pretty often there are few examples in which the change ratios are exhibited. The following (fol. 32, verso) is the most complete of such examples :

100	urdha chchhe	768000*	a° – yo°
6		1	
8*			
3			
4000*			
5			
24*			

which means 100 *yojanas* + 6 (*krośas*) + 3 *hastas* + 5 *aṅgulas*, the numbers marked with asterisks being change ratios. These numbers inform us that 24 *aṅgulas* = 1 *hasta*, 4000 *hastas* = 1 *krośa*, 8 *krośas* = 1 *yojana*, and that to reduce *yojanas* to *aṅgulas* the former must be multiplied by 768,000. The term *krośa* does not actually occur, but it, or an equivalent, must have been mentioned in the question to which the table was attached.

In another statement (fol. 32, recto) the equation 8,000 *dhanus* = 1 *yojana* is given, hence 1,000 *dhanus* = 1 *krośa*. This equation occurs in Hindu works but most of the texts give 2,000 *dhanus* = 1 *krośa*.

In another example (fol. 36) the statement

chhe 4,608,000 *ya° – yo°*

indicates that 4,608,000 *yavas* = 1 *yojana*, whereas Mahāvīra and Bhāskara give 6,144,000 *yavas* = 1 *yojana*. The difference in the change ratios is noteworthy.

The measures *gadyūti* and *yojana* are connected (fol. 12), but unfortunately the actual change ratio is missing. The term *gavyūti* is not common in Hindu works but it occurs in the *Mārkaṇḍeya Purāṇa, the Mahābhārata*, etc. Its value is variously given as 4000, or 2000, or 200 *dhanus*.

Of the Indian tables given below the last is what the *Sūrya Siddhānta* designates *a-mūrta* or 'unreal.' It was never used seriously but it is interesting as exhibiting the constant change ratio of 'eight.'

108. (*b*) Possibly a measure of area is referred to on folio 39' verso where a change ratio of 64 is mentioned ; and possibly *bhu* on fol. 26 also indicates an area.

Indian Measures of Length.

Mārkaṇḍeya Purāṇa.

	Ya.	Aṅ.	Pa.	Vi.	Ha.	Da.	Nā.	Ga.	Yo.
Yava* .	1								
Aṅgula .	8	1							
Pada .		6	1						
Vitasti .			2	1					
Hasta .				2	1				
Daṇḍa .					4	1			
Nāḍī . .						2	1		
Gavyūti .						4,000	2,000	1	
Yojana .						16,000	8,000	4	1

Kauṭilīya Artha Śāstra.

	Ya.	Aṅ.	Vi.	Ara.	Da.	Go.	Yo.
Yava* .	1						
Aṅgula .	8	1					
Vitasti .	96	12	1				
Aratni .		24	2	1			
Daṇḍa .		96	8	4	1		
Goruta .				4,000	1,000	1	
Yojana .				16,000	4,000	4	1

Śrīdhara.

	Aṅ.	Ha.	Da.	Kro.	Yo.
Aṅgula .	1				
Hasta .	24	1			
Daṇḍa .	96	4	1		
Krośa .	192,000	8,000	2,000	1	
Yojana .	768,000	32,000	8,000	4	1

Mahāvīra.

	Se.	Ya.	Aṅ.	Pa.	Vi.	Ha.	Da.	Kro.	Yo.
Sesamum* .	1								
Yava .	8	1							
Aṅgula .		8	1						
Pada . .		48	6	1					
Vitasti .		96		2	1				
Hasta .		192	24	4	2	1			
Daṇḍa .		768	96	16	8	4	1		
Krośa .		1,536,000	192,000	32,000	16,000	8,000	2,000	1	
Yojana .		6,144,000	768,000	128,000	64,000	32,000	8,000	4	1

Bhāskara.

	Ya.	Aṅ.	Ha.	Da.	Kro.	Yo.
Yava .	1					
Aṅgula .	8	1				
Hasta .	192	24	1			
Daṇḍa .	768	96	4	1		
Krośa .	1,536,000	192,000	8,000	2,000	1	
Yojana .	6,144,000	768,000	32,000	8,000	4	1

Amūrta (un-real) Table.

Paramāṇus .	1							
Parasūkhamas	8	1						
Trasareṇu .	64	8	1					
Reṇu . .	512	64	8	1				
Vālāgra . .	4,096	512	64	8	1			
Likshā . .	32,768	4,096	512	64	8	1		
Yūka . .	8^6	32,768	4,096	512	64	8	1	
Yava . .	8^7	8^6	32,768	4,096	512	64	8	1

Bakhshālī.

	Ya	Aṅ	Ha	Dha	Kro	Ga	Yo
Ya	1						
Aṅ	6	1					
Ha	144	24	1				
Dha	576	96	4	1			
Kro	576,000	96,000	4,000	1,000	1		
Ga	2,304,000	384,000	16,000	4,000	8	1	
Yo	4,608,000	768,000	32,000	8,000	2	2	1

* These tables are accompanied by the *Amūrta* table.

Measures of Capacity.

109. The following is the most complete table given (fol. 13).

ā°	2	
	4*	dro°
pra°	0	
	4*	ā° pra°
ku°	2	
	4*	prasthi

which means 2 *āḍhakas* + 0 *prasthas* + 2 *kuḍavas*. The figures here marked with asterisks are change-ratios and indicate that 4 āḍhakas = 1 droṇa

4 prasthas = 1 āḍhaka

4 kuḍavas = 1 prastha.

The terms *ghaṭaka* and *pala* are mentioned together (fol. 15), where possibly *ghaṭaka* may be equivalent to *droṇa*; and in another place (fol. 53) *ghaṭaka* is obviously used as a capacity term.

A comparison with the Hindu measures shows that the Bakhshālī measures of capacity are strictly orthodox; and it may be noted that the Hindu measures of capacity are generally more consistent in the matter of change-ratios than any other Hindu measures.

Hindu Measures of Capacity.

Atharva Veda.

	Kri.	Mā.	Pa.	Pra.	Āḍh.	Dro.
Krishṇala .	1					
Māshaka .	5	1				
Pala . .		64	1			
Prastha .			32	1		
Āḍhaka .			128	4	1	
Droṇa .			512	16	4	1

Kauṭiliya Artha Śāstra.

	Ku.	Pra.	Āḍh.	Dro.	Khā.	Kum.	Va.
Kuḍumba .	1						
Prastha .	4	1					
Āḍhaka .	16	4	1				
Droṇa .	64	16	4	1			
Khārī .	1,024	256	64	16	1		
Kumbha .	2,048	5,120	1,280	320	20	1	
Vaha .	20,480	51,200	12,800	3,200	200	10	1

Varāha Mihira—Dry Measure.

	Pa.	Ku.	Pra.	Āḍh.
Pala . .	1			
Kuḍava .	4	1		
Prastha .	16,	4	1	
Āḍhaka .	64	16	4	1

Varāha Mihira Liquid Measure.

	Pa.	Ku.	Pra.	Āḍh.
Pala . .	1			
Kudava .	8	1		
Prastha .	32	4	1	
Āḍhaka .	128	16	4	1

Mahāvīra.

	Sho.	Ku.	Pra.	Āḍh.	Dro.	Mā.	Khā.	Pra.	Ku.
Shoḍasika .	1								
Kuḍaha .	4	1							
Prastha .	16	4	1						
Āḍhaka .	64	16	4	1					
Droṇa .	256	64	16	4	1				
Mānī . .		256	64	16	4	1			
Khārī .		1,024	256	64	16	4	1		
Pravartikā .							5	1	
Kumbha .								5	1

Bakhshālī.

	Pa.	Pra.	Ku.	Pra.	Āḍh.	Dro.	Khā.
Pa.	1						
Pra.	2	1					
Ku.	4	2	1				
Pra.	32	8	4	1			
Āḍh.	128	32	16	4	1		
Dro.	256	128	64	16	4	1	
Khā.	4,096	2,048	1,024	256	64	16	1

Money measures.

110. Although the terms *dramma* and *dīnāra* occur pretty often their relationship with each other is nowhere indicated; and, indeed, there is very little definite information in our manuscript about money beyond the mere money names. In early times in India there were no special measures for money beyond the weight measures for different metals, and sometimes the difference in these is more apparent than real. Thus Mahāvīra's gold and silver tables given below have much in common.

In our manuscript we have only the following scraps of information—

(*a*)[1] *chhe* 80 *rakti°*—*su°*

(*b*) *chhe* 9 *gu°*—*va°*

(*c*)	1	1	1	1	1	1	1	108	*pha°* *dī°* 1 *dhā°* 8 *aṁ°* 1.
	1	2	2	2	2	2	2	1	

Of these examples (*a*) is orthodox, for according to Manu 80 *raktikās* or *guñjas* of copper or gold are equal to one *suvarṇa*. Example (*b*) is isolated and cannot be interpreted with any certainty. Possibly *gu°* stands *for guñja*, and possibly *va°* for *valla*. Example (*c*) means $1 : (\frac{1}{2})^6 : 108 : 1$ *dī°* + 8 *dhā* + 1 *aṁ°*, where *dī°* = *dīnāra*, *dhā* = *dhānakā*, and *aṁ* = *aṁśa*; and it is implied that 4 *aṁśas* = 1 *dhānakā*, and 12 *dhānakās* = 1 *dīnāra* (fol. 33) and the same relationship is given on folio 49 recto. The term *kākini* (cowrie) occurs once; *satera* also occurs (fol. 34) but with doubtful import.

The Guptas adopted the term *dīnāra*[2] from the Kushāns together with the coin of weight from 118 to 122 grains; and in a number of Gupta inscriptions certain gold coins are termed *dīnāras*.[3] But the *dīnāra* was not invariably a gold coin[4] and in the Bakhshālī text it probably is a copper coin, for a day's wages is stated to be from 1½ to 3 *dīnāras* (fol. 60); and also according to Mahāvīra (vi, 231) wages work out about 18 *dīnāras* a day per coolie.[5] The term occasionally occurs in Sanskrit works of a more literary character and has provoked some discussion.[6]

The term *dramma* (δραχμή) also occurs in various Indian inscriptions[7] and in Indian mathematical works. Bhāskara makes it one-sixteenth of a *nishka*[8] and in our text wages are about 1 *dramma* a day.

Such are the facts but Dr. Hoernle writes[9]—

> "The way in which the two terms are used in the Bakhshālī arithmetic seems to indicate that the gold *dīnāra* and the silver *dramma* formed the ordinary currency of the day. This circumstance again points to some time within the first three centuries of the Christian era as the date of its composition."

The subject of monetary measures is thus treated in a vague and unsatisfactory manner in what remains of the manuscript; but this is by no means a peculiarity: it is almost a characteristic of Sanskrit texts of mediæval times. The connexion between measures of money and weight was then fairly intimate and they can be hardly considered apart.

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[1] (*a*) 80 *raktikās* = 1 *suvarṇa*. (*b*) 9 *gu°* = 1 *va°*. (*c*) 108/64 *dīnāra* = 1 *dīnāra* + 8 *dhānakās* + 1 *aṁ°*.

[2] Gk. δηνάριον, Lat. *denarius*.

[3] J. Allan, *Catalogue of the Coins of the Gupta Dynasty, etc.*, p. cxxxiv; J. F. Fleet, *CII*, iii, nos. 5, 7, 8, 9, 62, 64.

[4] Compare our penny and read Sir A. Stein's note on the Kashmir *dīnāra* in his *Kalhaṇa's Rājataraṅgiṇī* ii, 308ff. See also sec. 133 below.

[5] We must not, however, place very much reliance on these text-book wages.

[6] *JRAS*, 1907, pp. 408 and 681, etc.

[7] *Epigraphia Indica*, I, 167; P. Vogel, *Chambā Inscriptions*, 204, etc.

[8] *Līlāvatī*, sec. 2.

Ind. Ant. xvii, 1888, p. 37.

Hindu Measures of Money.

Mahāvīra—Gold.	Ga.	Gu.	Paṇ.	Dha.	Ka.	Pa.
Gaṇḍaka .	1					
Guñjā .	4	1				
Paṇa . .	20	5	1			
Dharaṇa .	160	40	8	1		
Karsha .	320	80	16	2	1	
Pala . .	1,280	320	64	8	4	1

Mahāvīra—Silver.	Gr.	Gu.	Mā.	Dha.	Ka.	Pa.
Grain .	1					
Guñjā .	2	1				
Māsha .	4	2	1			
Dharaṇa .	64	32	16	1		
Karsha .	160	80	40	$2\frac{1}{2}$	1	
Pala . .	640	320	160	10	4	1

Śrīdhara.	Va.	Kā.	Pa.	Pu.
Varāṭaka .	1			
Kākiṇī .	20	1		
Paṇa . .	80	4	1	
Purāṇa .	1,280	64	16	1

Bhāskara.	Va.	Kā.	Pa.	Dra.	Ni.
Varāṭaka .	1				
Kākiṇi .	20	1			
Paṇa . .	80	4	1		
Dramma .	1,280	64	16	1	
Nishka .	20,480	1,024	256	16	1

Albirūnī.	Mdr.	Pā.	Ka.	Ya.	Aṇḍ.	Mā.	Su.
Mdrī . .	1						
Pāda . .	4	1					
Kalā . .	16	4	1				
Yava .	100	25	$6\frac{1}{4}$	1			
Aṇḍī . .	400	100	25	4	1		
Māsha .	1,600	400	100	16	4	1	
Suvarṇa .	25,600	6,400	1,600	256	64	16	1

Bakhshālī.	Ra.	Dhā.	Dra.	Dī.	Su.
Ra.	1				
Dhā.	5	1			
Dra.	30	6	1		
Dī.	60	12	2	1	
Su.	80	16	$2\frac{2}{3}$	$1\frac{1}{3}$	1

Measures of Weight.

111. Besides the examples already given (sec. 104), the following are noteworthy :—

(a)

dhā°	1		
aṁ°	1		
	4*		
ra°	1	1¼*	bhā°
ya°	1	3 *	bhā°
ka°	1	6¼*	bhā°

(b)

to°	3	
mā°	2	
	12*	
aṁ°	3	
	4*	
ya°	3	
	4*	
ka°	1	6¼*
pā°	1	
	4*	

(c)

ra°	1		
ya°	1	3¼ *	bhā°
ka°	1	6¼*	bhā°
pā°	1		
	4*		
śe°	¼		

(d)

chhe°	8*		
dhā°	2		
chhe°	12*		
guṁ°	3		
chhe°	5*		
ya°	2	3¼*	bhā°

(e)

216	bhā°
270	pa°
2000*	chhe°
6	to°
8*	chhe°
8	dhā°
12*	chhe°

These measures are combined in the following table.

	Mū°.	Pā.	Ka°.	Si°.	Ya°.	Ra°.	Aṇ°.	Dhā°.	To°.	Pa°.
Mūdrikā .	1									
Pāda . .	4	1								
Kalā . .	16	4	1							
Si° . .	40	10	2½	1						
Yava .	100	25	6¼	2½	1					
Raktikā .	320	80	20	8	3¼	1				
Aṇḍika .	400	100	25	10	4	1¼	1			
Dhānaka .	1,600	400	100	40	16	5	4	1		
Tola . .	19,200	4,800	1,200	480	192	60	48	12	1	
Pala . .	153,600	38,400	9,600	3,840	1,536	480	384	96	8	1

(a) In all cases the change-ratios are marked with asterisks (not, of course, in the original). The table 1 *dhā°* + 1 *aṁ°* + 1 *ra°* + 1 *ya°*.

(b) 3 *to°* + 2 *ma°* + 3 *aṁ°* + 3 *ya°* + 1 *ka°* + 1 *pā°*.

(c) 1 *ra°* + 1 *ya°* + 1 *ka°* + 1¼ *pā°*.

(d) ....2 *dhā°* + 3 *guṁ°* + 2 *ya°*.

(e) 216 *bhā°* + 270 *pa°* + 3 *to°* + 8 *dhā°*

To this we should add 12 *māshas* = 1 *tola*, 5 *tolas* = 1 *suvarṇa* and 2000 *pala* = 1 *bhāra*.

This agrees with the Hindu tables generally but more particularly with that of Varāhamihira as given by Albīrūnī, which, however, contains neither *si*° nor *raktikā* – both interpolations here, as their fractional change ratios show. Obviously Albīrūnī's *mdri* and our *mūdrika* are identical : the term occurs in no other Hindu work known to me. The remainder of the table—*dhānakas* to *bhāras*—does not seem so orthodox. Compare with Mahāvīra. As stated before there is some uncertainty as to monetary measures in our text and it is possible that some of the measures included in the above table should be treated as such.

The combination that occur in the text are as follows :—

mā, su, to	fol. 11
ya, ra, am, dhā	„ 49
mū, pā, ka, ya, ra, am, mā, to	„ 49
dhā, to, pa, bhā	„ 48
dhā, gum, ya	„ 48
mū, pā, ka, si, ya, ra, am, dhā, to	„ 55
ya, am, mā, to	„ 55

Possibly the term *dra*° should be included in the above table. It occurs only once (fol. 20) in the phrase *chhedaṁ* 6 *dhā*° - *dra*° which means 6 *dhā* = 1 *dra*°, and this must stand for 6 *dhānakās* = 1 *draṅkshana*; for Albīrūnī gives 6 *māsha* = 1 *draṅkshana;* and Mahāvīra gives 6 *bhāga* = 1 *draṅkshana*, and *dhānaka*, *māsha* and *bhāga* are synonymous terms as applied to measures of weight.

The term *si*° occurs thrice and the values given are 40 *si*° = 1 *mā*°, 2½ *si*° = 1 *ya*° and 480 *si*° = 1 *to*° – all of which are in agreement. The abbreviation *si*° probably stands for *siddhārtha*. Connected with it there is another measure *ku*° also unidentified. The statement in which both these occur is as follows :—

| ku° 1½ chhe° 128 mā° – ku° ½ mā° chhe° 40 si° – mā° | sa° 55 |

Here we have 128 *mā*° = 1 *ku*° which so far has puzzled me. *Sa*° is an abbreviation for *satera;* but its connexion with other measures is not clear. Mahāvīra makes 2 *dīnāra* = 1 *satera*. The equation 8 *suvarṇa* = 1 *ku*° can be deduced, but is not helpful. *Śāna* (Dict. = 4 māshakas) occurs.

The measures are generally expressed by the abbreviations *mū*, *pā*, *ka*, *si*, *ya*, *ra* or *guṁ*, *aṁ*, *dhā or mā*, *to*, *pa*, *bhā;* but occasionally the full terms are employed and in one place (fol. 49) a fairly complete set of terms is given, namely—

dhānakā aṁdika raktikā yavā kalā pāda mūdrikā

Hindu Measures of Weight.

1. Manu and Yājñavalkya, 2nd—4th Century A.D.

					Ya.	Ra.	Mā.	Ka.	Pa.	Dha.
Trasa-reṇu .	1									
Likshā . .	8	1								
Rāja-sarshapa .	24	3	1							
Gaura-sarshapa	72	9	3	1						
Yava . .				6	1					
Raktikā . .				18	3	1				
Māsha . .					15	5	1			
Karsha . .					240	80	16	1		
Pala . .					960	320	64	4	1	
Dharaṇa . .					9,600	3,200	640	40	10	1

(a) The Bṛihaspati Smṛiti gives 4 karshas = 1 ḍhānaka and 12 dhānakas = 1 dīnāra.

2. Nārada, 5th—6th Century, A.D.

	Kā.	Mā.	Aṇ.	Dhā.	Dī.
Kākaṇī .	1				
Māsha .	4	1			
Aṇḍikā .	80	20	1		
Dhānaka .	320	80	14	1	
Dīnāra .	3,840	960	48	12	1

Kauṭilīya.

	Mā.	Guñ.	Mā.	Su.	Pa.
Māsha .	1				
Guñja .	2	1			
Māshaka .	10	5	1		
Suvarṇa .	160	80	16	1	
Pala . .	640	320	64	4	1

3. Varāha-mihira, 6th Century.
(After Albīrūnī.)

	Md.	Pā.	Ka.	Ya.	Aṇ.	Mā.	Su.
Mdrī . .	1						
Pāda . .	4	1					
Kalā . .	16	4	1				
Yava .	100	25	$6\frac{1}{4}$	1			
Aṇḍī . .	400	100	25	4	1		
Māsha .	1,600	400	100	16	4	1	
Suvarṇa .	25,600	6,400	1,600	256	64	16	1

4. Mahāvīra, 9th Century.

	Pā.	Ka.	Ya.	Aṁ.	Bhā.	Dra.	Dī.	Sa.
Pāda . .	1							
Kalā . .	4	1						
Yava .	25	$6\frac{1}{4}$	1					
Aṁśa .	100	25	4	1				
Bhāga .	400	100	16	4	1			
Drakshuṇa .	2,400	600	96	24	6	1		
Dīnāra .	4,800	1,200	192	48	12	2	1	
Satera .	9,600	2,400	384	96	24	4	2	1

4 (a). Mahāvīra.

	Pa.	Pra.	Tu.	Bhā.
Pala .	1			
Prastha .	12	1		
Tulā .	200	16	1	
Bhāra .	2,000	160	10	1

5. Śrīdhara, 11th Century.

	Gu.	Mā.	Ka.	Pa.
Guñja .	1			
Māsha .	5	1		
Karsha .	80	16	1	
Pala .	320	64	4	1

6. Bhāskara, 12th Century.

	Ya.	Ra.	Va.	Dha.	Ga.
Yava .	1				
Raktikā .	2	1			
Valla .	6	3	1		
Dharaṇa .	48	24	8	1	
Gadyāṇaka	96	48	16	2	1

Bakhshālī.

	mū.	pā.	ka.	si.	ya.	ra.	aṇ.	dhā.	dra.	to.	pa.	bhā.
mū	1											
pā	4	1										
ka	16	4	1									
si	40	10	$2\frac{1}{2}$	1								
ya	100	25	$6\frac{1}{4}$	$2\frac{1}{2}$	1							
ra	320	80	20	8	$3\frac{1}{5}$	1						
an	400	100	25	10	4	$1\frac{1}{4}$	1					
dhā	1,600	400	100	40	16	5	4	1				
dra								6	1			
to	19,200	4,800	1,200	480	192	60	48	12	2	1		
pa	153,600	38,400	9,600	3,840	1,536	480	384	96	16	8	1	
bhā											2,000	1

CHAPTER VIII.

The sources.

112. In considering the question of exotic influence the period of composition of the work is of importance. We must forget for the time being Dr. Hoernle's suggestion that the work was written in the early centuries of our era and bear in mind that it possibly belongs to a much later period,[1] by which time western mathematics, although almost all traceable to Greek sources, had assumed new forms and had included some new notions. On the whole western mediæval mathematics tended to become less rigorous and more mixed than the mathematics of classical Greece. Practical calculation (logistic) altogether supplanted the earlier pure arithmetic, mensuration took the place of pure geometry, and algebra slightly developed. Such changes are, indeed, indicated in the later Alexandrian works, and what we sometimes term degeneration had already set in there. The introduction of a place-value arithmetical notation made calculation easier and more popular, and more intricate arithmetical problems than those, for example, exhibited in the Greek Anthology, appeared in the later mediæval text-books. The general body of popular mathematical knowledge became more diffused. (Identical problems occur in Chinese, Indian, Arabic and European text-books of a comparatively early period). Indeed the mediæval mathematical works of Asia and Europe had so much in common that at first it seems almost impossible to pick out that which is definitely western or eastern in origin. In this connexion it should be remembered that the early Hindu astronomers (Āryabhaṭa and Varāha Mihira) were among the first to exploit Greek mathematical learning; that later the Arabs, after sampling Indian works, turned to those of Greece; and that it was from the Arabs that Europe received once more the learning it had previously rejected.

113. Such facts indicate to some extent the difficulties of making a direct comparison of, say, a twelfth century work with those of the classical Greek period. A knowledge of the development or degradation of mathematics during the intervening period is obviously demanded, and without such knowledge sound judgment is impossible. That knowledge must be sought elsewhere, but I give here a summary chronological table of the period, that may serve to recall the chief mathematical writers and their works.

			A.D.
5th century	Hypatia		d. 415
	Proclus		410-485
	Boethius		b. 470
	Āryabhaṭa		b. 476
6th century			
	Eutocius		
	Damascius	visited Persia	529
	Simplicius	visited Persia	529
	Dominus		
	Chang ch' iu-chien		550
	Varāhamihira		d. 587
	Isidore of Seville		570-636

[1] Specific reasons for arriving at the later date are given in the next chapter.

7th century	*Brahmagupta*	b. 598
	Fall of Alexandria	640/1
	Papyrus of Akhmin	
	Bede	b. 735
8th century	Muhammad b. Mûsa	
	Alcuin	d. 804
9th century	*Mahāvīra*	
	Tabit b. Qorra	836-901
10th century	al-Battānī	
	Avicenna	
	Pope Sylvester ii (Gerbert)	d. 1003
11th century	Albīrūnī visited India	1017-1030
	Śrīdhara	b. 991
	al-Karkhī	
	Psellus	
	Omar Khayyām	b. 1046
12th century	Adeland at Cordova	1120
	Bhāskara	b. 1114
	Leonardo	b. 1175

114. Although the manuscript was found at Bakhshālī it cannot be assumed that it originated there. The evidence of the script, however, limits the area of origin to the neighbourhood of Gandhāra. Roughly the Śārada script area is limited by longitudes 72 and 78 east of Greenwich and north latitudes 32 and 36. This area includes, besides Gandhāra, Kashmir, Kangra and territories near by. Vogel enumerates six inscriptions from Gandhāra, some of which belong to the Swat valley. The evidence therefore does not militate against Bakhshālī itself as the actual place of origin.

115. The position is peculiarly interesting with reference to the routes of transmission of knowledge and the consequent liability to outside influence. Between the fifth century B.C. and the fifth century A.D. the country round Bakhshālī was actually under the dominion of seven different nations : the Persians, the Macedonians, the Mauryas, the Bactrian Greeks, the Scythians, the Parthians, the Kushans : "and" writes Sir John Marshall,[1] "it may be taken for granted that, with the exception of the Macedonians whose conquest was merely transitory, each of these nations in turn left some impress on the arts and culture of the country." From the sixth to the tenth centuries of our era Gandhāra was more or less subject to the Guptas and their successors and then came the Muhammadan invaders. The following are some notable dates relating to the country with which we are concerned.

B.C.	326	Alexander receives the submission of Āmbhi, king of Taxila.
	190	Demetrius of Bactria conquers the Punjab.
	85-50	Maues, the Scythian, conquers Taxila.

[1] *A Guide to Taxila*, p. 23.

A.D.	20 circa	**Gondopharnes the Parthian ruling over Kabul, Taxila and Arachosia.**
	60	**Hermaeus and Kujula Kadphises annex Gandhāra.**
	120	**Kanishka the Kushān, king of Gandhāra.**
	319	**The Gupta era begins.**
	400	**Fa-hien entered India by way of Gandhāra.**
	400-500	**Invasion of India by the Epthalities or White Huns.**
	500	**Sung Yun in Gandhāra.**
	630 circa	**Hiuen Thsang in Gandhāra.**
	720	**Kashmir subject to China.**
	753	**Ou-K'ong visited Gandhāra.**
	1001	**Mahmud of Ghaznī defeats Jaipal near Peshawar.**
	1175	**Muhammad Ghorī attacks Multan.**

116. The achievements of the Greeks in mathematics and art form the most wonderful chapters in the history of civilisation, and these achievements are the admiration of western scholars.[1] It is therefore natural that the western investigator into the history of knowledge should seek for traces of Greek influence in later manifestations of art and mathematics in particular. The position of Bakhshālī in the heart of Gandhāra, and the political history of that country are such, that not only warrant the search for traces of Greek influence, but make it practically imperative. Indeed the neglect of such an enquiry would stamp any investigation of this kind as incomplete.

Evidence of Greek influence in the realm of art has been discovered in profusion in Gandhāra and the surrounding country. Sir John Marshall writes[2] "The monuments and antiquities that have recently been recovered from the soil at Taxila and other places, all consistently bear witness to the strong hold which Hellenistic art took upon this part of India. This hold was so strong, that long after the Greek kingdoms of the Punjab had passed away, even after the Scythians and Parthians, who overthrew the Greeks, had themselves been supplanted by the Kushans, Greek art still remained paramount in the North West, and continued to exercise considerable influence until the fifth century of our era, although it was growing more and more decadent year by year."

It would not therefore be unreasonable to imagine that in mathematics also there was at least a possibility of Greek influence in the same country and at the same period ; and Sir John himself supplies a sort of connecting link between art and mathematics. He continues[3] "This persistence and this slow decadence of Greek ideas is best illustrated by the coins, the stylistic history of which is singularly lucid and coherent. In the earliest examples every feature is Hellenistic. The standard weight of the coins is the standard established by Athens : the legends are in Greek .... Later on, when the Greek power in India became consolidated, the old Attic standard gave place to one, possibly based on Persian coinage .. bilingual legends were substituted for the Greek ; and little by little the other Greek qualities gradually faded . . ."

[1] The modern Indian does not seem to be attracted in this way.

[2] *A Guide to Taxila*, p. 26.

[3] *Ib*, p. 27.

The case for mathematics is almost exactly parallel, allowance, of course, being made for the more abstract nature of the subject; and the history of mathematics gives abundant illustration of the same types of change as Sir John traces in the Indo-Greek coinage.

Enough has been said to show that it would be stupid to exclude the possibility of Greek influence in the realm of mathematics on general grounds; and if such influence is to be negatived finally it must be for special reasons to be discovered by subjecting our manuscript to a detailed examination.

117. It is unfortunate that the question of origin of the Bakhshālī Manuscript was discussed and judged upon before the work was thoroughly examined. The announcement made by Weber, on the authority of Bühler, at the fifth International Oriental Congress was based upon no real knowledge at all; and Dr. Hoernle himself only examined in detail about one-third of the manuscript. We do not, therefore, feel bound to treat the pronouncements of these eminent scholars on this particular subject with the reverence that is usually due to them. Also we know that Weber was misled by Bühler and Dr. Hoernle told me that he was prepared to modify his earlier views to some extent. With reference to the last point the quotations given below must therefore not be taken as expressing Dr. Hoernle's final views. However, they are the basis of all that has been subsequently written and it is on them that the position of the Bakhshālī Manuscript has been determined by the historians of mathematics. Dr. Hoernle summarised his views in the following words :—

"I believe that it is generally admitted that Indian arithmetic and algebra, at least, are of entirely native origin. While Siddhānta writers, like Brahmagupta and his predecessor Āryabhaṭa, might have borrowed their astronomical elements from the Greeks or from books founded themselves on Greek science, they took their arithmetic from native Indian sources. Of the Jains it is well known that they possess astronomical books of a very ancient type, showing no traces of western or Greek influence.[1] In India arithmetic and algebra are usually treated as portions of works on astronomy.[2] In any case it is impossible that the Jains should not have possessed their own treatises on arithmetic, when they possessed such on astronomy. The early Buddhists too, are known to have been proficients in mathematics.[3] The prevalence of Buddhism in North-Western India, in the early centuries of our era, is a well known fact. That in early times there were also large Jain communities in those regions, is testified by the remnants of Jain sculpture found near Mathurā and elsewhere. From the fact of the general use of the North-Western Prakṛit (or the 'Gāthā dialect') for literary purposes among the early Buddhists it may reasonably be concluded that its use prevailed also among the Jains, between whom and the Buddhists there was so much similarity of manners and customs. There is also a diffusedness in the mode of composition of the Bakhshāli work[4] which reminds one of the similar characteristics observed in Buddhist and Jain literature. All these circumstances put together seem to render it probable that in the Bakhshāli manuscript there has been preserved to us a fragment of an early Buddhist or Jain work on arithmetic (perhaps a portion of a larger work on astronomy) which may have been one of the sources from which the later Indian astronomers took their arithmetical information."

118. There is not the slightest evidence in the manuscript itself of its being connected either with the Jains or Buddhists. It is Hindu (Śaivite). The author was a Brahman (fol. 50); to Śiva is attributed the gift of calculation to the human race (fol. 50); offerings to Śiva are mentioned on more than one occasion (fols. 34, 44); references are made to certain incidents recorded and persons named in the Hindu epics (fol. 32, etc.);[5] and there is not a single reference that could be construed as indicating any connexion with Buddhism or Jainism.

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[1] The reference is probably to the *Sūryaprajñapti*. See my *Hindu Astronomy*, pp. 19-21.

[2] Not before the advent of western astronomy.

[3] Is Dr. Hoernle here thinking of the incident related in the *Lalitavistara* and popularised by Arnold? The legend, according to Weber, "carries no weight whatever."

[4] I should not myself have noted this as a characteristic of the Bakhshālī work.

[5] And these references occur in the portions of the MS that were not critically examined by Dr. Hoernle.

119. Further there are indications of a connexion with Muslim mathematicians (§ 120) and there is other internal evidence that points to a much later date than Dr. Hoernle's thesis allows. It is rather curious that the connexion between arithmetical and astronomical works that Dr. Hoernle points out only holds for those later works which distinctly show western influence. For the major assumption "that Indian arithmetic and algebra are of entirely native origin" Dr. Hoernle was not himself responsible. It was a common opinion, that still obtains to some extent. One point appears at first sight to be in favour of Dr. Hoernle's argument (but of which he could not have been aware), namely that in some matters of detail[1] the Bakhshālī work more closely resembles the *Gaṇita-Sāra-saṅgraha* of Mahāvīra than any other Indian work on mathematics.

120. Although the mediæval works of East and West have so much in common yet there are differences even in the later treatment of topics and notions that had originally a common origin. For example the Indians, although they adopted the western sexagesimal notation for astronomical purposes did not utilise this notation for purposes of arithmetical calculation. On the other hand the Muslim mathematicians commonly employed this notation to express ordinary fractional quantities. Now in the Bakhshālī Manuscript is an example of the transformation of a simple fraction expressed in the ordinary way to the sexagesimal notation. This transformation may be represented by

$$\frac{178}{29} = 6 + 8' + 16'' + 33''' 6'''' \frac{6}{29}$$

No such example occurs in any early Hindu work and there is not the slightest doubt that it indicates direct western influence. Indeed our author could have hardly provided us with a more conclusive piece of evidence.

Again our manuscript exhibits a method for finding approximate roots of surd quantities that is not Indian. The method may be represented by $\sqrt{A^2+b} = A + b/2A$ approximately, and closer approximations may be achieved by continuing the process. The *sūtra* embodying this method is given three times and a number of examples of first and second approximate evaluations is given. Indeed this square-root method is one of the most prominent topics of the work. Its history is quite well known (See §69). It occurs in many western works from the time of Heron onwards but it occurs in no Indian work earlier than the twelfth century: indeed the earliest record of this method in an Indian work (other than the Bakhshālī Manuscript) known to me is of the 16th century!

There is an interesting similarity between part of our text and the arithmetical papyrus of Akhmin (See §89). There are problems of the type of the Epanthem which can also be traced to a definite Greek source (See p. 40); but in this and other cases it is possible that the problems reached the Bakhshālī Manuscript by way of other Indian works.

121. But, of course, this evidence of western influence does not mean that the work was not Indian. It is, indeed, almost as Indian as any other mathematical work of the period. It contains references to Hindu mythology and to Hindu deities: and the language is Indian of a sort: the script is an off-shoot of the classical script of northern India; the form of presentation is Indian; and the material of most of the examples is Indian.

The general conclusion is that the work is mainly Indian, but that, as was to be expected, it shows signs of outside influence, and it gives rather special prominence to the non-Indian material used.

[1]But these have not the slightest Jaina significance.

CHAPTER IX.

THE AGE OF THE MANUSCRIPT AND THE AGE OF THE WORK.

122. Dr. Hoernle held that the mathematical treatise which is written out in the so-called Bakhshālī Manuscript was considerably older than the manuscript itself. Indeed he thought that the work was composed about six centuries earlier than the copy we are considering. He excluded the possibility of our manuscript being a translation for he largely based his estimate of the age of the work on the antiquity of the language employed in the manuscript. If Dr. Hoernle were right in his differentiation between the age of the work and of the Bakhshālī copy then we should have to consider the possibility of other copies being preserved and the probability of the work being known to other mathematical writers of the intervening centuries. Dr. Hoernle, indeed, did suggest that the work was one of the sources from which the early Hindu mathematicians drew inspiration, but without any justification. Unfortunately Dr. Hoernle's reasons for his views as to the ages of the manuscript and the work are not satisfactory and we are compelled to reject his conclusions altogether. Of course it will be impossible to say definitely that the manuscript is the original and only copy of the work but we shall be able to show that there is no good reason for estimating the age of the work as different from the age of the manuscript to any considerable degree.[1]

There are certain general causes for Dr. Hoernle's mistaken conclusions. He examined in detail only a comparatively small portion of the manuscript; the history of the Śāradā script was not well known thirty years ago; there has since that time been light thrown upon the type of language used in the manuscript, and the knowledge of mediæval mathematics has been extended.

It is proposed therefore to re-examine in detail the whole question of age and in the course of the re-examination Dr. Hoernle's arguments will receive due consideration.

THE AGE OF THE MANUSCRIPT.

(a) *The circumstances of the find.*

123. The circumstances of the find have already been described in detail (§3). They led Bühler and Weber to suggest that the manuscript might prove to be of the age of Kanishka, *i.e.*, of the second century of our era. We need not labour this point. The suggestion was based upon a misunderstanding and there is not the slightest evidence to support it. It was discarded by Dr. Hoernle, who, however, follows much the same line of thought in arguing for a slightly later date. He writes "The country in which Bakhshālī lies and which formed part of the Hindu kingdom of Kabul, was early lost to Hindu civilisation through the conquests of the Muhammadan rulers of Ghaznī, and especially through the celebrated expeditions of Mahmūd towards the end of the tenth and the beginning of the eleventh centuries A.D. In those troublous times it was a common practice of the learned Hindus to bury their manuscript treasures. Possibly the Bakhshālī manuscript may be one of these. In any case it cannot well be placed much later than the tenth century A.D. It is quite possible that it may be somewhat older."

Dr. Hoernle assumes that the manuscript could not well have been written after the time of Mahmūd. Regarding the alleged burial custom I can say nothing.

[1] There is evidence that the MS is not a copy at all. It is not the work of a single scribe: there are cross references to leaves of the manuscript; there is a case of wrongly numbering a *sūtra* and the mistake is noted in another hand-writing.

It may have been prevalent among 'learned Hindus'; but there is not the slightest evidence to show that the Bakhshālī manuscript was *deliberately* buried.

(b) *The material.*

124. The material on which the work is written is birch-bark, which was the common writing material for a considerable period of time in Kashmir and its neighbourhood. Unfortunately we have very few birch-bark manuscripts earlier than the fifteenth century preserved, so it would be rash to fix definitely the earlier limit of its use. However the earliest known birch-bark manuscript belongs to about the second century of our era and we know that this material was in common use in Kashmir until about the seventeenth century. There will be little danger in placing our manuscript within the limits here indicated.

But there was fashion in birch-bark manuscripts, and the process of preparation of the material developed. The strips of bark from which the leaves for writing upon are obtained can generally be split up into a number of laminæ. Some scribes were content with comparatively coarse material, obtained by dividing the original strip into two similar strips; but sometimes the sub-division was carried on much further and very thin strips were obtained, two of which cut to the required size were pasted together to form a writing leaf. Portions of the Bower manuscript and all of the Bakhshālī manuscript are of the cruder form, while the Kashmerian Artharva Veda consists of the more elaborately prepared and finer writing material. There might appear to be little doubt that the use of the cruder material denotes an earlier period, but there is a good deal of doubt really; and the scarcity of known specimens reduces the value of any criterion based upon this fashion.

(c) *Format.*

125. The shape and size of the birch-bark leaf might also be expected to give some indication of age. The format of the Kharoshṭhī Dhammapada from Khotan was probably due to early western influence while the Bower manuscript format was probably due to the Indian palm-leaf pothī. The Bakhshālī format differs considerably from both of these and is certainly of a later date. Dr. Hoernle thought the Bakhshālī manuscript was the prototype of the early Indian paper book, but it might have been the other way round (See §16).

The Script.

126. The script will be described in some detail later on and the chronological evidence it gives will be indicated. The earlier orientalists had rather inaccurate notions about its chronology. Bühler stated that the oldest Śāradā inscription was that of Baijnāth, which he dated A. D. 804 instead of A. D. 1204, while Dr. Hoernle took the Śāradā script back to A. D. 500. The main facts relating to the chronology of the Śāradā script are as follows: The earliest known examples are of the ninth century and are found on certain coins of the Varma dynasty of Kashmir. There are at least two inscriptions of the tenth century, namely the Sarāhan inscription and an inscription of the reign of Queen Diddā. Many dated inscriptions of the eleventh and twelfth centuries have been preserved. At the beginning of the thirteenth century is placed the Baijnāth inscription, with which, according to Vogel, the history of the Śāradā proper comes to an end.

Of the script of the Bakhshālī manuscript Dr. Hoernle first wrote[1]—"Some of the forms which very frequently occur in the manuscript, especially of vowels, very closely resemble the forms used in the Aśoka and early Gupta inscriptions." The implication here made he slightly modified later[2] by writing as follows :—"The Śāradā characters used in it exhibit in several respects a rather archaic type, and afford some ground for thinking that the manuscript may go back to the 8th or 9th century." "But," he wisely continues, "in the present state of our epigraphical knowledge,[3] arguments of this kind are always somewhat hazardous."

Every letter of the manuscript has now been examined and the script has been compared with other available examples and the following age criteria have been applied :—

(*i*) Vogel thinks that the form of the letter *n* is a fairly reliable test of age. Table I, Part II exhibits three distinct types. The Sarāhaṇ example has a horizontal connecting stroke in the middle of the letter, the Baijnāth example is without this horizontal stroke but has a tail turning inwards from the left. The Bakhshālī manuscript shows no examples of either the horizontal middle stroke or of the left hand tail, and thus seems to place itself between the Sarāhaṇ inscription (tenth century) and the Baijnāth *praśastis* (early thirteenth century).

(*ii*) The common method of forming medial *ā* is to add a knob to the top right hand of the mātṛikā and of this method there are hundreds of examples in our manuscript ; but in the case of *jā* modification of the mātṛikā was the rule. There is, however, an isolated example (folio 16, recto, the sixth akshara of the third line from the bottom) where the commoner method is applied for *jā*. According to Vogel this method of writing *jā* came into fashion about A. D. 1200. This test, if reliable, is rather intriguing, as is also the next one.

(*iii*) Medial *i* and *ī* are generally formed as in Devanāgarī, but there was an older practice of forming them by sickle-shaped curves above the mātṛikās and of this older type there are two or three in our manuscript (folios 1 verso, 52, recto et verso, 60 recto). According to Vogel these superscribed short and long *i*s dropped out of use about A. D. 1200.

(*iv*) The slanting superscribed medial *e* tends to become horizontal in later Śāradā, *e.g.*, in the Baijnāth inscription. In our manuscript the stroke is nearly always slanting but there are a few examples where it is horizontal. Also another method of expressing medial *e* is by a stroke behind the mātṛikā and this method is exemplified 269 times in our manuscript. The anecdote related by Vogel (p. 96) shows that this back stroke method was not in general use in the fifteenth century.

(*v*) Medial *ai* is formed in two ways and according to Vogel the change took place about A. D. 1100. In the Bakhshālī Manuscript there are 19 examples of the older method as against 61 of the more modern method.

[1] *Indian Antiquary* xii, 1883, p. 89.

[2] Ib. xvii, 1888, p. 36.

[3] Our epigraphical knowledge of the Śāradā script has been largely extended by the researches of Dr. Vogel, whose volume on *The Antiquities of Chambā* has been of the greatest help in the present enquiry.

(*vi*) Medial *o* is expressed in three ways of which the most modern method largely predominates in the text.

The following statistics are interesting. (See pages 95—98.)

MEDIAL E.

Older [·e]	Newer [è]		
70%	30%	Sarāhaṇ inscp.	xth cent.
57%	43%	Chambā, No. 25.	xith „
44%	56%	Bakhshālī manuscript.	
0%	100%	Ant. of Chambā, p. 63.	xiiith „

MEDIAL O.

Old [·o·]	Middle [ò]	New [ŏ]		
58%	12%	30%	Sarāhaṇ inscp.	xth cent.
10%	20%	70%	Chambā, No. 25.	xith „
13%	10%	77%	Bakhshālī manuscript.	
0%	10%	90%	Ant. of Chambā, pp. 64, 65.	xiiith „

The second table may be combined thus

MEDIAL O.

Older	Newer		
70%	30%	Sarāhaṇ inscp.	xth cent.
30%	70%	Chambā No. 25.	xith „
23%	77%	Bakhshālī manuscript.	
10%	90%	Ant. of Chambā, pp. 64, 65.	xiiith „

This scriptual evidence is rather remarkable and is as convincing as such evidence can well be; but I must add a word of caution. Fashion in writing is not altogether a matter of chronology: it is also largely a matter of locality. Indeed no single criterion based upon script is infallible, but it is significant that *all* the evidence that the Bakhshālī script gives points to some time about the twelfth century, and there is not a single item of evidence of any type against this conclusion.

Symbols and notation.

127. Not unconnected with the scriptual questions are the forms of symbols, and particularly those connected with the arithmetical notation.

The negative symbol, or minus sign, is a cross +, the use of which for this particular purpose is unique. The nearest approach to this use is the inverted ψ (ↈ) employed by Diophantus to indicate a negative quantity. Dr. Hoernle rejected the Diophantine origin of the Bakhshālī symbol on the ground that "the Hindus did not get their elements of the arithmetical science from the Greeks."[1] That such a sweeping assumption as this is altogether unjustifiable has been already shown, and the implication that the work is wholly Hindu in origin has never been proved. Dr. Hoernle attempts to trace the sign back to the Aśoka *ka* but, as he confesses, not with much success. However he suggests that the Bakhshālī symbol is "a mark of great antiquity." See § 61 for some further discussion. All that we can now say is that the use of this symbol cannot be traced to an Indian source.

The arithmetical notation employed will be referred to again (see § 130). Here we are concerned with the forms of the symbols only. In my paper published by the Asiatic Society of Bengal in 1912, I attempted to place these symbols epigraphically. Unfortunately we have very few examples of numerical symbols written in early Śāradā, but on the whole the Bakhshālī symbols resemble most closely those of the Devī-rī-koṭhī Fountain inscription[2] of A. D. 1159. The Śāradā numerical symbols are fairly consistent in form but there are rather peculiar variations of the "four" and "six." The Bakhshālī "four" most closely resembles the example in No. 15 Chambā which probably belongs to the eleventh century.[3] The evidence is really too scanty for us to form any definite conclusion.

Language.

128. Dr. Hoernle writes[4]:—

"The Bakhshālī arithmetic is written in that peculiar language which used to be called the Gāthā dialect, but which is rather the literary form of the ancient North-Western Prākṛit (or Pāli). It exhibits a strange mixture of what we should now call Sanskṛit and Prākṛit forms. As shown by the inscriptions (*e.g.*, of the Indo-Scythian kings in Mathurā) of that period, it appears to have been in general use, in North-Western India, for literary purposes till about the end of the third century A.D., when the proper Sanskrit, hitherto the language of the Brāhmaṇic schools came into general use also for secular compositions . . . . . Its use, therefore, in the Bakhshālī arithmetic points to a date not later than the 3rd or 4th century A.D. for the composition of that work."

It would be presumptuous for me to contradict Dr. Hoernle on linguistic matters, and his opinion here carries great weight. But from the evidence before me I am compelled to disagree here, as elsewhere, when he is speaking of the age of the work. He gives a number of examples of the "peculiar characteristics" of the language of the Bakhshālī work, on which he appears to have based his views. Now every single one of these "peculiar characteristics" is common in Śāradā inscriptions of the eleventh and twelfth centuries. Indeed their very occurrence helps to

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[1] *Ind. Ant.* xvii, 1888, p. 34.

[2] *Ant. of Chambā*, p. 212.

[3] The resemblance of some of the numerical symbols to letters is noted in part ii. Bühler (*Ind. Pal.* p. 83) states that symbols for 4 and 9 are 'ancient letter numerals'; but the 'letter numeral theory' has succumbed!

[4] *Ind. Ant.* xvii, 1888, pp. 36-37.

confirm the evidence supplied by the script (§ 126). As it is proposed to devote the second volume of this work particularly to linguistic matters I give here only a list of Dr. Hoernle's "peculiar characteristics" with references to the Chambā inscriptions [in square brackets] where the same peculiar characteristics occur.

(*a*) "Insertion of euphonic consonants." [No. 32.] (*b*) "Insertion of s." [Inscp. No. 32, p. 210.] (*c*) "Doubling of consonants." [Nos. 15 & 32.] (*d*) "Peculiar spellings." [Everywhere.] (*e*) Confusion of ṛi and ri. [Nos. 15 (p. 165), 24 (p. 184), 25 (p. 188), 26 (p. 198).] (*f*) "The *jihvāmūlīya* and the *upadhmānīya* are always used before guttarals and palatals respectively." [" It is one of the most notable characteristics of the Śāradā," writes Dr. Vogel, "that we find the *jihvāmūlīya* and *upadhmānīya* used with great regularity." (p. 58) See Nos. 13, 1·2; 15, 1·28; 32 (p. 210), etc., etc.] (*g*) "Irregular sandhi." [Nos. 14 (p. 161), 24 (p. 184), 26 (p. 198), etc., etc.] (*h*) "Confusion of the sibilants." [Nos. 14 (p. 161), 15 (p. 165), 24 (p. 184), 25 (p. 188), 26 (p. 198).] (*i*) "Confusion of n and ṇ." [Nos. 14 (p. 161), 15 (p. 165), 26 (p. 198).] (*j*) "Elision of a final consonant." [No. 14 (p. 161) where the same actual examples occur.] (*k*) "Interpolation of r." [?.] (*l*) "Etymological and syntactical peculiarities." [Passim.] (*m*) "Peculiar forms." [Passim.] (*n*) "Peculiar meanings." Dr. Hoernle's interpretation of the two examples he gives cannot be accepted.

My references to eleventh and twelfth century inscriptions here given make no pretence of being exhaustive. They are merely given to justify my non-acceptance of Dr. Hoernle's views. On this matter I shall await the verdict of those more competent to judge than I; but my tentative conclusion is that the language of the manuscript is not appreciably earlier than the script itself.

Metre.

129. In 1888 Dr. Hoernle wrote[1]:—

"It appears that the earliest mathematical works were written in the *śloka* measure; but from about the end of the 5th century A.D. it became the fashion to use the *ārya* measure. Āryabhaṭa c. 500 A.D., Varāha Mihira c. 550, Brahmagupta c. 630 all wrote in the latter measure. Not only were new works written in it, but *śloka* works were revised and recast in it. Now the Bakhshālī arithmetic is written in the *śloka* measure; and this circumstance carries its computation back to a time anterior to that change of literary fashion in the 5th century A.D."

This statement is altogether misleading. Mahāvira's *Gaṇita-sāra-saṅgraha* (9th century) is largely in *ślokas*, and the *Sūrya Siddhānta* (c. 1100 A.D.) was written in that measure; and a number of other works dealing with astronomy and mathematics written in the *śloka* measure and rather later than the *Sūrya Siddhānta* are known. Also we can point to a number of Śāradā inscriptions of the eleventh and twelfth centuries in which the *śloka* measure is employed. It is unfortunate that Dr. Hoernle's obsession regarding the age of the work led him to employ this rather disingenuous argument, for it was repeated and emphasised by M. Cantor, the historian of mathematics, in his great work.[2] The subject of the metres employed in the Bakhshālī manuscript will be more fully dealt with in the second volume of this work.

[1] *Ind. Ant.*, xvii, 1888, p. 36.

[2] *Vorlesungen über Geschichte der Mathematik* (3rd edition), vol. i, p. 598.

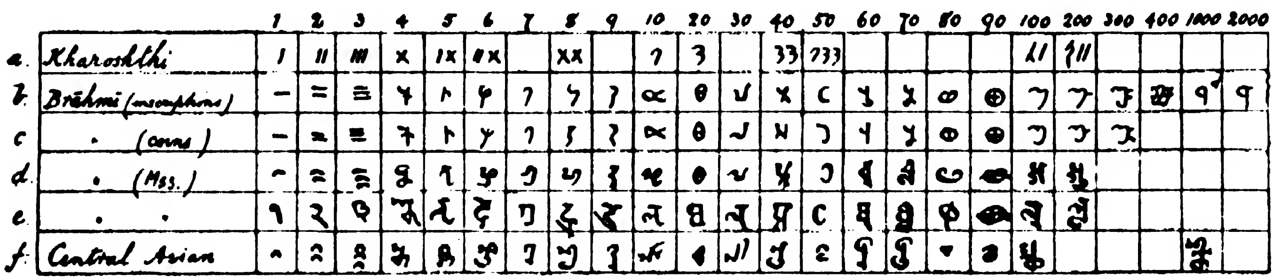
Arithmetical notation.

130. The arithmetical notation employed throughout the manuscript is the modern place-value notation. If we knew the period when this notation was first employed in the north-west of India we should have some criterion for the earlier limit of the date of the work. Note that we are not here so much concerned with the date and place of the invention of the modern notation as with its introduction and early use in the north-west of India. Obviously we should search for evidence of this in other manuscripts, on coins and inscriptions generally. Before we deal with such evidence, however, it may be as well briefly to refer to the broader aspect of the question.

According to the Hindus the modern place-value system of arithmetical notation is of divine origin. This led the early orientalists to believe that, at any rate, the system had been in use in India from time immemorial; but an examination of the real facts shows that the early notations in use were not place-value ones and that the modern place-value system was not introduced until comparatively modern times. The early systems employed may be conveniently termed (*a*) the Kharoshtī (*b*) the Brāhmī (*c*) Āryabhaṭa's alphabetic notation (*d*) the word-symbol notation.

(*a*) The "Kharoshṭhi" script, written from right to left, was in use in the north-west of India, Afghanistan and Central Asia at the beginning of the Christian era. The notation is shown in the accompanying table. The smaller elements are written on the left.

(*b*) The "Brāhmī notation" is the most important of the old notations of India. It might appropriately be termed *the* Indian notation for it occurs in early inscriptions and was in fairly common use throughout India for many centuries. The symbols employed varied according to time and place but on the whole their form was fairly consistent. They were written from left to right with the smaller elements on the right. Several false theories as to the origin of these symbols have



NUMERICAL NOTATIONS.

been published, some of which still continue to be recorded. The earliest orientalists gave them place-value, but this error soon disproved itself; it was then suggested that they were initial letters of numerical words; etc., etc.

(*c*) Āryabhaṭa's alphabetic notation also had no place-value. It was written and read from left to right but differed from the Brāhmī notation in having the smaller elements on the left. It may be exhibited thus :

Letters.	*k*	*kh*	*g*	*gh*	*ṅ*	*ch*	*chh*	*j*	*jh*	*ñ.*			
Values.	1	2	3	4	5	6	7	8	9	10.			
Letters.	*ṭ*	*ṭh*	*ḍ*	*ḍh*	*ṇ*	*t*	*th*	*d*	*dh*	*n.*			
Values.	11	12	13	14	15	16	17	18	19	20.			
Letters.	*p*	*ph*	*b*	*bh*	*m*	*y*	*r*	*l*	*v*	*ś*	*sh*	*s*	*h.*
Values.	21	22	23	24	25	30	40	50	60	70	80	90	100.

The vowels indicate multiplication by powers of one hundred. The first vowel *a* may be considered as equivalent to 1000, the second vowel $i = 100^1$ and so on. The values of the vowels may therefore be shown thus :

Vowels.	*a*	*i*	*u*	*ṛi*	*ḷi*	*e*	*ai*	*o*	*au.*
Values.	1	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}.

The following examples taken from Āryabhaṭa's *Gitiká* illustrate the application of the system :

$$\textit{khyukhṛi} = (2+30).10^4 + 4.10^6 = 4{,}320{,}000.$$

$$\textit{chayagiyiṅuśulchhḷi} = 6 + 30 + 3.10^2 + 30.10^2 + 5.10^4 + 70.10^4 + (50+7).10^6 = 57{,}753{,}336.$$

The notation could thus be used for expressing large numbers in a sort of mnemonic form. Āryabhaṭa's table of sines was expressed in this notation, which, by the way, was used only for astronomical purposes. It did not come into ordinary use in India, but some centuries later it appears occasionally in a form modified by the place-value idea with the following values :

1	2	3	4	5	6	7	8	9	10
k	*kh*	*g*	*gh*	*ṅ*	*ch*	*chh*	*j*	*jh*	*ñ*
ṭ	*ṭh*	*ḍ*	*ḍh*	*ṇ*	*t*	*th*	*d*	*dh*	*n*
p	*ph*	*b*	*bh*	*m*					
y	*r*	*l*	*v*	*ś*	*sh*	*s*	*h*	*l*	

Initial vowels are sometimes used as ciphers also. The earliest example of this modified system is of the twelfth century A. D. Slight variations occur.

(*d*) *The word-symbol notation.*—A notation that became extraordinarily popular in India was introduced about the seventh century A. D., possibly from the East. In this notation any word that connotes the idea of a number may be used to denote that number : *e.g.* Two may be expressed by *nayana*, an eye, or *karna*, an ear, etc. ; seven by *aśva*, a horse (of the sun) ; fifteen by *tithi*, a lunar day ; twenty by *nakha*, a nail (of the hands and feet) ; twenty-seven by *nakshatra*, a lunar mansion ; thirty-two by *danta*, a tooth ; etc. This notation, it is said, was used by Brahmagupta.

(*e*) *The modern place-value notation.*—The orthodox view is that the modern place-value notation that is now universal was invented in India; and until recently it was thought to have been in use in India at a very early date. Hindu tradition ascribes the invention to God! According to Masūdī a congress of sages, gathered together by order of king Brahma (who reigned 366 years), invented the nine figures! Patañjali and other early writers are supposed to make references to the place-value system. An inscription of A. D. 595 is supposed to contain a genuine example of the system;[1] according to M. NAU[2] the "Indian figures" were known in Syria in 662 A. D.; and certain other mediæval works refer to "Indian numbers"; and so on.

On the other hand it is held that there is no sound evidence of the employment in India of a place-value system earlier than about the ninth century A.D. The suggestion of "divine origin" indicates nothing but historical ignorance; Masūdī is obviously wildly erratic; the inscription of A. D. 595 is not above suspicion[1] and the next inscription with an example of the place-value system is nearly three centuries later, while there are hundreds intervening with examples of the old non-place-value system. The references to India in mediæval works do not necessarily indicate India proper but often simply refer to "the East" and the use of the term with regard to numbers has been further confused by the misreading by Woepcke and others of the Arabic term *hindasī* (geometrical, having to do with numeration, etc.), which has nothing to do with India.[3] Again, it has been assumed that the use of the abacus "has been universal in India from time immemorial" but this assumption is not based upon fact, there being actually no evidence of its use in India until quite modern times. Further, there is evidence that indicates that the notation was introduced into India, as it was into Europe, from a right-to-left script.[4]

131. As this subject is a matter of general controversy it behoves us to be circumspect about drawing definite conclusions from the occurrence of the modern place-value notation in our manuscript. As indicated above the proper procedure is to examine the evidence relating to the earliest appearance of this place-value notation in the north-west of India. Of manuscripts there are very few earlier than the twelfth century and none of these gives any example of the new notation; of coins there are plenty of examples but none earlier than eleventh century gives any example of the new notation; and the earliest inscription of this part of the world with any evidence of the new notation is not before the tenth century A.D. To suggest that the Bakhshālī manuscript was independent of all the circumstances that governed the epigraphical appearance of the new notation generally would be unscientific.

Dr. Hoernle was very dogmatic. He wrote, in 1888, "It is certain that the principle[5] was known in India as early as A. D. 500. There is no good reason why

[1] The figures were added at a later date. A cursory examination of the plate (*Epigraphia Indica*, vol. ii, p. 20) makes this obvious.

[2] Journal Asiatique, 1910, p. 209. M. Nau's 'authority' does not read well. He speaks of the Hindus' noble discoveries in astronomy—more ingenious than those of the Greeks and Babylonians.

[3] About nine hundred years ago Albīrūnī wrote (India, ii 211)—"They .... relate all sorts of things as being of Indian origin, of which we have not found a single trace with the Hindus themselves."

[4] For a more detailed examination of this question see my papers on (1) *Indian Arithmetical Notations*, *JASB* 1907, 475-508, (2) *The use of the Abacus in ancient India*, *JASB* 1908, 293-297; *References to Indian Mathematics in certain Mediæval works*, *JASB* 801-806. In 1917 in SCIENTIA (1917, pp. 273-282) Baron Carra de Vaux pursued a line of investigation completely different from that I had myself followed and came to the same conclusion, namely that the original home of the modern place-value notation could not well have been in India. For a popular exposition of the other side of the controversy see SMITH and KARPINSKY *The Hindu Arabic Numerals*.

[5] That is the place-value principle. The quotation is from the *Indian Antiquary* of 1888 (xvii), p. 38.

it should not have been discovered considerably earlier. In fact, if the antiquity of the Bakhshālī arithmetic be admitted on other grounds, it affords evidence of an earlier date of the discovery of that principle." Bühler is bolder still. "If Hoernle's very probable estimate of the antiquity of the arithmetical treatise, contained in the Bakhshālī manuscript, is correct," he writes, "its (*i.e.*, the place-value notation's) invention dates from the beginning of our era or even earlier."[1]

132. There are two other criteria suggested by Dr. Hoernle, namely (*a*) the length of the year mentioned in the text, and (*b*) the occurrence of the term *dīnāra*. Regarding the former he came to no definite conclusion but seems to have thought that there was some correspondence between the year of 360 days that occurs in the text, and the accepted estimate of the length of the year at the period of composition of the work. Had he gone into the matter further he might have come to the conclusion that the Bakhshālī work had been composed in pre-Vedic days! It seems hardly possible for any one to be misled by such "internal evidence" but it may be as well to point out that the practice of reckoning the year as consisting of 360 days for the purpose of arithmetical examples was quite common in mediæval Indian works. Mahāvīra and Śrīdhara actually give 1 year = 360 days in their tables of measures, and the latter adds the remark—"Time is calculated according to this rule in all arithmetical works."

133. Dr. Hoernle's argument regarding the use of the *dīnāra* is as follows :— In the early centuries of our era the *dīnāra* in use in India was the gold one only, and "the Bakhshālī arithmetic seems to indicate that the gold *dīnāra* and the silver *dramma* formed the ordinary currency of the day. This circumstance again points to some time within the first three centuries of the Christian era as the date of its composition."[2] The only reply that this statement calls for is to state that all the evidence of our text points to the use of a copper *dīnāra* and there is not the remotest indication of a golden *dīnāra*.[3] (See § 110.)

134. There are other indications rather than evidence of the age of the work in the material of the text. The occurrence of the square-root rule already referred to would not be an anachronism if it were found in any Indian text from the time of Āryabhata onwards—but it occurs in no known Indian text until very late indeed, and its appearance in the Bakhshālī manuscript is probably due to direct western influence, possibly to Muslim influence. Such evidence is valueless to the inexpert : it can only carry weight with those who have a very fair knowledge of the mathematical field of the period. Likewise the employment of the sexagesimal notation points in the same direction, but the general reader must bear in mind that this notation had been used by Hindu astronomers from the time of Ārya. bhaṭa, and that the arithmetical use of it in our text is rather a matter of western fashion than the introduction of a new idea.

The employment of the *regula falsi* is evidence of a slightly different character It occurs in no Indian work until the time of Mahāvīra,[4] and it was probably, even

[1] *Ind. Pal.*, p. 82.

[2] *Loc. cit.*, 37. It is noteworthy that the muslim arithmetical works often devoted a section to the *Dirham* and *Dīnāre*. See E. WIEDEMANN *Beiträge z. Gesch d. Natur Wissenschaften*, xiv, pp. 29 and 31.

[3] SIR A. STEIN, in 1900, wrote : "This word, undoubtedly derived from the *denarius* of the West, is well known to Sanskrit lexicography as the designation of a gold coin usually spelt *dīnāra*. But the manifest impossibility of accepting this meaning for the passages of the Chronicle which mention sums in *Dīnnāras*, has already struck Dr. Wilson. Noticing in two passages figures are given which, if calculated in gold would be large beyond all credence, he suggested that the 'Dinara' meant might have been of copper. Curiously enough, however, none of the subsequent interpreters seems to have followed up the suggestion thrown out by Wilson, or to have otherwise paid attention to the subject." *Kalhaṇa's Rājataraṅgiṇī*, vol. II, p. 308. He then goes on to prove that the *dīnāra* used was a copper one. The whole of the dissertation should be read, pp. 308-328.

[4] See his *Gaṇita-sāra-saṅgraha* vii, 112. His employment of the method is rather special and limited. In Northern India the first known use of the method occurs in the twelfth century.

in the west, a fairly close predecessor to an algebraic symbolism. There is also other evidence that our work was probably produced not long before an algebraic symbolism came into use.

135. Of the evidence as to age discussed some is of doubtful value; but there remains a good deal that must be considered as giving no uncertain indication, and the period indicated is in all cases about the twelfth century. The script, the language, the contents of the work as far as they can give any chronological evidence, all point to about this period, and there is no evidence whatever incompatible with it. Bhāskara was born in A. D. 1114, Omar Khayyām was flourishing in the early part of the century, Adelard of Bath visited Cordova in 1120, Leonardo was born in 1175; and it was during this period that the Bakhshālī work was probably composed. It is possible that in the future more light will be thrown upon the script and language of the locality and period of the Bakhshālī manuscript; and that, if the manuscript be required as suggested, further internal evidence will be forthcoming. Such further discoveries may modify the conclusions now drawn regarding the age of the manuscript; but they cannot very well put the date of composition of the work back to any great extent.

THE BAKHSHĀLĪ MANUSCRIPT

PART II.

i. – *The Script.*

The Bakhshālī manuscript is written in the Śāradā character, which is a descendant of the Brāhmī, in a line distinct from but parallel with the Nāgarī line of descent. With the last mentioned Śāradā has many features in common, but, generally speaking, it is much more archaic in appearance. The Śāradā script, although not nearly so well known as the Nāgarī, has been examined and written about to some extent. Leech's *Grammar of the Cashmeeree Language* (*JASB.* xii, 1894, 399 *sqq.*) gives the alphabet. Bühler's *Indische Palaeorgraphie*, gives useful, but sometimes misleading information; Vogel's *Antiquities of Chambā State* (1911) contains the most valuable contribution– and to this I am chiefly indebted; Ojha's *Bhāritīya Prāchīna Lipimālā* gives some useful tables, but, so far as the Śāradā script is concerned, is largely based on Vogel's work. Sir George Grierson's paper in the Journal of the Royal Asiatic Soicety (1916, xvii, 677 *sqq.*) contains tables of ligatures, etc., of modern Śāradā; and in his note in the *Linguistic Survey of India* (Vol. viii, Part ii, page 254) he states that the Śāradā character is the ancient indigenous character of Kashmir, and that it is still generally used by Hindus and is taught in their schools in that country. I have not yet been able to obtain a copy of Burkhard's tables,* which, according to the editors of the *Kashmirian Atharvaveda*, contain transliterations of 340 characters and ligatures.

The principal examples of the Śāradā script that are known come from Kashmir[1] and Jammu; Gandhāra[2] (in which the village of Bakhshālī is situated); Ladakh; Kāṅgrā, Kulū and Maṇḍi; the Shāhpur district of the Punjab, and Delhi. The distribution roughly corresponds to the area between longitudes 72° and 78° east of Greenwich and north latitudes 32° and 36°.

The earlier epigraphists had rather inaccurate notions about the Śāradā script. Cunningham described it as the Gupta character, which he thought had persisted in use in certain localities;† Bühler stated that the oldest Śāradā inscriptions were the two Baijnāth praśastis from Kāṅgrā and that their date was A.D. 804,‡ whereas it is A.D. 1204; Dr. Hoernle dated the development of the script from about A.D. 500 and laid down the maxim that Śāradā characters were no guide as to age, and Kielhorn seems to have entertained a somewhat similar view.

The conservative character of the script puzzled many investigators and it seems to have retarded the formulation of any chronology based upon palaeographic considerations. Vogel, however, in spite of the weight of the opinions of Kielhorn, Bühler and Hoernle, holds that the historical development of the Śāradā script can be traced. "I believe," he writes,§ "that a close examination of the characters will also enable us to fix the approximate date of any undated Śāradā record of the pre-Muhammadan period, provided it is extensive enough to lend itself to a detailed study." Whether or not Vogel has been perfectly successful in

* Proceedings of the Imperial Academy of Vienna, Vol. CVII, p. 640.

[1] Kashmir is, indeed, known as *Śāradā Kshetra*, or land of the goddess Sarasvatī.

[2] i.e., the Peshawar district and the surrounding hill tracts.

† A. S. R. xiv. 121.

‡ Ind. Pal. p 57.

§ *Antiquities of Chambā* p. 49.

formulating his chronological tests remains to be seen; but, at any rate, he has provided material and detailed notes that are invaluable.

Dr. Hoernle first gave 500 A.D. as the approximate period of the birth of the Śāradā script, but later he modified this view and stated that it originated directly from the Gupta script in the course of the seventh century. The earliest known examples are on the coins of the Varman dynasty of Kashmir which start from the middle of the ninth century; while previous to this the acute-angled script was, it is said, in general use in northern India. It is, of course, possible, as Vogel points out, that the Śāradā was employed as a literary alphabet considerably earlier than the ninth century; but there is no evidence. There is a Kashmir inscription of the reign of queen Diddā that belongs to the tenth century, as also possibly does the Sarāhan inscription: and there are many dated inscriptions belonging to the eleventh and twelfth centuries. The following table gives the dates of some of the more important Śāradā inscription :—

	A. D.
Coins of the Varman dynasty of Kashmir†	855-939
Inscription of the reign of Queen Diddā	992
Devi-rī-koṭhi fountain inscription	1159
Angom inscription	1197
Baijnāth inscription (Kāṅgrā)	1204

With the Baijnāth *praśastis*, according to Vogel, the history of the Śāradā proper comes practically to an end; and the script of the later records differs so considerably from the pre-Muhammadan inscriptions that he proposes to give it a special name. "The thirteenth century," he says, "forms a blank which separates the two palaeographic periods."

The relationship of the Bakhshālī script to other Indian scripts is illustrated in its main outlines in table i, which shows the Brāhmī, Western Gupta (Bower), Acute-angled, Śāradā (Nos. 4, 5, 6, 7) and the Nāgarī alphabets. It is at once seen that the Bakhshālī alphabet is more closely connected with numbers 4 and 6 than with 7; and that numbers 4, 5, and 6 are differentiated on the one hand from number 3, and on the other from number 7. Indeed numbers 4, 5, and 6 are examples of the Śāradā group proper, which is supposed to be a direct descendant from the acute-angled script (No. 3); and number 7 is a direct descendant from the earlier Śāradā script.

The most notable differences between the Śārada and the acute-angled are with respect to the *na*, *ta* and *śa;* while minor differences are seen in the *ga*, *na*, *pha*, *la* and *ha*, but in these latter cases slight variations occur in one or both of the scripts which, more or less, blot out the differences.

The differences in the cases of the *ga*, *ṭa*, *śa* are possibly due to the emphasis given in the later scripts to the overhead horizontal lines. The most important difference lies between the two example of *na*—the middle horizontal connecting line being consistently employed in the acute-angled script, and consistently omitted in all but the very early examples of the Śāradā.

Between the Śāradā proper (Nos. 4, 5 and 6) and number 7, which may be termed modern Śāradā, the differences are much more marked. The *i*, *e*, *ka*, *ja*,

† CUNNINGHAM *Coins of Mediaeval India*, plate iv.

ḍa, ya (Table I) are particularly noticeable; while the *ṇa* in both the Baijnāth example and in the modern Śāradā shows an inturned tail on the left.

The alphabet.

Further details of the alphabet are given in Tables ii to iv. Table ii shows the consonants and their more important combinations; Table iii gives the various methods of writing the vowels; and Table iv gives most of the ligatures, the numerals, etc. It is proposed to examine the whole alphabet more or less in the order of these tables.

The Consonants. *k-, kh-, g-, gh-, ṅ-*

The normal *k-* of the manuscript differs very little from other early Śāradā examples. Indeed the variation in the manuscript is almost as much as in the inscriptions. The right-hand curve ex hibits, on the whole, a tendency to close inwards towards the vertical like the modern Śāradā, particularly in the "M" section of our manuscript; but there are some half dozen examples where it is almost straight. The Sarāhaṇ and Baijnāth examples have a much shorter and more open right-hand curve, while in modern Śāradā the curve is quite closed. Vogel says that the left-hand loop is generally more rounded in the older inscriptions and in the Bakhshālī manuscript, but this is, at least, doubtful.

As the first element in a ligature *k* harks back to the Brāhmī type, *e.g.*, in *ku, kṛi, kt- kr-, ksh-, etc.* (Table ii, 1), as it does in all the Śāradā examples; but as the final consonant in a ligature it generally retains its normal shape.

In the Śāradā, *g* and *gh* have fairly constant forms, which show very little deviation from the Bower manuscript examples. The left-hand shoulder of the *g* is sometimes slightly rounded in the Bower manuscript and the acute-angled script; and in the Bakhshālī manuscript similar examples occur once or twice. In ligatures each of these letters retains its normal shape.

The letter *ṅ-* is only found in combination with other consonants, as is general in the Śāradā. The form of *ṅ-* is the same as in the Gupta period but it has developed a swelling at the right-hand end of the top horizontal stroke, and this is sometimes mistaken for the sign of *ā*. In the Bakhshālī manuscript such right-hand protuberances can, however, generally be distinguished from the *ā* sign. See *j-* and *ṭ-*

ch-, chh-, j-, jh-, ñ-

Bühler speaks of the "quadrangular *cha*" as characteristic of the Śāradā, but it can hardly be deemed to be such, for in the Sarāhan and Bakhshālī examples it is often somewhat rounded, or in the latter has the right side vertical, and in the Devī-rī-koṭhī inscription it is almost triangular. Bühler possibly had the Baijnāth and modern Śāradā examples in mind.

The akshara *chha* is also fairly constant in form, although the Bakhshālī examples are more cursive than in the inscriptions. The digraph *chchh-* frequently occurs. These letters are illustrated in Table ii, 2.

The *j-* is a conservative character in the Śāradā. In the Bakhshālī manuscript it differs very little from the Bower manuscript and acute-angled examples,

except that the top horizontal stroke has developed a much more pronounced knob on the right-hand. According to Vogel this knob or wedge disappears in the Muhammadan period, but Grierson gives it for modern Śāradā. The akshara *jā* differs considerably from *ja*. The top bar disappears and a nearly vertical stroke is added to the end of the tongue; but instead of this modification of the *mātṛikā*, in late inscriptions occasionally we meet with a *jā* formed by the usual medial *ā* sign added to the *mātṛikā*, and in our manuscript there is one such example (folio 16R.) The later script seems to revert to the Bower manuscript type.

The letter *jh-* only occurs in *jjhi* and *jjhya*. It is something like the *jā* with the left-hand stroke turned backwards. It shows little alteration since the Gupta period.

The *ñ* also only occurs in combinations (*jña*, *jna*, *ñcha*). The superscribed *ñ* differs considerably from the subscribed form, but both forms are almost identical with examples that occur in the Bower manuscript.

ṭ-, *ṭh-*, *ḍ-*, *ḍh-*, *ṇ-*

Of the linguals *ṭh* does not occur at all independently, and it is doubtful whether it is intended as a subscript. The *ṭ-* generally, but not always, has a top bar, and always it has a well developed right-hand knob. As the first element of a ligature *ṭ* occurs twice, but as a subscript to *sh* it is common. The scribe evidently made no distinction between *shṭ-* and *shṭh-*, for the open and closed letters are used indiscriminately for the same words. The normal *ṭ* or *ṭh* as a subscript to *sh* is generally accompanied by another curve attached to the right of the *sh* near the bottom. Usually this line curves slightly inwards towards the subscript: but in several cases it is a long stroke slanting outwards. This additional stroke is absent in the Sarāhaṇ inscription (and in the late Kashmir Śāradā) but it is fairly common in other early Śāradā inscriptions.

The *ḍ-* is almost exactly like the Baijnāth and modern Śāradā examples. It differs considerably from those in the Bower manuscript, and its development is somewhat uncertain. Many examples of *ṇḍ-* occur in our manuscript.

The akshara *ḍha* is very like the *pha* but has a kink in the downward stroke. Its form has not altered essentially since the Brāhmī period.

Table I exhibits three distinct types of the Śāradā cerebral nasal *ṇ*. The Sarāhaṇ example has a horizontal connecting stroke in the middle of the letter, like the Bower and acute-angled examples; the Baijnāth example is without this horizontal stroke but has a tail turning inwards from the left, and the modern Śāradā has the same tail somewhat shortened; the Bakhshālī example has no horizontal middle line and no left hand tail, and is exactly of the same type as that exhibited in the Devī-rī-koṭhī fountain inscription and other inscriptions of the twelfth century. Vogel seems to think that we have here a reliable criterion for age, and if so the Bakhshālī manuscript places itself between the Sarāhaṇ and Baijnāth praśastis. But like all other particular tests for the Śāradā this must not be relied upon with too great confidence, for in the later Śāradā we find the Bakhshālī and Baijnāth types used promiscuously.

In the Bakhshālī manuscript the form of *ṇ* is very constant but different sections of the manuscript show slight variations. The tendency is to make the left-hand stroke shorter and slightly thicker than the others, particularly in the " M " section ; the third example in the table is peculiar in having a long thin tail but it only occurs once or twice in some hundreds of examples. The principal conjuncts are *ṇḍ-*, *ṇy-*, *rṇ-*.

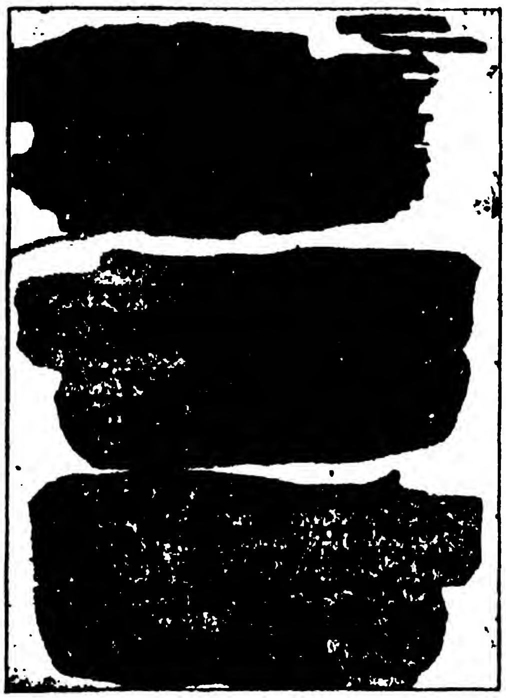
t-, *th-*, *d-*, *dh-*, *n-*.

The dental group is quantitively very important. It is, on the whole a conservative group, the chief changes being, perhaps, in the general shapes of the *th* and *dh*, which in the early inscriptions were, more or less, crescent shaped, *e.g.*, in the Sarāhaṇ praśasti. The employment of the top bar modified the crescent shaped letters into lozenge shaped. In our manuscript the crescent shape is partially retained.

As a subscript *t* is something like initial *u* but without the thin up-stroke (See Table ii, 4) ; *th* in ligatures has different forms as a second and third element, *e.g.*, as in *tthy-*, *sth-*, *rth-*; *dh* as a subscript usually has a triangular form (*ddh-*, *bdh-*, *rdh-*, *ddhv-*), but it is sometimes difficult to distinguish *ddh-* from *dr-*.

In the Devī-rī-koṭhī fountain inscription the *rth-* is rather remarkably different from the Bakhshālī examples and so it is in modern Śāradā. The examples in fig. 1, from the Suṅgal grant, are interesting as showing the transition stage in the development of *th* as a subscript and medial.*

Fig. 1. Suṅgal.



Hand made copies. *Indian Antiquary*, 1888.

* Bühler (Table VI, column viii, 50) gives an incorrect form for *rtha*. I may as well point out here that numbers 17 (*gu*), 24 (*jñā*) and 38 (*bha*) of Bühler's table are hardly representative examples. He had only Dr Hoernle's hand-made copies to follow, and those, as Dr. Hoernle was well aware, are not perfectly reliable. For example, the transcript of F14 recto given in the Indian Antiquary (xvii, 1888, p. 43) misrepresents *lyo* and *tu* in the penultimate line ; ¢ *paṁ*., *ddhyā*, etc., in the last line ; and that of F14 verso gives l. 1 ϕ *paṁ* ; l. 3 *uhya*, *tāṁ*, *stha* ; l. 5 *sta* ; l. 6 *tpra* ; last line *svame*, inaccurately.

p-, *ph-*, *b-*, *bh-*, *m-*.

The labial group is also very important quantitively, and this rather accentuates certain difficulties that present themselves. The principal of these are due to similarities between the *v* and *dh*, the *ph* and *ḍh* and the *m*, *d* and *s* under certain conditions. Generally in the Śāradā no distinction is made between the *b* and *v*, and in the Bakhshālī manuscript this lack of differentiation is almost, if not quite, complete. The resemblance between the *b* or *v* and *dh* in ligatures has already been noted upon; the *m* is distinguished from the *d* by having a complete vertical stroke on the right, but the ligatures *nd-*, *nm-*, *rd-*, *rm-* are sometimes rather difficult to differentiate; and badly written examples of *m* and *s* are also confusing. Final *m* (with the virāma) is very different from its mātṛikā and occasionally is only to be differentiated from the numerical sign for " six " by the length of the very long virāma stroke.

y-, *r-*, *l-*.

In table i are shown two types of Śāradā *y*, of which the Bakhshālī examples may be described as representing the transitional stage between the Bower manuscript example and the Sarāhaṇ example. Table ii,* however, shows that the open (Sarāhaṇ) type also occurs in the Bakhshālī manuscript, but it occurs with comparative rarity.* Most of the examples are of the type shown in table i and the first example of ii, 6; while the type illustrated as the third example in table ii, 6, occurs generally in the A_6 section of the manuscript. In most Śāradā scripts the latter (open) type prevails but in the Devī-rī-kothī fountain inscription there is exhibited a tendency towards the prolongation of the central vertical, although the left-hand loop is never actually closed, as it is in most of the Bakhshālī examples. As an element of a ligature the form of *y* is masked. Many examples are given in table iv. Occasionally the upward right-hand portion of the curve is lengthened as in *gyū* (iv, 1) and *ddhya* (iv, 5): otherwise there is little to note. As the middle element of a ligature *y* seldom occurs, unless we count *ū* as one of the elements.

The letter *r-* is one of the most consistent in form in this family of scripts. The bottom serif had developed in pre-Śāradā days. This serif is generally rather larger in our manuscript than in other Śāradā texts and occasionally develops into a loop (F 24). As a final element the *r* seems to be formed by a thinning and lengthening of this serif (table iv). For the peculiar shape of *rū* see page 95 and for other modifications see below.

Two types of *l* are shown in the Sarāhaṇ and Bakhshālī examples exhibited in table i. The former has the left curve attached to the vertical by a horizontal stroke, and this seems to be a characteristic of the earlier Śāradā scripts: the latter has the left curve attached by another curve—practically always in our manuscript (table ii, 5, etc.). According to Vogel the former type was still prevalent at the beginning of the twelfth century.

ś-, *sh-*, *s-*.

Table i illustrates fairly well the development of the Śāradā *ś*, which, in its later form, is exactly the Nāgarī *s*. In our manuscript the *ś* is always square in

* See FF 4, 6, 38, 41, 52, 53.

shape and generally has the triangular wedge on the left emphasized (See table ii, 6). This wedge is never open but always blocked-in solid. The *sh* is consistent in form (See table i), but in the ligature *ksha* the portion of the vertical above the cross bar is suppressed* (tables ii and iv) : See also *shsha* in table iv, 4.

The *s* is like the *ś* but open at the top. In the Bakhshālī manuscript it is quite common for *ś* and *s* and *sh* to be used indiscriminately.

The letter *h* is fairly consistent but occasionally tends towards angularity, like its Acute-angled prototype.

Visarga.

The visarga is represented by the two usual dots like a semi-colon. Vogel states that in several Chambā inscriptions " composed in corrupt Sanskrit " the real meaning of the visarga is misunderstood and that it is regularly used to separate words and sentences. The same remark applies to our manuscript.

The jihvāmūlīya (χ-) and upadhmānīya (ϕ-) are often employed. In one case (fol. 33) the former seems to be used in the middle of a word, *duχkha* (?) for *duḥkha*. It (χ) is similar in shape to the *b* or *v*, while the latter (ϕ) is very much the same as in the Chambā copper-plate grant of Somavarman (l. 2). Vogel states (p. 170) that the upadmānīya dropped out of use in Chambā after about 1200 A.D. Examples are shown in tables ii, 1 and ii, 5. See also page 79.

Virāma.

The final consonants *k*, *t*, *t*, *n* and *m* have each a very long virāma symbol drawn through the top of the letters on the right. See table iv, 6.

The Vowels.

a and ā.

Initial *a* and *ā* are fairly consistent in the Sāradā, and the Bakhshālī examples differ very little from the acute angled and early Gupta types. The *a* has at the bottom of the right vertical stroke a small triangular wedge, while the *ā* has in place of this wedge a small curve something like the bottom part of the *t*. Both *a* and *ā* have open tops, while in modern Sāradā the tops are closed, as in the Devanāgarī.

Medial *ā*.

The symbol for medial *ā* is a wedge or serif attached to the right of the mātṛikā at the top. Examples are seen in table iii, 1. In the Bakhshālī manuscript this symbol is generally rounded and is often written with a very small flourish at the top.

* Possibly to prevent confusion with *th* ; but in ligatures *th* is considerably modified.

There are three methods of forming medial *ā* to consider: (i) The archaic method of joining the symbol by a horizontal bar attached to the *left* top of the mātṛikā. Vogel thinks this method dropped out of use about A.D. 1000, and that it may have been merely a local fashion. When there is any choice this method is not employed by our scribes. (ii) The more common method is to join the symbol to the top of the right of the letter, and this is generally done in our manuscript with each of those letters that has a dexter upright or a top bar. (iii) The third method entails a modification of the mātṛikā, but this only occurs in the case of *j*. Examples of *jā* are shown in table ii, 1; but there is an isolated example* where *jā* is formed by method ii, and, according to Vogel, method ii, as applied to the *j* came into fashion about A.D. 1200.

It will be noted that each of the mātṛikās of *ṅ*, *j*, and *ṭ* already possesses a mark similar to the medical *ā* symbol. In the cases of *ṅ* and *ṭ* the usual *ā* symbol is added according to method ii, but, as already stated, to form the *jā*, method iii is generally employed. In the *ṇā* the symbol is often slightly curled inwards, and this hook-shaped symbol is said to be characteristic of early Śāradā.

Note that the *-ā* symbol is joined to the left upright of the *d-*, but this is because the dexter upright is not a complete one—and thus the *dā* is differentiated from the *mā*.

The vowels *i* and *ī*.

The initial vowel *ī* does not occur. The short vowel *i* is of the form that is, with fair consistence, employed in all Śāradā scripts: it consists of two dots placed above a, more or less, semicircular loop. (See table iii, 2.)

Medial *i* and *ī* are, almost without exception, of forms that are essentially the same as in the Devanāgarī, and consist of left (*-i*) and right (*-ī*) vertical strokes, sometimes considerably longer than the mātṛikā and joined to its top by a bend inwards.[1] But our manuscript contains three or four examples in which the *-ī* is formed by a sickle-shaped curve above the mātṛikā and with its convex side upwards and turned slightly towards the left. (See table iii, 2.) Vogel says, "We may assume that about A.D. 1200 the superscribed medial *i* and *ī* dropped out of use."

u and *ū*

Initial *u* is the same character as in the Bower manuscript. The left upstroke, which is generally continued to the level of the top of the letter, differentiates *u* from *ta*. The long *ū*, as an initial, is the symbol for the short vowel with a streamer hanging down from near the right top of the letter. It is distinguished from *rū*, which has a similar "steamer," by the long upward curve on the left.

Medial *u* is expressed in three ways: (i) By a triangular wedge† attached to the bottom left of the right vertical, or, where this vertical is absent, to a vertical attached to the bottom of the mātṛikā; (ii) By the addition of the initial *u* as an ordinary ligature; and (iii) by attaching a downward steamer to the right of the

* F 16 recto.

[1] The *ī* stroke is often longer than that of *-i*. Compare the aksharas *sī* and *ti* in table ii, 2: these are contiguous aksharas from F 33v.

† Often closed solid, but occasionally open.

letter near the top. Each of these methods generally applies to a definite set of letters only, as follows :—

The wedge is attached to	The curved u symbol is attached to	The streamer symbol to
d-, dh-, n-, p-, b-, y-, ṣ-, sh-, s-, h-	*k-, g-, t-, bh-, ś-*	*r-*

This practice is quite in accordance with that of the early Śāradā; but in our manuscript the *śu* has either form, and this is not surprising, since the *ś-* and *s-* are occasionally employed indiscriminately. According to Vogel the wedge symbol is earlier than the curve.

Medial -*ū* is formed most generally by a, more or less, horizontal "streamer" attached to the left bottom of the vertical or to a vertical provided for the purpose (*hū, dyū*). The *rū* and the *brū** are exceptions to this method of formation : the former is very like the initial *u* but without the left up-stroke, and the *brū* is sometimes formed by an angular attachment to the middle of the right vertical, and this is possibly a modification of the initial *ū*. In one case *brū* is formed by the addition of two hanging streamers on the right and the *r-* stroke is shortened.

ṛi

Initial *ṛi* occurs once (63 r.). Medial *ṛi* is invariably the initial *ṛi* sub-joined as a ligature. Examples are shown in table iii, 5. This symbol differs from other Śāradā examples by being less rounded. Bühler terms it angular, and Vogel states that this angularity is only found in the later inscriptions.

The vowel *e*

Initial *e* does not generally differ essentially from the Bower manuscript examples. There are, however, examples in our manuscript similar to the 12th century examples given in Bühler's table VI (11, x-xi) and these examples suggest one of the medial forms.

Medial *e* is formed in two distinct ways : (i) By a slanting stroke over the letter, touching or nearly touching the letter at the right top corner [è]. In late Śāradā this stroke tends to become horizontal. (ii) By a short thick stroke or knob attached to the left top of the consonant by a horizontal line [·e].† Vogel describes this as a wedge ; but in our manuscript it is never wedge-shaped. It is very like a reversed medial *ā*, but with two slight differences : the connecting horizontal stroke is comparatively long, and the terminating knob or stroke has no little flourish but is perfectly smooth.

These two symbols, so different in appearance, seem to be used indiscriminately, at least for certain letters ; and they seem to be very unevenly distributed. For example, F 11 exhibits the second method [·e] 11 times and the first or top-stroke method only twice ; while F 60 does not show the second.

* It is the *r* in the ligature that occasions the so-called exception.

† The symbols shown in square brackets in this and the following paragraphs are mnemonic only. They do *not* accurately represent the symbols employed in the manuscript. The tables and text should be consulted in all cases.

The table on page 97 shows that the two methods are used almost equally in the "M" section and that in the remainder the ratio is roughly 3 to 2.* The following table gives more details for sections A to L, and M :--

		ke	ge	chhe	te	ne	me	ye	re	śe	se
A to L	è Percentages	38	100	100	68	82	100	29	50	33	100
	·e "	62	0	0	32	18	0	71	50	67	0
M	è "	100	50	56	13	33	0	67	22	50	0
	·e "	0	50	44	87	67	100	33	78	50	100
No. of examples		*21*	*11*	*36*	*45*	*23*	*16*	*24*	*21*	*61*	*7*

The values of these ratios depend largely upon the number of cases. As a whole they seem to indicate an inversion of fashion. The M section, which is possibly the more ancient, generally shows a larger proportion of the second method [·e]; but notable exceptions are *ke* and *ye*. In one or two cases complete inversion is exhibited : for example, *me* is formed by the first method [è] only in sections A-L, while in "M" it is formed by the second method [·e] only; and the same is the case with *se*. The best test aksharas of those given in the table appear to be *ke*, *chhe*, *te*, *ne*, and *me*; although all may be used with fair safety. It should also be noted that *ne* is formed by the first method [è] only in all sections by reason of the shape of the mātṛikā. Similarly the first method predominates with *le*.

Vogel relates an interesting anecdote† of the 15th century which tells of a forger changing *me* into *dasa*. The original narrator writes : "In order to express *e* following a consonant the clerks used formerly to write a stroke behind the consonants. But as, in the course of time, the script became changed the writers of to-day write the stroke expressing *e* over the consonant."

Medial *ai*

Initial *ai* does not occur in Śāradā. Medial *ai* is formed in two ways : (i) By two top strokes [äi]; (ii) By a combination of the two *e* symbols [·ài]. In both cases the *ai* symbol is thus a symbol for *ee*.

Of these methods the latter [·ài] is the older. Possibly the earliest example of the former method [äi] occurs in Chambā No. 25, and Vovel assumes that the change took place about A.D. 1100. In the Baijnāth praśastis the double top stroke [äi] is the more common. Later it is invariably used and the top strokes are horizontal.

In the Bakhshālī manuscript both methods are employed, but the second [·ài] predominates : indeed out of 80 examples 61 or approximately three-quarters are formed by the second method [·äi], and 19 by the first or double stroke method [äi]; but in the M section there is not a single example of the first method [äi].

Medial *o*

Initial *o* does not occur. Medial *o* is formed in three ways : (i) By a large circumflex shaped symbol placed above the consonant [ŏ]; (ii) By a combination of

* *i.e.*, 3 è to 2 ·e.

† *Antiquities of Chambā* p. 63.

the *e* sign [·e] with the symbol for ā [·o·]; (iii) By the top stroke for *e* combined with the ā symbol [ȯ]. See table iii, 8.

Out of 252 examples the distribution is as follows :—

First method	ȯ	189 or 75 per cent.
Second method	·o·	38 or 15 ,,
Third method	ȯ·	25 or 10 ,,

But in the M section the percentages are remarkably different, namely 36 per cent., 32 per cent. and 32 per cent. as against 85 per cent., 14 per cent., and 1 per cent. in the remainder.

Medial *au*

Medial *au* is, in our manuscript, always expressed by the circumflex sign combined with the medial ā sign. See table iii, 9. The more ancient method, which combines the second *o* method [·o·] with the top *e* stroke [è] (and of which three examples occur in the Sarāhaṇ inscription) is not employed.

The following tables give statistics of the methods employed in the formation of the medials *e*, *ai* and *o*, etc.

Section.	Medial *e*.		Medial *ai*.		Medial *o*.			Jihvāmūlīya and upadhmānīya.		Style of writing.
	·e	è	·ai	ꞌai	·o·	ȯ·	ȯ	χ	¢	
A	39	46	13	9	6	0	42	1	7	a_1 a_2
B	22	28	6	1	0	0	35	3	1	a_2
C	28	29	1	1	3	2	26	0	0	a_2
D	22	28	2	0	0	0	10	1	0	a_2
E	8	10	2	0	0	0	9	0	0	a_1
F	13	28	6	0	0	0	5	0	0	a_1
G	52	36	1	1	2	0	20	2	3	a_1
H	10	24	8	3	1	0	11	1	1	a_2
J	5	7	0	4	0	0	3	0	0	a_1
K	1	4	0	0	0	0	3	0	0	a_1
L	11	41	4	0	0	0	4	3	0	a_1
M	67	77	18	0	26	23	23	6	4	β
TOTAL .	278	358	61	19	38	25	191	17	16	

Percentages.

Section M . .	47%	53%	100%	0%	36%	32%	32%
Remainder . .	43%	57%	70%	30%	14%	1%	85%
Whole MS . .	44%	56%	76%	24%	15%	10%	75%

A comparison with certain dated inscriptions.

Period.	Medial *e*.		Medial *o*.			References.
	˙e	è	˙o˙	ò˙	ō	
xth century . . .	70	30	58	12	30	Sarāhaṇ Inscription.
xith century . . .	57	43	10	20	70	Chambā Inscriptions, No. 25.
?	44	56	13	10	77	Bakhshālī Manuscript.
xiith century . .	0	100	0	10	90	Ant. Chambā pp. 63, 64.

Ligatures

The ligatures can usually be easily analysed into their constituent elements and only occasionally give difficulty. There are about 90 different digraphs and some 30 different trigraphs.* Some of these occur very frequently, while of others (*e.g.*, *ggh-*, *nch-*, *tp-*, *tph-*, *shn-*) there are only isolated examples.

The consonants, as elements of ligatures, may be classed as (i) those in which the mātṛikā is considerably modified and (ii) those which retain their original shape. Of the former are *k*, *ñ*, *th*, *y*, and in certain cases *r*.

As the first or middle element of a ligature the letter *k* reverts to the form pertaining to the Brāhmī script (Table iv, 1 and 5), but as the last element it retains its normal shape, *e.g.*, *lka* (iv 3). The letter *ñ* occurs only in ligatures and, except as a subscript to *j*, only in isolated examples (*ñch-* iv, 1, *shñ-* iv. 4). As the final consonant its form has little resemblance to the mātṛikā.

The letter *th* as the middle or bottom element is also very different in shape from the mātṛikā. In our manuscript it always takes the form of a spiral and there is no example of the S shaped *-th* (See page 91).

In *tthya* (table iv. 5) the spiral shape is lost but see the Suṅgal example given on page 91.

As a final consonant *y* is also disguised in form. See *kya*, *khya*, *gya*, *chya*, *jya*, etc., in table iv,[1]. Generally the final *y* is of the type shown in *khya* but occasionally the bottom curve is more prolonged upwards on the right as in the *gya* shown in the table.

As a first element *r* is often shortened and sometimes loses its bottom serif, *e.g.*, *rna*, *rtha*, *rya* (table iv. 3) ; and in *rva* it is marked only by a small excrescence on the left curve of the *v* mātṛikā. As the final element it is changed altogether and is something like the virāma but placed at the bottom of the ligature. Nearly all the examples in our text have the stroke continued left and right (table iv) ; but as a middle element the *r* consists of a stroke slanting upwards on the left of the vertical only, *e.g.*, *tryū* and *śrya* in table iv. 5. In *brū* it is treated as a middle element and also loses the right half portion.

* *i.e.*, counting consonants only, but when the vowel signs form part of the vertical scheme they are essential elements also. See for example, *truū* and *dhrī* in table iv. 5. Similarly the circumflex *o*, although detached, is an element of the vertical scheme. The ligatures are really examples of vertical writing while medial *i* and the left-handed ˙*e* are examples of right-to-left writing.

In *sht-* and *shth* the original forms of the *t* and *th* are, more or less, preserved, but a "streamer" is added to the right of *sh* (iv. 4). The word *ashta* "Eight" occurs some thirty or forty times but the form of the conjunct is not consistent. There are, at least, three types: (1) that shown in table ii, 3 as *sht-*, (2) that shown as *shth-* in the same table, and (3) that shown in table iii, 9. The last type occurs some half dozen times in the "M" section only.

The top bar of the second element (or third) of a ligature is often omitted. See *tta, nta, pta, ptra;* sometimes it is given in a shortened form (*śla* iv. 4); and sometimes in its full extent (*mbha* iv. 3).

Numerals.

The numeral figures are shown in table iv. 7.* There is some resemblance between certain of these symbols and certain letters, *e.g.*, the 2 is not unlike the *-ñ* in *jña*, the 4 resembles the *k* but instead of the top bar has a loop to the left, the 5 may be said to resemble the *p*, the 6 the final *m*, and the 8 is not unlike the *ṅ-* and is also rather remotely like the *h*. But this resemblance to letters has probably only a fictitious value—the letter-numeral theory having succumbed.

There are two types of symbol for unity—the semi-circular curve and the almost horizontal line; but generally these have separate functions. The curved symbol is ordinarily used while the straighter symbol is used in fractions. The other differences shown in table iv. 7 are a matter of style in writing. The examples in the first line are all taken from the "M" section, while those in the second line are taken from other sections. The example of 2 in the second line only occurs occasionally. The chief differences in form are shown in the cases of the 6, 5 and 9. The "six" indeed is a fairly safe criterion in differentiating between the work of the scribes. The symbol for zero besides being used as such is also occasionally employed as a sort of symbol for an unknown quantity.

The symbol for "minus" is a small cross placed after the numeral. Hoernle attempted to connect this sign with the Brāhmī *ka* and to show that its use was an indication of considerable antiquity. But he was hardly successful and the details of his arguments in this matter do not now call for any remark. See §§ 61 and 127, Part I. No other signs of operation are employed in connexion with the numerical notation.

Punctuation.

Besides the visarga the single bar | and double bar || are used, and the end of a sūtra is marked by The virāma appears to be used to mark a pause or the end of a verse. On folio 5 recto two punctuation signs, which are possibly cancellation marks, are employed.

Cancellation.

On several occasions letters and words are cancelled by marks like accents placed above the letters (See folios 8 and 9).

* The figures given in the *Indian Antiquary*, XVII, 1888, p. 36, and in Bühler's table IX are hardly representative.

TABLE I

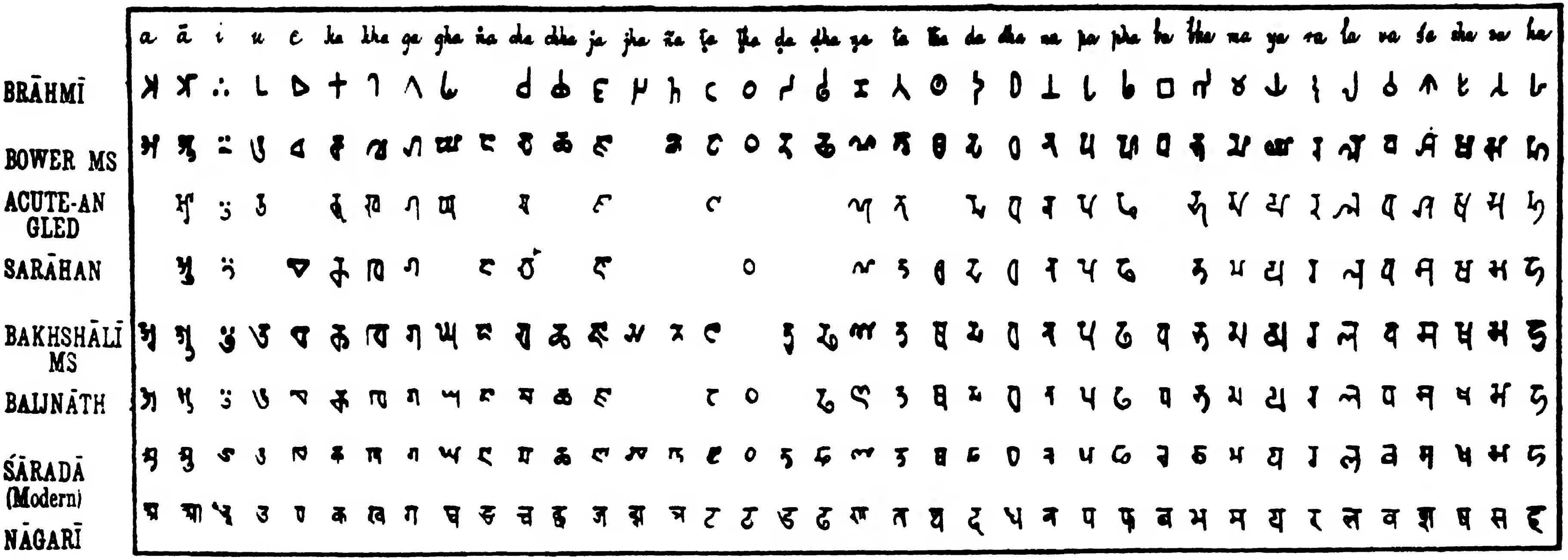
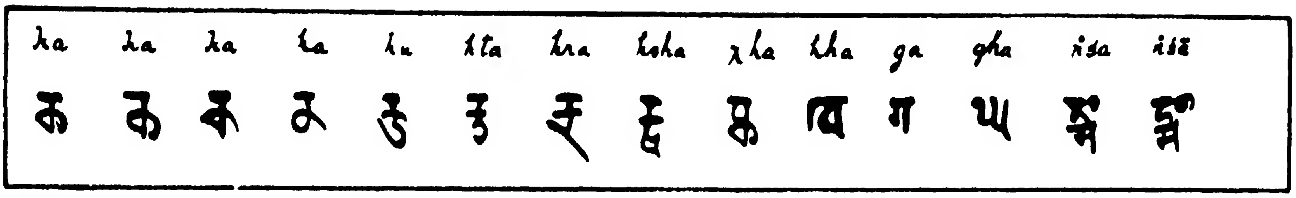

a ā i u e ka kha ga gha ṅa ca cha ja jha ña ṭa ṭha ḍa ḍha ṇa ta tha da dha na pa pha ba bha ma ya ra la va śa sha sa ha
BRĀHMĪ
BOWER MS
ACUTE-ANGLED
SARĀHAN
BAKHSHĀLĪ MS
BAIJNĀTH
ŚĀRADĀ (Modern)
NĀGARĪ

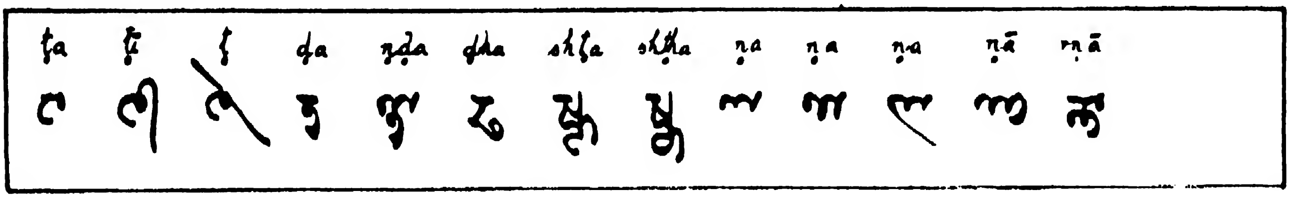
TABLE ii. Consonants

1. Gutturals

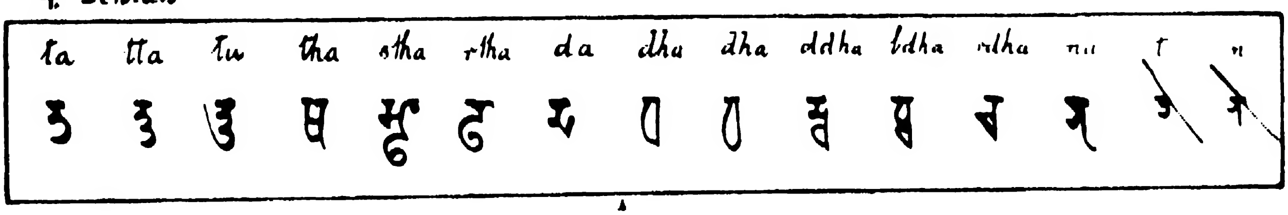


2. Palatals

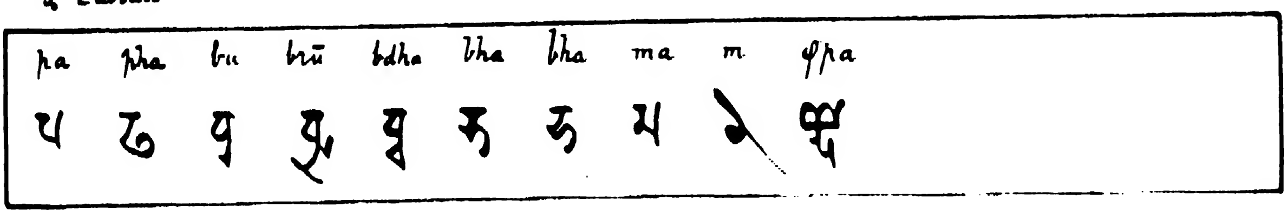
3. Cerebrals



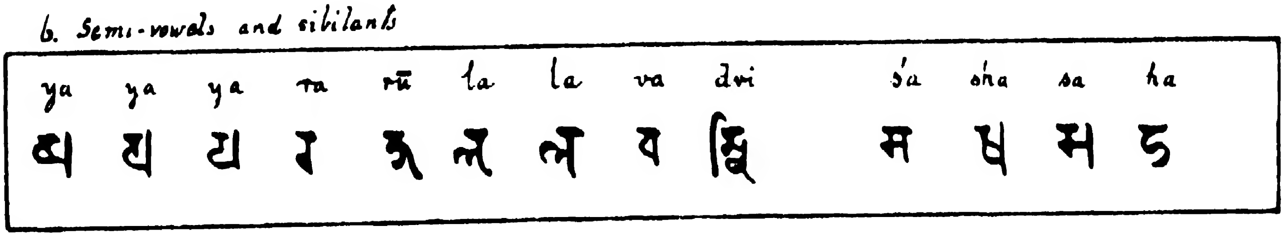
4. Dentals



5. Labials



6. Semi-vowels and sibilants



TEXT.

bhyāsa $\boxed{1}$ rū . . . . [1r.]

turguṇaṁpaṁchaguṇaṁhastagatamdhana*ṁ* | ja . . . .

paṁchaguṇaṁ **25** || navamasūtraṁ **9** || || sūtraṁ || guṇaupṛi

thagrūpayutauyāchanāyuktisaṁguṇauḥ || guṇaneṇaguṇe . ai

rūpahinenabhājitau | viparītayāchanākshiptauguṇasāsyorayaṁvidhiḥ || . .

evaṁsūtraṁ || dvitīyapatrevivaritāsti || daśamasūtraṁ **10** || ||

|| sūtraṁ || aṁśāṁviśoddhyachchhedebhyakuryātatparivartanaṁ || . .

sāsyaṁtataprojjhyadhanānviśavinirdiśet || udā | paṁchānāmvaṇijāma*dhye*

*vikrī*yatekilaḥ tatroktāmaṇivikrītāmaṇimūlyaṁkiyadbhavet . . . .

. . . . daṁ | ardhatṛibhāgapādāṅśaṁpaṁcha*bhā*

. . . . [1v.]

. . . . sya*ṁt*atoprejjhyaḥśadṛi*ś*aṁkṛiyate |

. . . . jātā	**120**	**90**	**80**	**75**	**72**	tatraprojjhyajātaṁ **120**	. **0**	
. . | eshāṁyogakṛi	**60**	**60**	**60**	**60**	**60**	tejātā **437** atosa . .	

. śeshaṁ **377** eśamaṇimūlyaṁ | chaturṇāṁsaṅkasarvasvaṁ || prathamasyasaṅka ar*dhaṁ*

80 | **75** | **72** chaturṇāṁyoga **317** prathamārdheṇaśashṭibhiryutaṁ **377** prath*am* . .

. *pra*thamadhanaṁ | tṛitīyachaturthapaṁchamasyadhanaṁsarvasvaṁ **347** dvitīyā tṛibhāgaṁ **3** .

. . y*u*taṁ **377** eśadvitīyasyadhanaṁbhavati || punaprathamadvitīya chaturthapaṁcha

. . svaṁ **357** tṛitīyasyapādaṁ **20** eshayutaṁ **377** eshatṛitī*yasya* . .

. . . ti || punarapiprathamadvitīyatṛitīyapaṁchas*ya* *362* . . . . . .

. . . **eshach** . . . . ***bhavati*** || . . . .

[2r.] syadhana*m*bhavati || athapratha . . . .

tyaṁśashṭiśeshaṁ 377 || athadvitīya | 75 / 72 | sya / ya | 120 / 30 / 80 / 75 / 72 | eva*ṁ*

. . . . ṁdvitīyasyabhavati || athatṛitī . . . syakṛi*yate* | 120 / 9. / 2. / 7. / 7.

. . 377 tṛitīyasyadhanaṁbhavati || chaturthasyakṛiya . . . te | 120 / 90 / 80 / 15 / 72

evaṁ 377 chaturthasyadhanābhavaṁti || paṁchamasyakṛiyate | sthā

panaṁ | 120 / 90 / 80 / 75 / 12 | evaṁpa*ṁ*chamasya 377 || eshamaṇimūlyaṁpra

|| udā || || anyonyaviditavibhavaṁ

. . . vaṇik . yaṁ | tṛi . . . . dalaṁ*ta*thā || .

[2v.]* . . . . ṛi . . . .

. . . . aṁśāṁ | 7+ / 12 / 12 | 3+ / 12 / 12 | 5+ / 6 / 6 | vi . .

. . . . . *śo*ddhyavi*so*dhayet riṇaṁsthitaṁ | e . . . sha . .

. yate | 19 / 12 | 7 / 4 | 11 / 6 | kuryātatparivartanaṁ | 12 / 19 | 4 / 7 | 6 / 11 | *ch*chhe*de*

. . ṁjātaṁ | asya | 924 / 1463 | 836 / 1463 | 798 / 1463 | projjhyajātā | 924 |

. . . . e . . . shāṁyutiṁkṛiyate |

. kṛit . . jātā | 2558 | chchhedaprojjhyaṁ 1095 etanmaṇimūlyaṁ

* The bottom portion of this page is blank.

tvāssamadhanājātāprastamūlyaṁtaduchyatām | 4 . || 5 go || 6 śā | e [3r.]

. prastamūlyaṁ 2 | 3 | 6 | dattaissamadhanā jātā 17 | 17 | 17

śamasūtraṁ 13 || ✣ || sūtraṁ || ekayutānāsaṁkhyādvi

. . . *hinā*cha || evaṁtāvatkāryaṁyāvatpurushaisamābhavati

. *ptam* . *pa*trebhilikhitasthitachaturdasamasūtraṁ 14 || ✣

. *traṁ* | *ga*tisyaivaviśeshaṁchavibhaktaṁpūrvagaṁtunāḥ tenaivakālaṁbhavatis*tha*

. . . . kenatu || udā || addhyardhayojanagateśatahaṁtasa . . .

*ya*syahayānnavaḥ ūshṭrādaśatṛitī*ya* . . . . [3v.]

pradattaṁchaparasparaṁ | pṛithagdhanaṁtuvaṇijāṁmūlyaṁvāprāṇināṁpṛithakyad . . . . .

. vaktuṁtatomechchhindhisaṁśayaḥ | 7 a || 9 ha || ū 10 vaṇijjakā 3 d . ya .

7	a	9	ha	ū	10
		1			1

. ṇikpiṇḍahataṁ | piṇḍa 7 | 9 | 10 | deyaṁ 3 śuddhaśeshaṁ 4 | 6 | 7 *tata*

*pa*rasparakṛitaṁguṇitajātaṁ 168 | 168 | 168 | svaśeshenatuvi*bhak*

168	168	168
4	6	7

labdhaṁ 42 | 28 | 24 | eshapratyaikamūlyaṁekaikasya

*gu*ṇitājātāni aśvaihayai ūshṭrebhyaḥ 294 | 252 | 240 | ekai*ka*

. . . . ṁjātā 262 | 262 | 262 |

[4r.] . t . . y . . . ch . . . .

yojanapaṁchakaṁ | saptadinānitasyaivagatasya | paratadvitīyanavayojan*aika gatake*

. . tām | $\frac{1}{1}$ di $\frac{5}{1}$ yo | dina $\frac{7}{1}$ gatasya | gatayojana $\frac{35}{1}$ | dvi $\frac{1}{1}$ di 9 yo | **gatisyaiva viśeshaṁchaka**

. . yate | gati 5 | 9 | viśeshaṁ | 4 | vibhaktaṁ | $\frac{1}{4}$ | **pūrvagata 35 eshapādergunitaṁ 3**

. . bhirdinaisamagatībhavantinavayojanaṁ || pratyayatrairāśikena

$\frac{1}{1}$ di	$\frac{5}{1}$ yo	$\frac{35}{4}$	
$\frac{1}{1}$ di	9 yo	$\frac{35}{4}$	pha 31

. . udā || ashṭādaśayojanā ekenadineyāti | tasyāshṭadi

. **gatasya | dvitīyapaṁchaviṁśeyojanādineyāti | kenakāle**

. sāsyatāṁ || evaṁekādaśamapattrebhilikhitapūrvepi || **paṁchadaśama sūtraṁ 15**

. . . . **yorviśeshakartavyaṁ uttarasyaviśeshataḥ vibhaktamsuttare**

. . . . **lṇasāsyaṁg** . . i . . .

[4v.] . . . .

. . taraṁ | 2 | vibhaktaṁ | $\frac{1}{2}$ | **ādiśesha** $\frac{2}{1}$ | **jātā** | $\frac{1}{1}$ | dviguṇaṁ | 2 . rūpasaṁyutaṁ | 3 | **esha**

. . . **saṁkalitepratyayāpadaṁhīnā ubhayesthāpitavyā rūpoṇākaraṇephalaṁ** | **21 .**

. . . . . . **kiṁprabhūtepilikhite || shoḍaśamasūtraṁ 17 ||** **217**

sūtrobhrāntimasti

. . . . *ā*dyorviśeshadviguṇaṁchayaśuddhirvibhājitaṁ **rūpā . i**

. . . . . *g*atisāsyaṁtadābhavet || || **udā || dvayāditṛi chayaschaivadvi** . .

. **dikottaraḥ dvayochabhavatepaṁthākenakālenasāsyatāṁ ||** sthāpanaṁkṛiyate | e . .

$\frac{3}{1}$ | pa $\frac{0}{1}$ | dvi | ā $\frac{3}{1}$ | u $\frac{2}{1}$ | pa $\frac{0}{1}$ | **karaṇaṁ | ādyorviśesha** . .

. . . . i . . . . . .

ā 5/1	u 6/1	pa 0/1	dha 0/1
10	u 3/1	pa 0/1	dha 0/1

karaṇaṁ | ādyorviśeshaṁādi . . [5r.]

chayaśuddhichayaṁ 6 | 3 | śuddhi | 3 | ādiśesha 5

dviguṇam 10 uttaraviśesha 3 vibhaktaṁ | 10/3 | sarūpaṁ | 13/3 | anenakā . .

samadhanābhavanti // pratyayaṁ rūpoṇākaraneṇa phalaṁ | 65 / dvi 65 |

ashṭādaśamasūtraṁ 18 || dinagamana mādirahitaṁ . .

ṇaṁtachchhottareṇa ✣ saṁyutaṁ | pratinihita ātmaguṇaṁjñeyaṁ

. . . . . o . i | ashṭhottaraguṇitekshepasaṁjñakodatvāmūlaṁprati . i

. . . . hataṁ | 30 | [5v.]

. . . . dinagamanamādirahitaṁdinagamanayojana ∅ pañcha | 5 | ādi .

. . . . 3 | rahitaṁjātaṁ | 2 | dviguṇaṁ | 4 | tachchottareṇasaṁyutaṁ | 8 |

. . .ātmaguṇaṁ | 64 | eśakshepasaṁjñakorāsi | ashṭottarasaṁgu . i . .

. . labdharāshi | 30 | ashṭaguṇaṁ | 240 | uttareṇaguṇaṁ uttaraṁ | 4 | .

. . . . guṇitaṁjātaṁ | 960 | kshepasaṁjñakodatvā | tatrakshepasaṁjñ . .

4 | yutaṁjātaṁ | 1024 | asyamūlaṁ | 32 | pratinahita . . . .

i . m 8 yutaṁjātaṁ | 40 | u . . . . ṁ . . . .

śikepratyaya || 1/1 | 5/1 | 5/1 | phalaṁy . . . . [6r.]

nenassaha 55 eśasamābdhānaṁ || udā || ādipaṁcha uttaraṁtr

ṇinaroyojanagamyate | dvitīyapratidinaṁssaptagatasya .

napaṁchakaṁ | kenakālenasamatāṁkatthyatāṁgaṇakotta . .

. . 5/1 | u 3/1 | pa 0/1 || pratigati 7 | dina | 5 | paṁchadinaga . .

yojanikaṁyojana | 35 | karaṇaṁ | dinagamanamādiraṁi

tatradinagamanaṁ | 7 | ādirahitaṁ | ādi | 5 | rahitaṁ . . .

[6v.] . . . . | 8 | anenaguṇitaṁjātaṁ | 84 | . . .

. saṁjñakodatvā | tatrakshepārāshi | 49 | datvājātaṁ | 889 | .

. . . dānadadātisamaṁ | karaṇokriyate | sūtraṁ || akṛiteśli . .

. . . . chhedodvisaṁguṇaḥ tadvargaḥ dala | saṁślishṭha | hṛiti .

. . . yaḥ anenasūtreṇaslishṭhamūlaṁānayasvamatimā . . .

. *telabdhaṁ* mūlaṁ | 29 48 58 | pratinihitaṁ | 7 | anenayutaṁ | 36 48 | .

. . . | 2136 | dviguṇottarabhājitaṁ | ta . o . .

. . . . . . e . . . .

[7r.] . . . .

7 dalitā | 16 60 cha | 7 58 | . . . .

9 sāsyeyu | 33 60 li | taṁ | 737 | padaghnā | tatrapadaṁ | 178 29

. anenaguṇitaṁ | 6 60 vi | jātaṁ | 58 | | 65593 841 | *slish* . . . .

. tyaśeshaṁkṛiyate | śe 6 29 | 65569 841 bhā | gehṛite 7 . . .

. . . . pratyayaṁtrai rā ___ śikena | 1 7 yo 1 1 | 17 . .

phalaṁyojana 42 śe | 28 29 | niyataṁtena

| 77 | e . o . viṁśatima *sūtraṁ* 19

[7v.] . . . . 1 . . . .

. . matāndanaḥ | ā 3/1 | u 4/-1 | pa 0/1 || nityadatta 7/1 | ādiṁviśoddhya . .

| 3 | niyataṁ | 7 | viśeddhya | 4 | uttarārdhenabhājitam | uttaraṁ | 4 | a . .

. . . . . jitaṁ | 4/2 | jātaṁ | 2 | labdhaṁsarūpa | esharūpādhikaṁ | 3 | eśakāla

. . . . 3/1 | u 4/1 | pa 3/1 | ruponākaraṇenaphalaṁrū 21 || dvitīyasyatrairā .

| 1/1 di | 7/1 | 3/1 di | pha rū 21 || eshassamadhanājātā || *udā*

. . . . . . ka uttaradvayaṁdvitīyapaṁchapratyahaṁ | kenakālenasama*tām* .

. . . ṇakottama || ā 1/1 | u 2/1 | pa 0/1 || niyatanityaṁ 5/1 || ādiṁviśo*ddh*

|| sūtraṁ dviguṇaṁprabhavaṁśuddhādviguṇā daya . [8r.]

ttareṇabhajechchheshaṁlabdhaṁrūpaṁvinirdiset || udā || va .

bhṛitakaχkaśchitatraikodashamāśakaṁ | pratyahaṁkurutetatrakarmaṁbha . i

mānavaḥ dvitīyaṁkṛiyatekarmaṁdvyāditṛitayaruttaraṁ | padaṁtatra .

. kenakālenasāsyatāṁ | ā 2/1 || u 3/1 || pa 0/1 || prati 10 || . .

dviguṇamprabhavaṁśuddhā | prabhavaṁ | 2 | dviguṇaṁ | 4 | niyatapunadvi .

. . . . . . o . . taṁ | 16 | (uttarārdheṇabhājayet)* uttaraṁ . .

*The portion within the brackets is deleted in the MS.

gantata | ekā —śchagamanajñeya yutāssaṁguṇya . . . . . . [8v.]

|| niyorathośvairdaśabhiryujyatehayapaṁchakaṁ | gaṁtavyaṁyojana . . . .

. .ośvakimudbhavet || ha 10/1 | hayalagnarathasya 5/1 | gantavyoyojana . . . . . .

. . yorvibhajyagantavyaṁ | tatrahayā | 10 | gantavyaṁyo | 100 | atobhāga . . .

. . bdha | 10 | tatrayuktāśva | 5 | etaissaṁguṇyapariyogajātaṁ . . .

. . . yojanānyaikośvarūḍha | pratyayaḥ paṁchabhiśśatasaṁguṇyajātaṁ . .

. . . kṛiyate || yadida . . shayojanā ɸ paṁchas . . . .

. . . . r . shṭh . . ch . . . . udā || [9r.]

. . . . . māptaṁdvijanmabhi | tatpunastesamaṁbhaktvāda .

. ssamāptavān saṁkhyāyaχ katimmāchakshukativiprāχkati . r . . ṁ*

. 1/1 | u 1/1 || pa 0/1 || labdhaṁ 10/1 || karaṇaṁ || labdhaṁdviguṇitaṁkṛitvātatra .

bdhaṁ | 10 | dviguṇaṁ | 20 | tathādyūnaṁ | 18/1 |

. . . . vibhājitaṁ | atrottaraṁ | 1 | anenabhaktvājātaṁtade

. . kaṁ | 19/1 | ayaṁprashṇāprāhūṇā ekonaviṁśati | sthāpa

. *pratyayaṁ* | ā 1/1 | u 1/1 | pa 19/1 | rūpoṇākaraṇenaphalaṁ | 1

*The first m is deleted in the MS.

[9v.] *yojana* . . . . . . *serathagama* . . . .

. yo $\frac{6}{1}$ | śayo $\frac{1}{1}$ || yo 70 gantavyaṁ | adhvasaṁ*yoga* | $\frac{7}{1}$ | *bhaktaṁ* | $\frac{1}{7}$ |

. enaguṇitājātānlabdha | 10 | dviguṇaṁ | 20 | eshālpasyaḥ || atha

. nā ayaṁkālojñeyaḥ anenakālenashshaṭ yojanānigantavyaṁ | ā

. . yamekayojanikasyasamāgamobhavati || tadyathātrairāśi . e . . .

. . . yadyekasyashaṭ yojanātadāviṁśānāṁkiṁ | $\frac{1}{1}$ | $\frac{6}{1}$ yo | $\frac{2}{1}$ . . . . .

. . . saptatiśoddhyaśesha atrassaptati | 70 | āgatapaṁchāśa | 50 | *adhva*

. . . . . . i . . ṁ || $\frac{1}{1}$ di | $\frac{1}{1}$ yo || $\frac{20}{1}$ di | pha yo . .

[10r.] sūtraṁ 24 . . . .

. . *dhānta* ✣ saṁguṇa . . . . pravṛit*tirgu* . . . .

vinirdiset || udā || tṛibhāgamalada*gdh*asyatridhāntasyaiva . .

*ash*ṭottaraśatānidattaṁkiṁśeshaṁvadapaṇḍita | 108 $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$

kṛitvārūpakshyaṁpārthajātā | 32 | śesha || pra | 3+ 3+ 3+

thamabdhāntekshayaṁ | 36 | śeshaṁ | 72 dvitīyabdhāntekshayaṁ | 24 | śe*sham*

tṛitīyabdhāntekshayaṁ | 16 śeshaṁ | 32 | pratyayaṁkṛiyate | sthāpanaṁ

0	1	1	1	bhā śeshaṁ	32	phalaṁmūlā 108 || atha .
1	1	1	1		1	
	3+	3+	3+			majātikṛiyā*kara*ṇaṁ . ī . .

. . . . pratyayaṁ [10v.]

. . . . tṛibhitryashṭabhāgasaṁyutaṁ .

. . . . taḍā | 3+ 1 3+ | shṭottaraśatā χ kiṁ | 27 8 | 1 1 | 108 1 | pha śe .

. . . . yadyekasyatrayastraya ashṭha

bhāgātadādvātṛiṅśānāṁkimiti | 1 1 | 3 3 8 | 32 | phalaṁ 108 1 | udā ||

. . kṛiddhāntasyalohasyadaśāṁshākshīya | testrayaṁ | saptatedviguṇa

. . śchakiṁśeshaṁvadapaṇḍitaḥ || 3 10 | 140 1 | kṛitvārūpakshayaṁpāsthamiti

. . rūpaṁ | 1 | 3 10 | kshayaṁkṛitvā | jātaṁśesha | 7 10 | mūlaṁ

40 | anenaguṇitaṁjātaṁ | 98 | kshayaṁ | 42 | evaṁ 140 | *pra* . .

. . . . na | 7 . . | 1 1 | 98 . | phalaṁ 140 . . . .

. . pratyayaḥ | 0 1 1 3+ 10 | . . . . [11r.]

palākrīte pa | latṛibhāgaṁkshayavrajati | ashṭādaśa . . . . .

. . thatāṁbrū | hi || 1 3 3 bhā 2 | 18 1 | karaṇaṁ | addhyardhapalaṁśchhe.

idaṁ | 2 9 | kṛitvārūpaksha | yaṁrūpaṁ | 1 | kshayaṁkṛitvājātaṁ | 7 9 | 18 1 |

guṇitaṁjātaṁ 14 | kshayaṁ | 4 | pratyayatrairāśikena || addhyardhapala

krītetṛibhāgaṁkshayagachchhati | ashṭādaśapalakrītākiṁkshayaṁvad.

ṇḍita | 1 1 2 | 1 3 | 18 1 | phalaṁ 4 1 | punatṛibhāgādivardhaṁtadāchatu

kimiti | 1 3 | 1 1 2 | 4 1 | phalaṁ 18 1 ||| udā || chaturbhāgamala . . . .

ṛṇaśatapaṁ . . . . . .

[11v.] . . . . . | . . | athapratyay . .

. . . . 1 5 1 64 | 4 1 4+ | phalammūlā 500 || punarevaprastārakra

maṁ || 500 1 | 1 1 4+ | 1 1 4+ | 1 1 4+ | 1 1 4+ | phalaṁ tediti

śhaṁsthāpya

4+ 1 4+ 1 4+ | śesha 158 to 1 śe | 1 64 | anyaṁchaturthapratyayaṁkṛiya

te | 0 1 | 1 1 4+ | 1 1 4+ | 1 1 4+ | 1 1 4+ bhāśesha 158 1 5 1 64 | phalaṁ 500 ||

*ādyaṁkshaya*ṁ 125 śeshaṁ 375 || dvitīye kshayaṁ 93 to

. . . āśa 9 śeshaṁ | 281 1 4 | kshayaṁ | 70 5 16 | śeshaṁ | 210 15 16 | kshayaṁ | 52 47

. . *shaṁ* | 158 | eśasarva trakartavya ||

. . .

. . .

[12r.] . . . . . *tuprasthama* . . . .

. dhunāstathāḥ ambh*asa* . . . . | . . . .

kṛitvārūpakshayaṁpāstamiti. tatrakshayaṁ.pāstamiti .tatrakshayaḥ rū . . .

nyaśeshaṁ 3 3 3 3 4 / 4 4 4 4 1 | gadyūtigadyūtigatvātprasthaṁ . . t . . . .

gadyūti | yojanaṁ | chatuprasthai āḍhakaṁ | tadādhāntaśorguna

. . tataḥ | 81 256 | āvṛittipravṛittirguṇanaṁ*ta** *taḥ || 4 || anena . .

ṇ . taṁjātaṁ | 81 64 | eśamaddhubhāgābhāgehṛitelabdhaṁ | madhuprastu 1 .

śe | 1 16 | aṁbha bhāgāprasta 2 kuḍava 2 śe | 15 16 | . . . . .

. ktiprakshepake āḍhakāśo*ḍ*ashakuḍavābhavanti | 16 | atoma . . . . .

. . . . śeshaṁ 12 . . . .

. . . . 16 e [12v.]

. . . . prasthakuḍavā | 4 | 3 śeshachatvāra .

. . kuḍavaḥ $\frac{2}{1}$ $\frac{2}{1}$ śeshāchakuḍavāpītā | ma 7 9 punachatvār .

kuḍavābhuktaṁśe 4 4 shaṁ $\frac{81}{16}$ | $\frac{175}{16}$ | jalabhā 1 1 gaṁ | madhukuḍava 5 4+ 4

jalakuḍava | $\frac{10}{1}$ śe $\frac{15}{16}$ | evaṁkuḍava 16 || || udā || datvāśulkaṁ

chaturbhāgaṁashṭau āṇītakuṁkumā | chatuśulkaśālaistukiṁśeshaṁvada

. ṇḍ . ta 8 1 | 1 1 | karaṇaṁ | kṛitvārūpakshayaṁpāstaṁ | pāstaṁ 8 3 gu . i 1 4

. . . . taṁ 4+ | 6 | śulke | 2 | śeshaṁ | 6 1 | 1 1 4+ ane | nagu

. . . . ṁ . kshayaṁ 1 1 2 | śesheṇa | 4 1 2 | 1 1 4+ | datvāguṇitajātā | 27 . . .

. . . . . ta [13r.]

. 1 1+ 4 1 1+ 4 1 1+ 4 1 1+ 4 | guṇitaṁjātaṁ | $\frac{81}{32}$ | || . u . y . | 8 4 | 4

phalaṁ | $\frac{81}{32}$ | phalaṁ || punānyaṁ | 8 1 1+ 4 1+ 4 1+ 4 1+ 4 | phalaṁ | $\frac{81}{32}$ | punapratyayaṁ | 0 1 1+ 4 1+ 4 1+ 4 1+ 4 | bhā 32

phalaṁkuṁkuma 8 || udā || tribhāgashaḍbhāga | paṁchā . .

. guḍapiṇḍāshṭabhārakaṁ | kiṁśeshaṁdattabhirbhavet . .

8 | 2 | 5 | 4 guṇitaṁjātaṁ | 32 | etatphalaṁ || || u .
1 | 3 | 6 | 5 | 9

chatu ϕ paṁchakalā | bhenadaśadroṇātprayojita

dvaitṛibhistukiṁlābhaṁkatthyatāṁguṇakottama || 10 | 5 | 5 | .

ṁ . . . 125 |

[13v.] . . . . 0 phalaṁdro 19
1
. . . . 4 | 10 | 1 || udā || kasyā
1 ā 2 1 4
. . . . jjakasya 4 4 dro shashṭhisvadale 1 nakshayaṁgata
1 pra 0 4
u . vṛiddhyātribhā, 4 gena 4 ā pra svapādena 1 tatojjhitaṁ | .
ku 2 4
ddhyātupaṁchabhāgenastathā ku 4 prasthi vṛiddhidvayogataṁ | kāvṛiddhi .

. . . . . syākiṁvāśeshaṁtaduchyatāṁ 60 1 1 1 1 rūpalā .
1 1 1 1 1
. . . . . tā 36 || pratyayaṁpu 2+ 3 4+ 5 nāsyaiva

1 1 1 1 bhā 36 phalaṁ 60 || punānyaṁpratyayaṁ 60 phalaṁ . .
1+ 1 1 1 1
2 3 4+ 5 1 1+
. . . . syevaṁmūlaṁnajñāyate || 0 1 1+ 2 1 3 1 4+ 1 5 pha 2 *laṁ*

[14r.] *la*t . dhan . | yasyatanmayatāchakshu . . y . . . . . . d . | . . . .
. pahṛitaśulkapiṇḍaṁ 24 || karaṇaṁ || kṛitvārūpāksha . . 3 . . .
. pāsta | 2 3 4 | jātusaṁguṇyajātam | 2 | etāvadapirūpasaṁśu
3 4 5 5
. äjātaṁ | 3 | anenabhaktvāśulkapiṇḍaṁguṇitaṁjātaṁ | 40 |
5

eśapiṇḍaṁpratyayaṁ 2 40 guṇitajātaṁ 16 śeshaṁ | 24 |
5 1
. ṁ 40 | | anyaṁ svapratyayaṁ 40 phalaṁ 16 kshayaṁ 2 . .
1
. vaṁ 40 || || udā || guḍapiṇḍa 1 jñātatulyośchatvari . .
3+
1+
. avyeguḍaṁ | trichatu ɸ paṁchashaḍvṛiddhyā 4 chatvāriṁśavekshaya .
1
. . . . e . . 5+ . . . . e . .

. . . . 3 u . . . . [14v.]

. . . udā || ajñātarambhalohasyatri chatu φ paṁchakākshaye | sapt .

viṁśatipiṇḍasyatridhāntaśeshyadṛishyate | kiṁsarvaṁvadatatvajñakshayaṁcha .

makatthyatāṁ | 1/3 | 1/4 | 1/5 | śe 27/1 | karaṇaṁ | kṛitvārūpakshayaṁpāstha | 2/3 | 3/4 | 4/5 |

guṇitaṁjātaṁ | 2/5 | rūpakshayaṁ | 3/5 | anenaśeshaṁbhaktaṁśeshaṁ | 27 | bha

. ṁ jātaṁ 45 asyasaptāviṅśa | patyaśeshaṁ 18 || etakshayaṁ || udā ||

. rikshīnasyalohasyatṛidhāntaṁpaṁchamāśakaṁ | najñāyatetpravṛittikāṁna

. . shapradṛiśyate | pravṛittiśeshaṁyopiṇḍaṁkevalaṁviṁśatisthitaṁ | a .

. . . ṁpravṛittisyākiṁvāśeshaṁvadaśvame | 1/3 || 1/4 || 1/5 | kṛitvā . . .

. . . . *kshayasyacha* . r . . . . . . śaḥ . . . . [15r.]†

yaṁkaśchiyadiśakyastaduchyatāṁ || etanmesaṁśayaṁprājñaddhdhāntaksh*ayaṁ*

vichāraṇāḥ | 2/3 | 3/4 | 4/5 | ksha śe 32/1 | karaṇaṁ || bdhāntasaṁguṇyaguṇi

. jātaṁ | 3/5 | rūpaṁdadyā 8/5 | bhāgehṛitelabdhaṁbhaktve*ti*

. jātā | 5/8 32/1 | phalaṁ 20 eśasāpravṛitti | śeshaṁ 12 . .

. . 32 || paṁchaviṁśatima sūtraṁ || 25

† The bottom portion of 15 recto is blank.

. . . . y . . pravṛittibhav*etsakhe* || [15v.]

. 3 | . 3 | 1/3 | 1/3 | śe 16/1 | karaṇaṁ || dhāntaśoghātitaṁtena | rūpa

kshayaṁkṛitvājātaṁ | 2/3 2/3 | 2/3 | 2/3 | guṇitaṁ | 16/81 | bha.

81/16 śeshenaguṇaye | śeshaṁ | 16/1 | . guṇitajātā | 8 .

. . pravṛittirityarthaḥ || athānyavidhikalāsavarṇe | chaturdhānta

. . lohasya ekāśītischadattavān kiṁśeshaṁvadadharmajñaya .

. . ṇitekṛitaṁśramaṁ | 81/1 | 1/1/3+ | 1/1/3+ | 1/1/3+ | 1/1+/3 | phalaṁśe 16 || pu .

. . . yaṁkriyate ||| mūlaṁnajñāyate

. . 1/1+/3 1/1+/3 1/1+/3 1/1+/3 bhāśe 16/1 | phalaṁlohapala 81 ||

[16r.] *vi*bhakt*am*jāt*am* | 2/9 śe 10/1 | dh . . . . . || . . . .

| 9/7 | anena guṇitaṁjātaṁ | 90/7 | 2 bhāgehṛitelabdhaṁ 12 ||

. asyapratyayatrairāśike | na | 6 | 1/1/2 | 10/1 | pha 12/6 ||

|| udā || mākshikagghaṭakasyaivadvitṛibhāgapravardhi | 7 | . .

. īyedvipaṁchamobhāgotṛitīyedvisaptako*dbhavaṁchaturthe* . .

. vaṁbhāgaṁevaṁjātapalatrayaṁ | babhūvāsaulkikaihṛitvā . .

. rvaṁvadapaṇḍita | 2/3 | 2/5 | 2/7 | 2/9 | śe 3/1 | dhāntaśo*ghā* .

. ātimiti | kṛitvā . . . . . .

[16v.] . . . . *sūtraṁ* || idānisuvarṇakshayaṁvakshyā

. yasyeda*ṁ*sūtraṁ || kshayaṁ ✢ saṁguṇyakanakāstadyutirbhāj*a*

yettataḥ saṁyutairevakanakairekaikasyakshayohisaḥ || udā || *e* .

. . . ssaṁkhyāsuvarṇāmāshakairiṇai | ekadvitṛichatussaṁkh*yā*

* *rahitāsamabhāgatāṁ || stāpanaṁkriyate | eshāṁ

. . + | 2+/2 | 3+/3 | 4+/4 | karaṇaṁ || kshayaṁsaṁguṇyakanakādi . i

. . kshayenasaṁguṇyajātaṁ | 1 | 4 | 9 | 16

. . . ti | eshayuti 30 | kanakāyuti 10 anenabhaktvā la . .

[17r.] || udā || ekadvitṛi | 1/10/1 | 1/30/1 | ./4/1 | phamāśe 12/1 | . . . .

*va*rṇaprojjhitā ime māśakadvi

tṛitāṁchaivachatuḥpaṁchakarāṁśakaṁkiṁkshayaṁ | 1/1/2 | 2/1/3 | 3/1/4 | 4/1/5 | k .

. kshayaṁsaṁguṇyakanakā eśasthāpayate |

| 1/2 | 2/3 | 3/4 | 4/5 | stadyutirbhājayetatataḥ haraśāsyekṛiteyut*aṁ*

| ·63/·0 | saṁyutaiḥkanakairbhaktvātadākanaka | 10 | anenabha*ktaṁ*

. . . jātaṁ | 163/600 | eśa ekaikasuvarṇasyakshayaṁ || . . .

. . . śikenakar . . . ||

.	.	.	. 200	. . *srūṇuśvame* | kr*a*m*e*n*a*dv*a*y*am* [17v.]
·0	163	4	pha 163	shādi uttare ekahīnatāṁ | suvarṇaṁ*me* .
1	60	1	150	sammiśryakatthyatāṁgaṇakottama || sthāpan*aṁ*

4+	5+	6+	7+	8+	9+	1+	2+	3+	kshayaṁsaṁguṇya
.	6	7	8	9	10	. 2	3	4	jātaṁ |

. . . 30 | 42 | 56 | 72 | 90 | 2 | 6 | eśāṁyuti | 330 || kana

. . . ṁyuti 45 | anenabhaktvā labdhaṁ $\frac{330}{45}$ paṁchadaśabhāgechchhe*da*

. | phalaṁ | 7 śe $\frac{1}{3}$ | eka ekaikamāśa kakshayaṁ | pratyaya . .

. . . 330 | $\frac{1}{1}$ | phalaṁ 22 | evaṁsarveshāṁpra*t* .

. kaḥ kā*ṁ*chanaiyadbhavelabdh*a*sakshayajñātamāśakā || udā || . . . . [18r.]

. shakoprāptodvau . prāptaṁchapaṁchabhi | tryaśchakatibhi ϕprāptāshaḍeva

. nikevalaṁ | chaturbhimāshakairhīṇaṁkaṭidṛishṭhvāmayāsakhe | trayaścha

katibhi ϕprāptāsuvarṇāṁmaśakovadaḥ

1	2	3	6
2	5	0	4+

karaṇam

aprāptasaṁguṇākaṭīditi | $\frac{6}{1}$ |

. i . tvāra | 4 | saṁguṇyajātaṁ | 24 | kāṁchanānitatojjhitaṁdvābhyāṁ

. . kapaṁchabhidvayaṁsaṁguṇyajātam 2 | $\frac{10}{1}$ | tadyuti 12 | hi . ā 2 . .

. . hitvājātaṁśeshaṁ 12 | aprāptagaṇḍikai . . i . . . .

R 2

[18v.] , ... shṭaviṁśatimasūtraṁ

|| sūtraṁ ūnaissaṁguṇyakanakātatpiṇḍaṁchaviśodhayet su

varṇakanakābhyastārāśisheshaṁvibhājayet aprāptagaṇḍi

. śaśuddhenakanakenatu | yallabdhaṁtatpramāṇaṁtugaṇḍikāyā vi . i

. . t || udā || ekadvitrichatussaṁkhyā aprāptamāśakāni tu

. kadvitrichatuśsaṁkhyā ekatrāvartitākilaḥ gaṇḍikājñā . .

. . kā ūnaikādaśamāshakai | aprāptajñātakanakaipra . .

. . . . yaḥ | 1 1 | 2 . | 3 . | 4 . | 0 . || karaṇaṁ || ū . ai

[19r.] *Folio 19 is blank.*

[20r.]

. . . . . †. | asyaivaϕprashṇasyā . . rovi

. . 20 rakti 1 . 4 | dhā 1 a 1 0 ya 4 1 4 | su 1 chhe 80 raktisurakti 1 / 1 1 1 | pha 4 y . dhā

. pā 1 || punatṛi | tiyāsyaiva | 2 20 / 1 1 / 4 | 1 3 1 2 1 4 | 1 2 chhe | 1 chhe 1 4 | 1 1 1 2

. chhedaṁ 6 dhā | draphadhā 4 ya 1

pā 2 mū . || suvarṇasyamāṇaṁsama

. dā || sapaṁchanavabhāgāṇidināni trayodasaḥ

. . . . ṇāṁkiṁ || ||

[20v.]

. . . . y . | ·4 / 1 | 5 / 1 | . 1 chhe 7 *chhe* . . . .

pa 12000 | . . . .

. . *araṇaṁ* | sarposhṭādaśahastopraviśatyārdhāṁgulaṁśanava

. . . i ekaviṁśatibhāgaṁmapaharaṁti | pratidinenaḥ kiṁkāle

. . vilaṁsaṁprāpyate | 1 2 1 9 | 1 21+ | 1 1 | 1 360 | 18 / 1 chhe 24 / 1 aṁha

. . . 2 mā 4 di 10

. . . || kiṭa χ kilārdhāṁgulaṁdivasedivase

. . . . *paṁchāśā* . . . ke . .

[21r.]

. . . . *tṛibhāga*

. . . *dine*tatha | *tṛirūpapaṁchabhidinai* | . shāṁda

. 1 . 1 | rū 1 / 1 | rū 3 / 1 | dṛishya 100 / 1 | karaṇaṁ ||

. . . | *di* | 5 di / 1 | kṛitvā | 3 / 1 | 2 / 1 | 3 | dṛi 100 / 1 |

. . . . *bhaktaṁ* | 5 | *dṛishyena*

†This leaf consists of six scraps besides the portion marked (a).

[21v.](a)

. . . .

. . . vārdhaṁtṛitīyasya jīvalokāt eshāṁdīnār

. . . . kasyakiṁbhavati

. . rivartanaṁkṛiyate ||

| 36 dṛi 500 | prakshe
| 10 1 |

2 dī 1 2	3 dī 1 2	4 dī . .
1 1 di 2	1 1 di .	1 1 *di* 4

[22r.] . . . . | 47 . 5 / 947

rtitajātāphalaṁdī 500 || asyapratyayatrai

śikena

2 dī 1 2	1 di 1 2	100,000 947	phalaṁdi 60,000 947
3 dī 1 2	1 di 1 3	157,500 dī 947	phalaṁdi 60,000 947
4 dī 1 2	1 di 1 4	216,000 dī 947	phalaṁdi 60,*000* 947

22v.] . . . . dviguṇaṁdvitīyasyaprathamā

. . . . . | prathamāchaturguṇaṁchaivachaturthechaivadattavān cha

. . . . *aṁ*śatamekaṁdvayānugaṁ ||| vadasvaprathamedattaṁkiṁpramāṇaṁ

. sya . | 0/1 | 2/1 | 3/1 | 4/1 | dṛishya 200/1 | śūnyamekayutaṁkṛitvā 1 | 2 | 3

. kshepayuktyāphalaṁ | 20 | 40 | 60 | 80 | evaṁ 200 ||| eshāṁ

. . . | ā 20/1 | u 20/1 | pa 4/1 | rūpoṇākaraṇenaphalaṁ 20 .

. . | '|| sūtraṁ || yadṛich*chhāpi*nyaseśū*ṇye tadā* . . ṁ

. dāchatriguṇa*ṁ*d . . . . [23r.]

*pratha*masyatukiṁbhav*et* | 0 over 1 | tadā 2 over 1 | *tadā* . . | . | .

yadri*ch*chhāvinyaseśūṇye

. . chchhā || 1 || tadāvargaṁtukārayet | . 1 | 2 over 1 | 2 over 1 3 over 1 | 6 over 1 .

. kshipeguṇitaṁ || 1 | 2 | 6 | 24 |

*pra*kshiptaṁ 33 || drishyaṁvibhajet | 132 over 33 | *v*artyaṁjātaṁ | 4 over 1 | . . . . .

. ṇadattaṁ || atonyāsaḥ || 4 | 8 | . . 24 | 96 | . . . .

eshavargakramagaṇitaṁ || || athayutivargaṁ*kri* . .

s . taṁ || . ā . i . aṁś*ūṇ*yevinyastaṁtadāchaivakrameguṇaṁ

. . . . kritvāchaturtha [23v.]

. . . . prathamasyatukiṁbhavet . .

. . 3 over 1 3 over 1 | 12 over 1 4 over 1 | dri 300 | kāmikaṁśūnyepiṇyastaṁ

mikaṁ 1 ||eshaṇyastaṁ*pra*

rāś . tadāchaivakrameṇaguṇitaṁ | 1 | 2 | 9 | 48 | eshāṁyu .

. . | 60 | anenadrishyaṁbhājitaṁ | 1 over 60 300 over 1 | jātā | 5 | e .

. . syadhanaṁ ... anenakshepaṁguṇaye | 5 | 10 | 45 | 240 | e .

. . yutivargagaṇitaṁ || || udā || prathamasya .

. . . thaṁdattaṁchavaidhanaṁ | ‸sachadvārdhay*ut*odattaṁ . . .

*ś*ata*ṁ* chatuśchatv*aliṁśādhi* . . . . [24r.]

dattaṁchaivachaturguṇaṁ | *kiṁ*prathamasya . . . . .

| 0 1 over 1 1 over 2 | 2 2 over 1 1 over 2 | 3 3 over 1 1 over 2 | 4 4 over 1 1 over 2 | dri 144 over 1 over 2 | . . . . .

śūṇyeśu 1 over 1 .

yutaṁchaivaguṇaṁtataḥ yutaṁchaivaguṇaṁkritvākārayegaṇa . . . .

. . natu | 5 over 2 | guṇaṁ | upare uparaṁadhe adhaṁguṇaye | 10 over 2 | sārdhadv . . yutaṁ

. . tīyarā | śyāguṇaṇaṁ | sārdhaissaptabhitrīṇi | 45 over 2 | sārdhatrayayut*aṁ*

. chaturtharāśiguṇayeshshaḍviṁśatibhi | jātā | 208 over 2 | | sārdhachatvāriyu

. . . . i 289 *evaṁ* dri śyaṁ | sarvaṁtadevajātaṁ

* This phrase is deleted in the MS.

[24v.] . . . . trisārdhayu .

. . . . . . *urguṇaṁcha*turthenanavārdhayutaṁdattaṁ |

. . . tādvāviṁśādhikākimatraprathamasyadattāsit

. $\frac{3}{2}$ | $\frac{2}{1}$ $\frac{5}{2}$ | $\frac{3}{1}$ $\frac{7}{2}$ $\frac{4}{1}$ $\frac{9}{2}$ | ekatraṁdattaṁ **222** | śūṇy*a*

. *erū* . datvā || 1 || yutaguṇitayutakrameṇajātaṁ || sthāpa .

. . | $\frac{\cdot}{2}$ $\frac{67}{2}$ | $\frac{357}{2}$ dṛishya **222** | prakshepenajātāṁ **222** || . .

ntāḥ dṛiśyāḥ **222** || jā .

|| udā || prathamaṁnajānāmi | divardhayutaṁ .

ṇaṁpaṁ*chārdhayu*taṁpratha*mā* . . . .

[25r.] . . . . . . || $\frac{3}{1}$ $\frac{7+}{2}$ || . . | dṛi $\frac{78}{1}$

. . . .

: . *rūpaṁdīyate* | yutaṁjātaṁ | $\frac{5}{2}$ | dvitīyaguṇaṁ | $\frac{10}{2}$ | pārya

. *tritīya* ekatreguṇitaṁ | yutena | yutaṁ || $\frac{10}{2}$ | $\frac{23}{2}$ |

. yutaṁ | | $\frac{33}{2}$ | guṇitaṁ $\frac{132}{2}$ | riṇaṁjātaṁ | pā*rya* .

eśanyāsa | $\frac{5}{2}$ | $\frac{5}{2}$ | $\frac{23}{2}$ | $\frac{123}{2}$ | dṛishya $\frac{78}{1}$ | e . .

. . . . . $\frac{6}{2}$ | vibhaktavyaṁ | $\frac{2}{1}$ . . | 78 . . . .

raṇ . . śūṇyasthāne . . . . [25v.]

rūpaṁdatvā | 1 | yutājātā | $\frac{5}{2}$ | . . . .

. | $\frac{15}{2}$ | prathamātṛitīyasyatṛi guṇaṁyutaṁjātaṁ . . . . .

. rguṇaṁnavārdhayutaṁjātaṁ | $\frac{29}{2}$ | ekatranyāsā | $\frac{5}{2}$ | $\frac{15}{2}$ | .

. 9 | dṛi $\frac{71}{2}$ | prakshiptaṁ | $\frac{71}{2}$ | bha ktaṁdṛishyaṁjātāṁ 1 .

. anena sarvaṁguṇitaṁtadeva | $\frac{5}{2}$ | $\frac{1}{2}$ | 22

ekatraṁ || uda ||

. shāṁaparovidhiḥ || prathamadhanaṁdattaṁnajātaṁkiṁtudivardhayutaṁ | tadādvi

*igu*ṇaṁdattaṁpaṁchārdhahīnaṁ | tadātṛitīyenatṛiguṇaṁdattaṁsa*ptārdh*a

. . thāchaturtheṇachaturguṇaṁnavārdhahīnaṁdattaṁekatraṁta . . .

. . . . . . im . . ṁ . . . . || $\frac{2}{1}$ $\frac{5}{2+}$ || $\frac{3}{1}$ gu 7 . || $\frac{4}{1}$ *gu*

. haraṇ*aṁ* || śū*nye* . . . . [26r.]

. . rūpaṁdatvāḥ yutaṁjātaṁ | $\frac{5}{2}$ | p . . . .

. ṁ | $\frac{5}{2}$ | prathamātṛitīyaṁ trigu*ṇaṁ* . . . . . .

mā chaturthaṁchaturgaṇaṁnavārdharahitaṁ | śeshaṁ | $\frac{11}{2}$ | *e* . . .

$\frac{5}{2}$ | $\frac{5}{2}$ | $\frac{8}{2}$ | $\frac{11}{2}$ | dṛi $\frac{29}{2}$ | prakshepayuktiḥ | $\frac{29}{2}$ |

bhaktaṁ | $\frac{2}{29}$ | $\frac{29}{2}$ | jātaṁ | 1 |

. . guṇitaṁtadeva | evaṁriṇarāśībhavanti | triprakāraṁ

. samāp*taṁ* || śūṇyasthānerūpaṁdatvā | tadanuyuktaṁ | guṇita

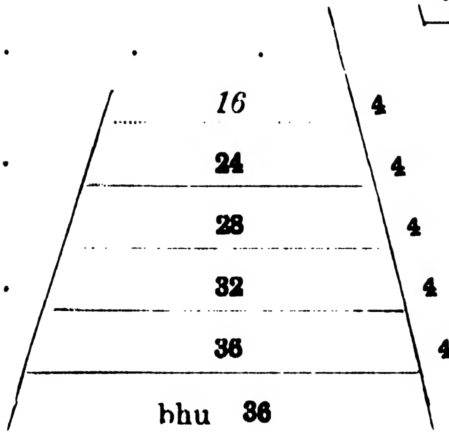
[26v.] . . . . phala . .

. . . athadvau **4** | **30** asyadalam*pha* .

. . *athā*shṭha **8** | **32** | dalaṁ *pha* .

. . . bhu **36** | **16** | **28** dalaṁ *pha* .

24 | **16** dalaṁ *pha* .



athatrīṇi usārā da .

36 | **20** | **4** | asya tṛi .

32 | **20** | **8** atho chchhed .

28 | **20** | **12** puna .

24 | **20** | **16** .

[27r.] . . . . chh . y . *tas*mātki . .

. .. . kaṛaṇaṁ | pṛithakrūpaṁvinikshipya | pṛithak rūpaṁ kshiptaṁjātaṁ .

. 63 bhyāso | tatraguṇa | 3 | 4 | abhyāsaṁ | 12 | rūpahīnaṁ | 1 . .

*abhāgā*chatu*ϕ*paṁcha *45* | atrakshiptaṁjātaṁ | 15 | 16 | éśatṛiguṇ .

. . . tāmūla . . nichatu*ϕ*paṁcha || **5** | **4** | esha

[27v.] . . . .

. masyadhanaṁ | eshaṁanukkrameṇapūrvokt . sth . . ṛi . . .

9 pra	**7** dvi	**10** tṛi	**8** cha	**11** paṁ	yutaṁjātaṁpratyai*k* . .	
. ṁ	**10** tri	**8** cha	**11** paṁ	**9** pra	**16** | **17** | **18** | **19**	

. . . . aṁ sarvatra kārayet ||

[28r.] (a)*

17
25 dā

taṁdviguṇaṁdviguṇaṁbhāraṁas

labdhaṁ **14** || punakṛiya

*This is not homogeneous. There are portions of at least two leaves. There are other small scraps.

(a) . . . . vet || . . . . . [28v.]

. . . guṇaye | $\frac{1}{16}$ | $\frac{1}{8}$ | guṇi . jātā $\frac{1}{9}$

. . . . āhuṭva aḍho . . shaguṇa

sṇ . abhāgasyadivardhāχkiṁ $\frac{1}{96}$ $\frac{11}{2}$ phalaṁ

phalaṁ 5 . . o an

(a)* . . || sūtraṁ || gu*ṇau* . . . . . [29r.]

. . . kadhanaṁ || guṇanyāsorūpahīnaṁlabdhaṁrūpaṁ

(b) prathamaṇyasya | tatrechchhāpaṁchaḥ | 5 | tatprathamacha .

. 4 | 15 | tadādī*śś*odhayetkramāt *ādina* . . . *paṁchaśadv*

(c) | $\frac{1}{1}$ 4 | 1 . | . . . . . kr . . . t . i . ch . ś .

*eta*chaturdaśabhiśoddhyaśeshaṁ | 6 | e . *npaṁcha*

(d) . . diset || udā || dhanā . . . .

. syadvitīyayonmiśraṁdhanaṁtatrattrayodashaḥ dvitīyatṛitīyayonmi .

. . . *śa* | ādyatṛitīyayonmiśraṁdhanaṁpaṁchadaśasmṛitaḥ ekaikasyadh*a*

. . . . chchhichekatthyatāṁmamaḥ | $\frac{13}{1}$ | $\frac{14}{1}$ | $\frac{15}{1}$ |

(b) 1 | $\frac{19}{1}$ | $\frac{2}{1}$ | karaṇaṁ || ichchhā [29v.]

. . . dānīśodhayetkramāt tatrādī 16 śud . s . e .

(c) . tīyāyaṁśoddhya 7 chaturthāyaṁśoddhya 12 paṁ .

(d) y . . *d*vitīyay . . *miśraṁdhanaṁ*

yatṛitīyay . nmiśraṁdhanaṁsaptādashasmṛitaḥ tritīyaśchaturthay

. . . chatu*ϕ*paṁchakamiśraṁtudhanaṁekonaviṁśati | *prathama*

. . titatracha ekaikasyadhanaṁkiṁsyādvechchhi . .

. . 17 || 18 || 19 || 20 ||

(a) . . . . tr . bh .

*Besides the pieces (a), (b), (c) and (d) are two other small scraps in the envelope. It is doubtful whether (a) is in the proper envelope.

[30r.] . . 1 . . . . . .

. . pra . . . .

sūtraṁ | ekayutanarasa . . .

sarv*a*shshaḍbhipala . . . an*e*nala*bdh* . .

hītāp*ra*thaṁ | 36 | 42 | 48 | 54 | 6.

. . . . . | | *sh*. | 78 | 7. | .

[30v.] . . . kā . . | . .

. . *śdā*tsadṛiśakṛi | . .

. . bhāgahāraṁkṛiyate | 234 70

*tatu*lāḍhe | 3 24 70 | mudgāḍhe 1 47 . . |

. *kri*yate |

[31r.]* . . | 8 10 | . . . .

trairāśikena | 3 dine 1 || 2 1 2 || *168 dinā* 11 | . . . .

īyasya kṛiya te | . . .

. . *draṁ* | 168 dinā 11 || phalaṁdraṁ | 140 11 | prathamenadattaṁ .

. . ssaptaḥ dattaissamadhanā *jātā* | h .

. shadṛiśaṁ | 77 11 | 294 11 | pātyaśeshaṁ | 217 11 | dvitīyasyadatta . .

. . 77 eśassamadhanājātā || || punānyaṁ*sarvabhā* . .

. . bhā 4 1 4+ dine | draṁ 15 jīvyā || dvitīyasya | bhā 3 1 3 dine | draṁ

*F. 31 consists of two leaves stuck together. The transliteration given is that of the outside surfaces of the two leaves only; but it is not always easy to differentiate.

[31v.]

. . . . 2

. . . . bhāga

. . . kāranaṁ || 2 | 1 ||

. . . chchhedasaṁguṇe | draṁ 1 4 taṁ 1 1 2 | draṁ 16 mudgā 1 2

.

. . *rdha*yutihṛitaphalaṁ || asyaguṇākāradvay $\frac{2}{9}\,\frac{1}{13}$

. . uparaṁgunave | *ā*dautava *5* | dva 2 *2*

. . guṇaya | 2 | . . . . . dvinavabhāgesu . . .

[32r.]

chanmanibhāṇa . . . .

. tugaganaṁnita* *rāvaṇe | rā . . . . . .

. yaṁtyaktasutayaśetayā | sākaikenaparāvartaṁdhanurbhāg*aśapa*

. . vaṇepatamāṇāsaudaśabhāgaṁnidhāryate | evaṁtatparimāṇa

. hiyamānaṁtunityaśaḥ kiyatastuparāvartaibhūmiṁprāpyavateja .

dha 1 1 1 10+ 5 | parā 8 | yoja 30 1 chhe 8000 1 yoja | phalaṁparā varta

. 18181 śe $\frac{9}{11}$ udā | nāgaśvachchhandagāmi*dratachadaśa*

. . i . . . naṁ . . . . . .

[32v.]

. . . . . *cha* . . *ḍ* . . . *ḍ* . .

. . . . . *yāsvaga** *ndh*i*te etadviko . .

. . . . 100 6 8 3 | ūrdhachchhe 768000 ayo | adhachchhedaṁ 1 .

phalaṁva | 429867 mā 1 di 4 || udā || vr*a* . .

*cha*riśvā | 4000 | ktāpatitaṁbhūmitalepaṭaṁ || triśatāṁ . .

nāṁtusa | 5 | ptayojana*h*īyate | chaturdaśaśtukoṭṭ*i* . .

yatapaṁ | 24 | chaśashṭicha | kaidinaibhūtaleprāpyava

. ko *ttamā* || nyāsasthāpanaṁkṛiyate . .

[33r.] . . . . *bhi*sārdhaivā*

. . . . 1 chhe 9 guva | gha . . . .
. . . 2 1 udā || dīnārakonāmaviś*ā* . .

. trīduχkhārjanīyaṁsukhabhojanecha | tasyārdhamardhaṁchayadardhamardhaṁ*tane*

. guru prasādaṁ | 1/1 | 1/2 1/2 1/2 1/2 1/2 1/2 || 108/1 | phadī 1 dhā 8 . .

. *pana*dhanabhu ktaṁ || ardhaṁ . .

stāraṁnavaromaśatanicha | dvādaśastīticharmāṇikatiromā . . . .

. . . . | 900 | 12/1 24/1 | 12/1 24/1 | 12/1 24/1 | pharoma

[33v.] . . . . . . * nyā . sth . .

. . . | *1* 36*0* *gha 12/1 chhe 33/1 . .
. . . 1

. 112 udā || sumerupṛithivīśaṁkusurāṇāṁparimāśrayaṁ || āru

. . . χkaśchitarasāsuramadiraṁ || satataṁsaptasārdhānāṁsapā*ga*dhyase .

. . || satṛibhāgatṛipaṁchāṁśanityamevaṁchagachchhati | yojanānāṁsa*taguṇa*

. . chaturāśītiruchchhṛitām | kenakālenasaugachchhevadametaśuniśchi .

. . 7 || *di* 1/1 | yo 84000/1 || adhachchhedaṁ 360 di .
. . . . | . .

[34r.] . . . . sasya . jat

. . . . rthinaḥ khage . k . d . *bh*uktaprasṛitiṁchaivamevacha | .

. . | shṭauvadasakhekiṁ*kha*gaṁvadasundari || pra 1/1 | kha 11/1 | khā

phalaṁkhaga 63360 eshabāhupramāṇaṁ ||

kaśchitpumāṁsuvarṇastukalāpādayutaṁyavaṁ || pratyahaṁsūlinesū .

kiladattavāṁ | paṁchā*bd*aimāśamevaṁtu diṅaipaṁchadaśastathāḥ datvā

syasarvāyajñātumichchhāmitatvata || di 1/1 || 1/1/1/4 6/1/4 bhā || 5 . .

. . chchhedaṁ 192 yavatola |

. . . *yukta* . . . . [34v.]

chattṛitāṁgai | tānī*yatāṁsaraparaya*ā*r*ju*n*enagṛiddhra . r . .

tayāspṛiśaṁnti | 1 8 1 9 1 5 | śa 1 / 1 śakā | yo 777 / ja 222 | 8 / 1 / 1 chhe / 7 | phala .

940 || mā | rdhayutovya*kt*

. stapaṁchapaṁ chāśasaterenavajramaṇa ilabdham

. trakathayaśva mūlyaṁśāṇachaturbhāgasyasiddhārthapaṁcha

. . gasya || ku 1 1 2 chhe 128 1 māku 1 2 māchhe 40 simā || sa 55 1 ||

. . . . . i . . || ū . . ṁ || . . . .

(a) . . . . jātā | purushaḥ $\frac{1}{4}$ | $\frac{3}{2}$ | $\frac{3}{1}$ eshā*ṁ*sadṛiśe [35r.]

. dhanaṁ 19 1 | anenaguṇitaṁjātaṁ | 4 eshaprathamasya*dhana*ṁ

. * dviguṇaṁ 12 dviyutaṁ | 14 | *etadvit*īya*sya*

. guṇaṁ | 21 | dviguṇaṁ 42 | tryūṇaṁ | 39 | eshaḥ nyāsaḥ pratye*ka* . .

. daśamagravṛindānāṁ chaturdaśa ekonachatvāriṁśa | ta*t*pādārdhatṛibhāgān . ṁ

. | 4 | 1 4 | pha 4 1 | *evaṁ*dī 21 | eshaprashṇa etainmī . . . .

(b) . . . . . t . . . .

. smātkāraṇā | tayoryogaviyogaṁsvāviy*o*gas

. . . *ru*śabhājitāpurusha 15 anenabhaktvā dhanaṁ | $\frac{9}{15}$.

. . . . . tad*nū*paddhayasahitaṁ || . . . .

. . . .

[35v.] (a) . *ushaḥ* 4 | 6 | 5 | sa*dri*shayutikṛi . *ājātā* | 60

. . . . || eshagavāśvamahishīpratyaikaśāleshu . *bhāga*

. . .	. . śā		180	gā	1 4	phalaṁ 45 . .	
. . .	1 1	śā	180 1	aśvā	1 6	phalaṁ 30 4+	26
. . ṁ	1 1	śālā	180	mahi	1 5	phalaṁ 36 5+	9

(b) . . . mūleṇa | 1
1
2 | etadviguṇaṁ 3 | dviyuta .

yasyadhanaṁ | tadevasārdhaṁ 3 | asyārdhaṁ | 1
1
2 | yutaṁnyāsa ||

. . . . d*vi*tīyas . . . . .

[36r.] . *yā*chatu*r*da . . . .

hyāpaṁchatṛiguṇi * *tāsakhe | 1
1

r*o* 21 eshadeśapramāṇaṁ samāptaṁ || || sa . i

lavanasyarāshekoshṭhatāṁvākṛitāṁrharai | eshāṁchaikāṁrāśipunasu .

*pta*dhānītā | saptāṇāṁmapichaikārāśistulitāṇi | paṁchasaptatyā .

hasraṁbhavet saptāshṭaguṇaṁkiṁ rā 1
1 | 1075
1 | 56
1 | adhachchhedaṁ

2000 pabhā | phabhā 30 pa 2 00 || esha

. . śilāvaṇapramāṇaṁ || kākinīdaśabhāgasyadadyādashṭādaśī .

. . . ā | tasyaṁviṁśatibhāgasvaśatabhāgaṁprayachchati | narovaksha .

. . *l* . . [36v.]

. . yāchayacha * * 1 . 1*5002* .

. . yojapaṁ*chak*aṁ | nyāśa*sthā*

| .*2* / 1 || 1 / 1 | 1 / 360 || yo 5 / 1 chhe 4608000 yayo || phava 2 . . / mā 4 . .

yojanasyatṛibhāgārdhaṁsatṛibhāgapadonakaṁ | yānaudinattṛibhāge .

genagachchhati | śāpuna ϕpaṁchabhāgārdhaṁyojanasyatathāshṭamam

tinivartaṁtevāyuvegavalāhatā | yojanānāṁshṭootaraśataṁ

. . lenagach*chh*ati | di 1/3 bhā 1/3 1/2 gu 1/5 1/2+ 1/8 . . .

. . . . . . k . . . . [37r.]

. . . . . sūyamāṇasyadivākarasyaghaṭikai χkiṁprayātasyava*da*

. suniśchitaṁ | 30 / 1 muchhe 2 / 1 ghamu | 500000000 / 1 | gha 1 / 1 || *pha*

yo 8333 *3*

. 73 | rbhāṇorathaṁsuramahoragasiddhasaṁhaividyādharaiϕparivṛita*ṁ*

. rātrau | koṭīśatārdhaṁsarathaṁprāyāsyāt tadbrūhiśāstrakuśalo

. . vartaṁ || muhūrtamekenakiṁgachchhebrūhimeganakottamā ||

. . . . || 500000000 || *gha 2* || . *yo* 16666 . .

. . . . . . | stubhāga . . . . . . [37v.]

bhāgebhavedrāśi | ūrdhachchhedaṁ 108000 viliptāṇāṁ . .

liptā 5 paṁchārdhasamvatsarebhukterāśaikāyadibhānujaḥ brūhi

ka tatvajñasamaśvavāsareṇakiṁ | 2 / 1 / 2 || rā 1 / 1 || 1 / 1 aṁ 1 / 360 |

ūrdhachchhedaṁ 108000 viliptāṇāṁ

rāśi | adhachchhedaṁ 1 / 60 viliptālipta || phalaṁviliptā 2 || e

shagrahagatiṁ || udā || rājāyudhishṭhironāma ϕ pāṇḍuvaṁ*saśa*

. . . . ṇripat . ṁ . m . . . . . .

[38r.] (a) 14

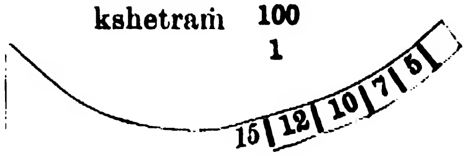
dvitīyapaṅkyāyoga $\frac{75}{15}$ | vartyaṁ $\frac{5}{1}$ pānīyaṁ || tṛitīyapaṅktyā | $\frac{24}{11}$ | 20

kṛiyate yogaṁ $\frac{90}{15}$ | vartyaṁ jātaṁ 6 payasaṁ | 1 . . .

. . ṁ . | 1 . ṁ . . . .

(b)* kshetraṁ $\frac{100}{1}$ | 300 *bh* . v . d

vaipulyādyogaṁ



(c) bhāgā vimsas chadasaguṇā | saptam . . ksh . . . jñā . . .

. . . . dṛishṭhaṁchaśatāni . . ai

. . . . o

[38v.] (a) | $\frac{7}{1}$ | $\frac{9}{1}$ | $\frac{. .}{.}$ | dṛi 60 | prakshepayukti 30 *vibhaktaṁ* $\frac{1}{30}$ dhanaṁ

ṇitājātā 14 | 18 | . . . . e . . *60*

(b) k . mūlyaṁ : . . . tall

1 . ā || 1 *jāru* . mūlye | 6210 maṇale ā iṁ || pha

(c) | pha 140 |

| pha *160* |

*(b) Should go with 99 (a); and (a) possibly with 39 (e).

(a)*. śadvādaśanriśakaśtathā | saptap . ch . . ch [39 r.]

dhāsaptapaṁchānāṁtṛidvimekaφprakalpitaṁ | tasyavāhasya kiṁ k . m .

. *tatr*āmama | kshetrasyasthāpanaṁkriyate | | karaṇaṁ *kshettra*

(b) dṛi*shṭhṭha*ksh . . .

. . . daśa | chaturdaśatṛitīyasyachaturthasya *n* .

. . . . . . *sṛi*tushṭhā . . s . shṭh .

(c) *tt* . sth . *n* (d) *shatkasāne* $\frac{5}{15}$ $\frac{9}{15}$ ksh

(e) ṁsthāpya	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{.}{15}$	
. . syakṛi	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$	yate || chaturbhipaṁchabhishshaḍbhi *gu*
. *thamarā*	$\frac{4}{..}$	$\frac{5}{15}$	$\frac{6}{15}$	śi yoga $\frac{60}{15}$ vartyaṁ 4 madhūgha

15 20 . 000
1. 15
15 *15 12 a*

(a) . . . . chh . . u [39v.]

. . ll . . . . . *sabhi* maṇḍalaka*i*ṭallikā | eśachchhedaṁbhava*ti*

yathechchh . . kāryā | śuthaṭalakṛiyā udāharaṇam ṭalasya*y* .

. . mekaṁ ta*dv*ā śa*shṭiśatānāṁdaśā* . i . *ā* . *ṁ* . *iṁ* .

(b) . . . . *dhanaṁ* 1 . . . . 10 || esha ekaikabhāgāguṇitajā

. . . . *160* | 180 | 200 | 300 | evaṁ

. . . . dhanaṁ 1200

(d) pha $\frac{180}{200}$ (c) . . . iṁ

(e) *yet śesheksheпа 16 anena trabhāga $\frac{32}{16}$ *labdha* 2

40 |

12 $\frac{\text{pha}}{\text{pha 120}}$ ev labdhe*r*bhāga*ṁ* $\frac{28}{2}$ | jātā 14 lab*dhakshepaṁ vi*

*The portion to the left of *yet* possibly belongs to another leaf.

40r.] (a)* . ch . . yatra . . . . . *ś*

. āgaṁchaivakārayet kshetravaipuly . . . t . . . . y

. || pṛishṭhaśatadvayaṁchaiva uchareśatamekataḥ *vaipulyādvi*

ṁ . . e . . ṁ . i

(b)* āgā*viṁśaścha*d . s . . t (c)* sy*a*tudv .

yaṁśataṁ | sarvemiśrāpi sh . ṁbhāgāstasyaivapaṁcha .

(d)* ṛi . . . .

bdhāṁ bupayasoghaṭaḥ ekamiśrīkṛit . ri . . r . . k . . . . . .

karaṇaṁ | havyatulyaṁdhinikshipyaḥ | $\frac{4}{4}$ | $\frac{5}{5}$ | $\frac{6}{6}$ | kuruprakshepakaṁ tata | praksh

*This should possibly go with 39r (a).

[40v.] (a) . . . . . | . h . ntyachaivatatphal .

guṇit*ā*jātā | 6210 eshavāhasy*a*kāṇḍapramāṇaṁ

ś . kemūlyaṁ kartavyaṁ | *a*dhach*chhe*daṁchat . . . i . . i

(c)* *dhanaṁ* . . 00 pratyayatrairā (b)* [Blank]

(d)* *k*ṛitva ṇetu | eko .

kritaṁ | 1 śatatrayarpichabhi*ϕ*purushai*rl*abdhaṁkimādyaṁprathamādhanaṁ

| 120 || d . *225* || . . ṇaṁ . . . ṁ

* These consist of portions of two, or more, leaves stuck together. The knot on b verse does not appear on b recto.

[41r.]* *drammā* . shṭh . dvāchatvaliṁśabhirdinai | tatsaptat

. sya $\frac{42}{1}$ dine | draṁ $\frac{8}{1}$ jīvyā | $\frac{70}{1}$ purushā 42

. drammā 560 || yadipaṁchaśataśashṭyādhi*ka*

. ālimśabhi | tadrammai *ashṭa*bhi katidinām .

. . . . ṁ . . . .

*Besides the portion transliterated the envelope contains six small scraps.

. . . . . . uparād . . . . 2 / 1 / 4 . . [41v.]

. ya 2 adhedāpayedattaḥ 17 / 8 / 2 / 3 adheno parisaṁ .

. parimarāśīdvayaguṇaye 51 / 6 / 2 / 3 .

. *yet* upariyuktakṛiyate ekapaṁchāśānāṁ

. . . sthāpanaṁ 1 / 1 53 || 2 . ṁ phalaṁ ā 17 u 2 || 5 / 6

1 / 3 | 1 / 1 / 2 / 1 / 2 pha 54 . . kṛiyate | i | i . . [42r.]†

saṅsvāsārdhayuktetrayodaśasārdhaṁbhavati 40 / 1 bhā 160 / 1 13 / 1 / 2 .

. . pi eshāṁchchhedāṁkṛitājātā ekeṇa . .

. *ri* sārdhatrayodaśabhikimiti 1 / 1 | 4 / 1 | 27 / 2 pha 54 eshāṁapa .

. . . ekenalabdhachatvārishshaḍbhi saṁpadyatekathaṁ 1 / 1

. . || *tā* . . ekola*bhati* chatvāriśaṅsardhasyatukiṁbhavet

. . . . ṇi . m . .

. . . 2 [42v.]

saiātā 54 | śaḍtri 24 | . . 12 || ardhā 18 || ekattraṁ 54 || esha*mū*

tr*ai*rāśikakaraṇapratyekamūlyavidhi || aparaṁvakshyāmi | viṁśano*ṁ*di

. kiṁprathamekhandhakeśvayo'bhilikhita | apasyaprashṇāvidhi 20 / 1 | 1 / 1 / 2 | 1 / 3

. śiguṇaye | guṇitājātā 20 / 1 | 3 / 2 | 1 / 3 chhedaṁ 20 / 1 | 1 / 2 | 1 / 1 bhā*ge*

jātaṁphalaṁrū 10 || eshaviṁ śānāṁ *di* . .

bhavati | atra uparimāśkhandhakasya eshaguṇākāraṁbhavati | *asya*

. *khiśyā* . . . . *laṁ* . . .

†Besides the portion transliterated there are five very small scraps.

[43r.] va .g

. . . ārayet sārdhadva . ś . tubhoja*n*amadyamuttamet .

satṛibhāgatrayastṛiṁśaidinaidvāṇijyakasyatu | bhāṇḍāredvāda

śaśatava*j*ārāṇāṁsthitāsyavai | eshāvyayasamutpattaukaχkālaṁ brū

hipaṇḍitakaraṇavidhānenadvādaśaśatasyabhāṇḍāresthitatā

10	2	bhā	13	3	bhā	13	8	bhā	1	3	bhā	1	1	bhā	1	5	bhā	2
1	1		1	1		1	1		1	1		2	1		1	1		1 1
2	3		3	8		4+	2		3	5			2			3		4 4

2	33	bhā	1	1	bhāṇḍā	1200	guṇita	. i
	1		1	360		1		
	3							

[43v.]

	2	10	800 .	a . .	chchhedaṁ 360	di*va* ‖ bh
	1	1				
re	3	2	727	ṇasaha ‖	āyupiṇḍaṁ	2982

adhunāvyaya piṇḍaṁ	di 1	1807	800		486
ūrdhachchhedaṁ 360	1	240	727		727

diva	2982	puna	800	2982	1	a . . .
daṁ	486		727	486	1	360 di*va*
pha	727	laṁ		727		pratidina .

. . . 1807 evaṁsarvatrairāśikena | udā ‖ . .

[44r.]

2559	di 1	. . . .	223	evaṁ . . . .
1	1		144	
61		vyayapra		mānaṁ ‖ udā ‖ eka .

*ś*ārdhamu*tp*atisatṛibhāgadinadvayet pūjārthaṁ satṛibhāgaṁcha .

tāśchayet sāshṭabhāgadinātrīṇivā*s*udevasyachārchane .

pādonatrayoda* **ś*āṇāṁcha ashṭasārdhadināni *kh*et ‖

brāhmaṇābhojanedadyā paralokahitārthinaḥ satṛibhāgaṁ

* * . jjāraṁsapaṁchabhāgadinattrayet pa .

. . . *lachendaṁ*chakaXk . lakhanavalik*ā* | *sardhaṁsārdhaṁdine*

. . . . [44v.]

. . . ptaśatānāṁkaχkālaṁ ārjanabhakshaṇe ||| nyāsasthāpanaṁkṛ*iya*

bhā $\frac{8}{1}$ di $\frac{5}{1}$ / 3 bhā $\frac{1}{1}$ pa $\frac{32}{1}$ bhā śū $\frac{2}{1}$ / 2 $\frac{1}{4}$ $\frac{36}{1}$ bhā | $\frac{1}{1}$ $\frac{1}{360}$

. rāśi | $\frac{223}{144}$ | āyarāśi | $\frac{280}{61}$ | etatkālena ārjanabha . .

. di $\frac{1}{1}$ | $\frac{223}{144}$ | $\frac{280}{61}$ | urdhachchhedaṁ 36 .

phalaṁva*jj*

. . 2559 śe $\frac{1}{61}$ || eshavyaye ||* *

. va 4 mā . *di* 2 śe $\frac{28}{61}$ atha āya | di $\frac{1}{1}$ || $\frac{1}{1}$ || 28 .

. *225* / *32800* dalitā | $\frac{10225}{65600}$ | . . . . [45r.]

diyutaḥ | $\frac{108625}{65600}$ | padaghnāpa . .

. *tā* | $\frac{6455040625}{3227520000}$ | a | topaṁchavi*ṅśā* .

. . ti uparāḥ | $\frac{6455040}{3227520}$ . . .

. . te labdhaṁ 2 eshadhanaṁ || ||

. ṁ | ā $\frac{1}{1}$ / 2 || u $\frac{1}{1}$ / 2 | padu $\frac{0}{1}$ | dhanu 7000 | ashṭo .

. i . . o . i . .

. . . . . chaturṇāṁ . . . . . .

. . | 7 . | . y . *384* | asyavarga 147456 | as*ya* [45v.]

. *43* 271936 | eshasarvaguṇitākaraṇipā*ti*

. . kṛitvābhājitajātaḥ | $\frac{1158+}{671250}$ | aṁśairaṁśāguṇayet .

. . . . āśivarjyajātaḥ

. . | 579 | 579

. . | 515520000 | 294912 + | 768 29* *4912 .

. . | 777307500 | 777307500 | 1158 777307500

śeshaṁ | 579 / 515225088

śaṁ

. . . . . 67588 || dvayenamūle | 0

[46r.] . . . 4 6 6 3 8 4 5 . . .

. . 2 1 7 9 0 2 5 0 0 4 4 | dalitā e | 4 6 6 3 8 4 5 0 0 0 |
. . 5 5 4 6 1 5 0 0 0

1 1 0 8 9 5 1 2 5 0 2 2 || ādisaṁyuta . .
1 5 5 4 6 1 5 0 0 0 || padaghnāk . .

. . 3 2 2 7 0 4 7 5 2 2 |
. . 1 5 5 4 6 1 5 0 0 0 |
5 0 7 5 3 3 8 3 7 6 2 7 4 6 7 4 3 2 7
7 2 5 0 4 8 3 3 9 4 6 7 5 0 0 0 0

. . rākaraṇipāta

. . 2 1 7 4 3 2 7 1 9 3 6 | pātitajātā uparānyāsasthāpa

. . . . . 2 5 0 0 0 0 0 0 0 0 0 || bhā

[46v.] . . . .

. . | *maḍe* 8 || maḍe 6 || maḍe

. . kā 20 |

. aparaprashṭaḥ pārā ā i e vīhujaṇavihānhai . .

. ṇa | gorejāmacha uppaṇesāmale a . . *dha*pa

ḍhale āpotḍīṇe āgaṇevihujaṇa ehuvī

karaṇaṁ | traigo*d*evārehahipaṇehi sā . .

. . . . . a *īśūḍhed* . *hivo*

[47r.] . . . . ṛi . . r . . . . . .

vichakshaṇaḥ chamūs*tu*pṛitanāstisrastisrasścha*mū* . . . .

nikīnidaśaguṇāmāhurakshohanībudhaḥ || akshohi . . .

ra	1	e	3	3	3	3	3	3	3	10 gu		guṇitajā
ga	1	sha	1	1	1	1	1	1	1	1		
na	5	pa										
tu	3	ti										

ṇī

ratha	21870	esha akshohi
gaja	21870	pramāṇaṁ || ||
nara	109350	kumāraśatruda*ma* |
haya	65610	. i . . o . .

dā || kaśchidrāja

. . . . . . 1 | 15 . | 10 śi | phalaṁ . 160 || udā [47v.] .
. . . tāvyū | 1 | 1 | 1 | hapārthaṁhehayakī

. *na*ghnatasāyakaiśchaiva⌀pattisvapādadalaśoḍasai | a .

*an*yāchatasrāvaihatātenamahāṅsavāṁ || śarānāṁchaparīmāṇaṁ . .

*tra*viśārada | . śi 1 / 1 | 16 / 1 / 1 / 4 / 1 / 2 | 4 achhe 21870 / 1 1 | phalaṁśarā

2824400 || anyā

. īpramāṇaṁ || sūtraṁ || ekorathogaja .

ki | di . | ra . 3 bhā ksha 8 / ya 1 1 ya 1 / 5 3 / ka 1 6 bhā / 1 / 4 / pā 1 / 4 / śe 1 / 3 | 36 pala | mā 1 / 1 / 3 / 1 / 12 / *di* 1 / 30 | chhe . / raktipa / ṇitaṁjātaṁ / 419942 / 115200 [48r.]

daṁ 480

. . gu

. . ṁpa

. . . to 8 pale to 3 *to*lenā *stidh*ā 12 .

. . . ā 7 dhāṇenāsti aṁ 4 dhā || aṁ 2

. . . . phalaṁbhā 2 . . [48v.]

. . . . *sti*pala 2000 bhā | pa 270 || pa

. . . | dhā 8 / 2 | to 8 tolapal to 6 tolenāstidhāṇa 12 *dh* .

. . 8 | chhe 12 / *aṁ* 3 | yadidinamekena eshadataṁtadvādaśavarsheṇa

. yāsa | chhe 5 | || | di 1 / 1 | 216 bhā / 270 pa / 2000 chhe / 6 to / 8 chhe / 8 dhā / 12 | varshe 12 3 / 1 1 / 93 . . / ṇaṁ / . .

phalaṁ | ya 2 / 1 | 3 bhā / 1 / 5 | bhāra / varṇapramā

. . . . śiraseśi

. . . .

[49r.] . ya 8 yava*nāstika* $\begin{smallmatrix}6\\1\\4\end{smallmatrix}$ . . . . . da 4 kalānāstipā . .

pādanāstimūdri 4 pāmu 8 mū 2 || udāharaṇaṁ || . .

śūkhyairyajaṁtidevīpratimahnikechit dadāmidevyā*rva*

kaṁ*cha*ḥ krītvādīnāraśatānichatvāritadhānakā aṁ*ḍ*ikā*rakti*

kāyavākalāpādamūdrikācha | etatramūlyaṁvadametatra

syakiṁ | 1 to 12 mū 400 | *dhā* 1 | phalaṁdī 50 dī*nāra*
1 1 1

aṁ $\frac{1}{4}$ stidhāṇe 12 dhā

ra 1 $\begin{smallmatrix}1\\1\\4\end{smallmatrix}$ bhā

10 dhāṇe anāsti aṁ 4 | *ya* 1 . . . . aṁ 1 ||

[49v.] . . . . *hāsobh* . . . .

. . . . . . raktikshaya . . . th . . . . . *yasyati* paṁchaguṇ*aṁ*

. . . . sa*rv*apaṁchchassamaṁ | divasāviṁśatikaṁkiṁ*śuṁdya*timahāṁ

vadaniśchayaṁ | $\frac{1}{1}$ | to 3 | ksha 4+ | va 25 | chhe $\frac{360}{1}$ | ma*dh*ya

rāsiguṇitaṁ | mā $\frac{2}{12}$ | ya 60 | mā $\frac{5}{12}$ | jā

| 6 2 3 2 1 / 1 9 2 0 0 | ksha | aṁ $\frac{3}{4}$ | si $\frac{4}{8}$ | di $\frac{20}{30}$ | yaṁśodhya

ya $\frac{3}{4}$

| 6 0 8 8 1 / . 9 2 0 0 | a sa | ka 1 pā 1 | 6 bhā | dhachchhe*daṁ* | 2
rvaguṇitaṁ || 7 tolepalaṁ : 558 . 78

[50r.] . . . . . . | 1 . / 1 . / 4 4 | . . . .
ntikāvati . . . . *vaśishṭaputraha*

sikasyārtheputra pautra upayogyaṁbhavatuḥ likhi-

taṁchchhajakaputragaṇakarājabrāhmaṇena | sarveshāṁmevaśāstrāṇaṁ

*gaṇ*itaṁ*mūrdhni*tishṭati | ādyāvasānesaṁsāre utpaṁnna gaṇi

. . hit paśchāśṛishṭitadākartuṁśivenaparamāt*man*

*y*ādyaṁchasutpaṁnnaṁgaṇitaṁsakhyakāraṇaṁ | yacha

. . . . hi*naṁ* . . *yadive*t *k* . *āṁ* . i . . . . . .

guṇayebhājaye *ekar* . . | udā || dramme . . pusaśataṁn *labdham* [50v.]

ardhe*ṇa*labhyateχkati | eka*rāśi*stukalanāgaṇitaprakṛiyā

kuruḥ | 1 dramme | phalaṁ 50 aparaṁ | udā || sārdhad*v*aye

traya | 100 dra pusā | sārdhadivardhelabhyateχkati | 2

sūtraṁ | $\frac{1}{2}$ | || ardhenoparisaṁguṇya | 1 2 1

. . . . *varṭh*akrameṇacha | ardheṇa ūrdhaṁguṇayema | 2 1

. *paṁ*chasaṁguṇe | bhājayelabdha*panya*ṁ . . e | 2

(a) . . . . [51r.]†

rā | 1 | 2 | 3 | | $\frac{1}{1}$ | $\frac{1}{1}$ | $\frac{1}{1}$ | $\frac{1}{1}$ | eshāṁyu*kta* | $\frac{6}{1}$ | .

sha | 48 | śeshāϕpurushasa 4 || anenabhajitārlabdhā *1* .

. . . sya bhavati | 12 | 13 | 14 | 15 | ekatraṁ 54 || . a

udā || kaśchidrājādadedānaṁsaptapaṁchāśakaṁbudha | paṁchā

*cha*pravakshyā*m*ya*nupūrvaśaḥ* dviguṇadviguṇaṁchaivarūparūpottare

prathameprāptaṁkiṁprāptaṁaparejane || $\frac{0}{1}$ || $\frac{1}{1}$ | . . . .

(b)* . bhāja*to*hitvā | tatrottarā 1 | 1 | yutaṁ | 2 | | . | $\frac{3}{1}$ | $\frac{3}{1}$

. . *9* | . eshāϕ*p* . . sh . . . .

(a) . | 81 | *dri 329* | karaṇaṁ | uttar*ā bha*jya . [51v.]

. 27 | | 1 | tatrottararāśīnāṁyoga 87

eshadhanādṛishyāśodhanīyājātā 24*2*

. . jatā | purusha || *1* | 9 | 27 | 81 | yoga 121 ane .

. . jātā | 2 | eshadvauprathamasyadhanaṁ || 2 | 6 | 18 | 54 | 16*2*

uttararāśīsaṁyutaṁjātaṁ || $\frac{2}{1}$ || $\frac{15}{2}$ || $\frac{48}{2}$ || $\frac{147}{2}$ | $\frac{444}{2}$ | eshāṁ

. . . . || udā || . .

(b)

. . 6 || . . || . . sy . . yoga 1 1 1 śeshā ϕpurushabhāji . . .

. . . . *jātā* | *37* | bhājitā | *60*

† ?Two leaves.

* The left portion or (b) is not homogeneous. There are also three other small scraps that are possibly out of place.

[52r.] . . a . . . . . .

. nā $\frac{57}{1}$ *ts*edaṁjātaṁ 3 anenachāl*iṁś*aguṇayaṁjātā 12

. . *sv*avaṁśurāṇāṁ || pratyayatrairāśikena

|| udā || dhanāsvamardhosaṁśoddhya

chaturīyakaṁ | tatseshāpaṁchamobhāgote

$\frac{1}{1}$ vaṁ	$\frac{120}{1}$	$\frac{1}{10}$ 1	
$\frac{1}{1}$ vaṁ	$\frac{120}{1}$	$\frac{1}{8}$	$\frac{1}{1}$.
$\frac{1}{1}$ vaṁ	$\frac{120}{1}$	$\frac{1}{4}$	30 a

śatadvayaṁ | aśītyādhikaṁdhanaṁchaivakiṁādyaṁprathamaṁdhanaṁ |

[52v.] . . ū . . . *200* || asyadvayānāṁśatānāṁpā*da*

. . *dhik*aṁśataṁbhavati 150 atrāpipaṁchabhāga 30 || evaṁ .

. . . $\frac{1}{2}$ | $\frac{1}{4}$ $\frac{1}{2}$+ | $\frac{1}{5}$ $\frac{1}{4}$+ $\frac{1}{2}$+ || phapiṇḍu 280 || paṁchamījātīkaraṇaṁkṛi

pi 280 || aṁśayuti | $\frac{28}{40}$ | bhaktaṁ | $\frac{40}{28}$ | dhanu 280

400 eshaphalaṁbhavati ||

[53r.] *y* . *tā* || $\frac{1}{1}$ yo *1* di || *6* . . . . .

ṇaṁ || 2 1 || 1 . . . . .

syai . . viśeshaṁtu | tatragati | $\frac{3}{2}$ $\frac{2}{1}$ | viśeshaṁ | $\frac{1}{2}$ | vibhaktaṁ | $\frac{2}{1}$

sarvagatī | yojana | 9 | a nenaguṇaye | 18 | anena . .

. *gata* | *bha*vishyati || pratyayatrairāśikena | 1 di / 1 |

. yo 27 *pūgata*dina 6 a . oyojana 9

. *ya* $\frac{1}{1}$ || $\frac{1}{1}$ di | $\frac{2}{1}$ yo || . . di . phalaṁyojana

. . . . 1 | $\frac{pha}{12}$ || 6

. . . ṛi . . y . 2 . [53v.]

	18 / 1	yojana	20	dina	phalaṁyo	25 / 2
			20	ghaṭike		
.			35	ghadina		
3	25	yo	20	. .	phayo	36 / 7
			20	ghaṭike		
			35	ghaṭikedina		

. . . . . . kaṁsya . . . .

(a)*. . . . ksh . . y . . χk . . y . . ksh . ri . . [54r.]

yasya ‖ . . . yadhanaṁ | *dh*anateyātā | tayoryogaviyo

1 / 1	2 / 1	4 / 1	. / 1

. . . kṛitāṁrāshayaḥ

. . . . bhājyahi

2 / 1	1 / 1	2 / 1+	9 / 1+
1 / 1	2 / 1	4 / 1	8 / 1

dṛi 82 tveti | tatra uttararā . i

. . . . . . uttaraṁriṇaṁjātaṁ

(b) sūtraṁ jātaṁ 76 eshaprathamasya

(a) . . dviguṇaṁchatṛiūnachatṛitī*ya*syadhanaṁbhavet tryūna [54v.]

. | . saṁyutaṁ | ekaviṁshati*bhi* χ*krī*todīnā*raittara* . .

āyatudaṁsāpṛithag*va*daḥ ‖ karaṇaṁ ‖ yasyapadaṁnajñāyate

. . . . . yā *1* / *1* ‖ eta*dpra*thamasyadhanaṁ |

(b) . . . . . | 4 / 1 / 2 | et . . . . . chadattavān hasta

. . . rveshāṁ ‖ 0 ‖ 2 ‖ 2+ ‖ . . . . . .

*Besides (a) and (b) there are eight small scraps; and (b) appears to consist of two unrelated pieces stuck together.

ū . . ṛik .

[55r] . . va 5 1 3 | to 1 | dhā 1 12 | aṁ 1 4 | pha va 6 . . . . . śe 9 10 asya . .

jjāradvidhāna | ra 1 1 bhā 1 4 | ya 1 3 bhā 1 5 | si 1 2 bhā 1 2 | ka 1 2 bhā 1 2 | pā 1

guṇitaṁ 7227 1200 athaśaḍdrammako kaidramam śa

. . viṁśatipā

lāhataidhānakā | asyaivaska

to 1 1

va 5 1 3 | to 1 1 | dhā 1 1 1 12 | aṁ 1 .

ra 1 1 1 60 ya

1 1 1 192 | si 1 1 1 480 | ka

. . .

. . .

[55v.] . . . . kasyaya . . .

paṁ | 10 | chatṛ*iṁśataṁ* | *divardhatolaṁkasyadivardhamāṁśa*kasyadi

rdha*m*āṇḍikādivardhayavasyakiṁmūlyaṁ | nyāsa ||

to 1 1 | 35 1

to 1 1 | 35 1

. ṁ . 58

. . .

1 1 2 to | 1 1 2 1 6 mā | 1 1 2 1 2 . ṁ | 1 1 2 1 2 ya

phadraṁ 58 śe 31 128 punā*nyaṁ*

1 1 2 | 1 1 2 | 1 12 | 1 1 2 | 1 48 | 1 1 2 | 1 192

śe 31 128 tathā . . ā . ṁ

. . . .

. . y . |___| . . | 20 / 21 | . . . . aḥ tastāt . . . [56r.]

. kṛityūnāṅśeshachchhedodvi | saṁguṇaṁ | tadvarga . *dala*

. shṭhaḥ hṛitiśuddhikṛitikshayaḥ || śeshachchhedodvisaṁguṇakṛi

. | 21 / 20 400 dala 1 saṁslishṭhaḥ 20 / 21 441 2 | 21 bhā / 20 / 21+ | śeshaṁpānya .

. tvābhāji . .

. . dhaṁupare uparaṁguṇitavyaṁvargaṁyāvarjaye . .

. . ṁ | 4 2 5 0 4 2 | 400 . . śeshaṁ 4246

. . . . .

. 880 / 84 | 964 / 168 | guṇitajātaṁ | 848320 / 14112 | chatvāriṅśa [56v.]

pṛithaksthānāṁva rgaṁ | 160 | . . .

. sha uparāpātyaśeshaṁ | 846720 / 14112 | vartyajātaṁ | 60 |

||

*The lower half of this page is blank.

| 5 / 8 | kṛitikshya | 11833 / 1848 | hṛi | 184 / 1848 | [57r.]

kṛitaṁ : eśamūlaṁ || tan*mūlaṁ*

. . nmūlaṁ:ekaṁ 1 eshasadṛiśepātitajāta | 9985 / 1848 |

. . saṁbhaktaṁ uttaraṁdviguṇaṁ 2 anenabha

. 9985 | eshapaṁchakasyapadaṁ || || asyapratya

. 3696 | sūtraṁ || ekonarāśidvidhāsthāpya ūpase . .

[57v.] . . . . . . r . . . . .

sh*ṭo*ttaraghneguṇite 40 dvighnamādicha yā

. . nikshipya | 41 | mūlaṁ | 6 5 6 | śeshachchhedodvi saṁguṇi

śuddhaḥ tas*mā*t akṛiteślishṭha kṛityūnāśeshachchhedo dvi

. . . dalasaṁślishṭhahṛitiśuddhikṛitikshayaḥ akṛiteślishṭha

. dādvisaṁguṇakṛita | 6 5 12 | ta*dvarga* . 12 | 6 5 12 | 25 144 dala

[58r.] udā || śaḍviṁśaśchatripaṁchāśa ek*o*natrimśevacha | dvāśa

śhaḍviṁśachatuśchatvāliṁśasaptati | chatushshashṭina*va* .

. . . . ṁśanaṁtaraṁ | trirāśīti ekaviṁśa ashṭa . .

. . . . pakaṁ |*

. . . . *2653*2962264470649 94 . . . *83218*

. . . . eśa . . . . .

[85v.] . . . . sthāpanaṁkṛiya*te*

. shya 1 1 | yuvī 1 1 1 | sūḍha 1 1 | dṛishya 20 ja

. . ḍo | maṁ 1 2 | maṁḍa 1 2 | maṁḍe 2 .

. tadattajātaṁmaḍa 2 | ya 5 | sū*ḍhe* . .

. . . || udā || korāsipaṁchayutā . ū . . sā rāśissapta . [59r.]*

mūladakosorāśiritiprashṇaḥ | 0 5 yumū 0 | sā 0 7+ mū 0 |

| 1 1 1 | 1 1 |

ṇaṁ | yutahīnaṁchamekatvaṁ | 12 | ta

laṁ | 6 | dvihrīṇaṁ | 4 | dalaṁ | 2 | vargaṁ | 4 | hīneyutiṁchakartavyā

. . | 7+ | anenayuti | 11 | eśasārāsi || asyapratyānayane .

. 11 yu 5 mū 4 | 11 7+ mū 2 | paṁchāśamasūtraṁ 50

1 1 1 | 1 1 1 |

|| sūtraṁ gavāṁviśesha kartavyaṁ dhanaṁchaivapuna . .

*The left-hand page of this leaf is blank.

. . . . rū . . . . [60 r.]

. ekonaviṁśatima | gāvī 10 | rūpa 8 | . v . |

| 1 | 1 | |

. vivaritāsti ||

. chāśamasūtraṁ 51 || sūtraṁ || āyavyayaviśeshatuvibha

. dṛishyasaṁgu ⸭ ṇaṁ | yallabdhaṁsābhavetkālaṁayaṁprashṇe

ya . vidhi || udā || dvidine ārjayepaṁchatṛidinenavabhakshaye

āṇḍāgāraṁtasyatriṁśakiṁkālaṁārjabhakshaṇaṁ | dī 5 | dīnāra 9 | dṛi

. naṁ | āyavyayaviśeshantu | tatrāyaṁ | 5 | | di 2 | dina 3 | 20

. . . . e . ṁ | 2 | . . . .

[60v.] dvipaṁchāsamasūtraṁ 52 ‖ | . . . 60 di | phalaṁ 180

‖ sūtraṁ ahadravya harāśautatadviśeshaṁvi

yallabdha | dviguṇaṁkā laṁdattāsamadhanāprati ‖ *udā*

. . dine ārjayepaṁchabhṛitakomekapaṇḍitaḥ dvitīyaṁpaṁcha .

. *va*seva*samārjayatebudhaḥ prathamenadvitīyasyasaptadattā . i

. . taḥ datvāsamadhanājātākenakālenakatthyatāṁ 5 rū ‖ 6 . .

. . . . ṁ . . ṁ . *au* . . . .

[61r.] . . *anenakālenasama*dhanābhavanti ‖ pra*tyayaṁtrairāśik . kṛi* . .

. 5	30	pha 50		
. 1	1			
. 6	30	36	43	43

prathamedvitīyasyassaptadattā | 7 | śeshaṁ 43 |

| 43 | 43 | etesamadhanājātā ‖ udā ‖

. japutrodvayokechinṛipatissevyasan tivaiḥ mekāsyāhnedvayashsh . .

. gā | prathamenadvitīyasyadaśadināra | dvitīyasyadivardhakaṁ

. ttayān kenakālenasamatāṁgaṇayitvāvadāśūme ‖

. . ‖ 3 ‖ dattaṁ $\frac{10}{1}$ karaṇaṁ ‖ ahadravyaviśeshaṁcha | ta*trā*

. . . *2* | *3* | *pha* . .

[61v.]

:.	1 / 1	13 / 6	30 / 1	pha *65*
rdattajātā | 55 |	1 / 1	3 / 2	30 / 1	pha 45

prathamenadvitīyas .

55 ‖ samadhanājā .

sūtraṁtripaṁchāsamaḥ sūtraṁ 53 ‖

yenakrayaṁ bhājyaṁrūpahīnaṁpunarbhajet la . e .

guṇayetatranīvībhavatitatracha ‖ udā ‖ dvibhi *x*kṛiṇāti . .

ptavikṛiṇātitṛibhishshaṭ ashṭādaśabhavedlābhā *x*kā . . .

. . katthyatāṁ ‖ $\frac{7}{2}$ ‖ $\frac{6}{3}$ ‖ $\frac{18}{1}$ lābhā | karaṇaṁ | vi .

. . *guṇitaṁ* jā*taṁ* . . . . nīvījātā | syapra*tya* . . . [62r.]

. na || yadidvibhissaptalabhyate | tadāchaturviṁśatibhiχkiṁ | 2/1 | 7/1 | ./1

laṁ*rū* 84 || asyavikrayaṁkṛiyate | yadishshaḍbhitraya

. *ṇā*labhyatetadāchaturāśītibhiχkiṁ | 6/1 | 3/1 | 84/1 | phalaṁ 42 ||

*mū*laṁ 24 | pātyaśeshaṁ 18 eśalābhāḥ chaupaṁchāsamasūtraṁ 54

*vi*krayaṁbhājayechaivaguṇayetkrayapiṇḍatāṁ | rū

. mūlaguṇayelabdhalābhaṁchaprāpyate || udā

. *bhi*krīṇātiyassaptavikṛiṇātitṛibhishshaṭ mūlācha . .

. . . . || 7 6 . . . .

. . 2/1 | 7/1 | 24/1 | pha 84/1 | *athavikrayaṁ* | 6/1 | 3/1 | 84/1 | pha 4 . [62v.]

. 24 | pātyaśeshaṁ 18 | eshalābhaṁ || paṁchapaṁchāsamasūtraṁ

vikrayaṁbhājayechaivaguṇayetkrayapiṇḍavat vi*bhaktam*

sachakartavyaṁguṇayemiśrakaṁbudhaḥ yallabdhaṁsābhavenmū

. śeshaṁlābhapiṇḍatāṁ || udā || tṛibhiśchalabhaterashṭaucha

*vikra*yaṁshshaṭ samūlalābhamutpaṇṇāśataṁsashthivimiśritaṁ | *kiṁ*

*laṁ*kaśchalābhaṁchakathayedgaṇakottamaḥ || 8/3 || 6/4 | miśra 160/1 || .

. . . krayaṁbhājayechaivaguṇayet

. . . . | 1 | [63r.]

. . *pratyaya*ḥ | 6/1 | 4/1 | 240/1 | phalaṁ 160/1 | mūlaṁ | 90 | pātyaśesh*aṁ* | 7· |

. . chāsamasūtraṁ 56 || vikrayaṁchavibhaktavyaṁguṇitaṁkra

. śivat kṛitvārūpakshayaṁchaivavibhaktaṁmūlam*a*

. yāt || udā || paṁchabhiśchatuvargaṁtugṛihītaṁkenamāna*ve*

. kenashshaṭ vikrītaṁshshaṭptaṁchāśaṛiṇaṁkṛitaṁ | .krayavikrayasaṁ

. nvanīvistasyaivakatthyatāṁ | 16/5 || 6/1 || ṛiṇaṁ 56+/1 ||

. . bhājayechaiva | 1/*6* |

x 2

] 63v.] . . . *84* punāsyavikraya | $\frac{6}{1}$ | $\frac{1}{1}$ | 384 | *phalaṁ* | *6*

mūla*ṁ* | 120 | chatushshashṭi . pātyaśeshaṁ | 56 | eśariṇaṁkṛi .

. . ptapaṁchāśamasūtraṁ 57 || vastraśulkaṁyadbhavatita .

. . *hṛi*tavastratāṁ | trai . rāśikavidhānenaśulka .

. . yatatvataḥ || udā || paṭasyaśulkaviṁśāṅśaṁka . i .

. . . *triśśa*taṁ | paṭakānāṁpaṇakṛitedvaupaṭauhṛitaśau . i

. . . . paṇadaśastathāḥ kiṁmūlyaṁ

[64r.]

. . 5 2 8 0 . 8 7 2 4	4 4 4 0 0 4 · 3 8 7 2 4
. . 5 2 8 0 *3* 8 7 2 4	4 4 4 0 0 4 7 7 4 4 8

1 7 9 9 4 5 9 4 1 1 2 0

2 9 9 9 0 9 6 3 5 2

1 6 0 0 0 0 + |

tayardhe . tus . . . . . .
ardhamkartavyaṁ . ta . .
saṁguṇyajātaṁ | a . .
hrarāhareshugu*n* . . .
asya ūrdhāṁka . .
shashardhaṁ . . .

[64v.] . . . shachchhedodvisa*ṁ*guṇaṁ | $\frac{6}{5 \over 12}$ | śeshapaṁchakaṁpṛithak . .

. . aṅśasvaṁśaṁ | $\frac{77}{12}$ | tanmūla varjitaṁ | tanmūlaṁ 1

$\frac{5}{1}$ | dviguṇottarasaṁbhaktaṁ | $\frac{65}{24}$ | eshapadaṁ ||

. . . nayanaṁ | ā $\frac{1}{1}$ || u $\frac{1}{1}$ || pa $\frac{65}{24}$ || rūpoṇa $\frac{41}{24}$ . . . .

. . . . ḥ dalitā | 41 | ādisaṁy . . . .

. ___.__| . . . . etatk*āla* [65r.]

*bhir*manushya a . . . . . . lagyantiḥ

|| aparaprashṇaḥ yadyekapurushasyadrammāshshaṭ

bhirdinaijīvalokā | tatkāryaṁprastu*tā* . śsaptatīnāṁ .

. . *ṇaṁ*pākarākshakānāṁdrammaishshaḍbhikatidinājīvalokaṁ*bhav* .

. . karaṇaṁ | ādautāvayadyekapurushasyadrammāshshaṭ triṇśa .

. . jīvyā | tatsaptatīnāṁkiṁ | 1 pudraṁ 6 / 1 1 | 30 di | 70 pu / 1 | pha .

. . . didrammāṇaṁt*r*īṇiśatāsār*dh*āsa . . . . ṁ . . ṁ . .

. . . . *u*

. chyate || ā 1 / 1 || *u* . . . . . 6 . || *karaṇaṁ* [65v.]

. shṭhottaraghnegaṇite | ashṭhaghanaṁ | 480 | uttaragha*na*

. . va | dvighnamādi | ādidviguṇa | 2 | chayojjhitaṁ | cha

. uttaraṁ | ato uttaraṁpātayitvā ekaṁbhavati | 1 | var*dh*i

. *tra*nikshipyadhanasya | 481 | mūlaṁślishṭhakaraṇyā | 21 / 40 / 42 |

. . vaṁśaṁ | 882 / 40 / 42 | śeshaṁchatvā*rī*ṁśapṛithaksthāpya | 40 |

. . yojyaṁ | 922 / 42 | tanmūlavarjitaṁ || tanmūlaṁ

. . . . ṇottara . aṁ . . . 8 8 0 . .

. . . . . . yukt*abhāgaha* . . . . [66r.]

*nye*kārghaṁtupaṇyānāṁekadvitṛichatushshaṭ *sa*

*mpā*ṇāṁsa . . ṇyānimānayaḥ sthāpanaṁkṛiyate

1 1 / 1 1	draṁ 1 2 / 1 1	draṁ 1 3 / 1 1	draṁ 1 4 / 1 1	draṁ 1 6 / 1 1

. . . . . . ṇetu*sa* . * * saṁ

. . . . . . 1 6 / 1 3 | 4 . . . . 2 || śe *draṁ*

[66v.] . . śikena | 1 dram 1 | 1 rū 1 | 12 dram 1 | phalam rūpa 1 2

. dram | 2 rūpa 1 | 6 dram 1 | phalamrūpa 12 | . . . . . .

. dram | 3 rūpa 1 | 4 dram | phalamrūpa 12 | . . . . . .

. . . *rū* . | . *2 dram* . . rūpa 12 . . . . .

[67r.] . . . . 8 8 | jātā | 61 8 | .

. . sadṛiśam 1 2 11 8 | . u . . . | 16 8 61 8 | jātā | 77 8 | .

. . *sadṛ*iśamekasya | 16 16 | yutam | 77 16 | jātam | 93 16 | eshaphalambhavati | . r . . y

. . . pratyayaḥ | 93 16 | 1+ 1 2 1 2+ 1 | 2+ 1 2 1 3+ 1 | 3+ 1 2 1 4+ 1 | 4+ 1 . .

. || huṇḍikāsamānayanasūtram || dinabhaktaviśeshamchavi .

. . . . *kṛi*yatechaivakālameshāmvinirdiśet trairāśikavidhānena . .

. | . . ādattamchapātavyamsūkshmedattamchatatsamam || udāharaṇam || dvi

. . . . m . . e . . . . o . . . *dhanam*

[67v.] . . *re*

. *nam* . . . . y . ksh . . . *viśe* . .

. achchheshamtadvigunaj . *tā* | nirgachchha . . . .

. *n*pr . viśamānechatvāridattaḥ punadviguṇa*mjātam* . .

. . āsūṇyahaśtamgatamtasyakimatumūla pana*sya* . .

. . bhā 2 1 | 2 1 2 1 bhā 3 1 || 3 1 2 1 bhā 4 1 || 4 1 2 1 bhā *5 1* | *mū*

. . . . . . ś . *yutam* y . . m . . guṇam*chā* . ri . . .

(a) 168 / 4 . (b) . . deśadv. . . . || [68r.]*

. pātyajātāśesha*ṁ* 21 ||

ekatraṁ 29

. . hārika . . draṁ 2 .

. *diy*

*The reverse of this is blank.

. yojana . . . . [69r]

| 2/5 | 2/6 | 2/7 | dṛishya . . . . || a*to*sadṛiśa hakaṁ | uparimāṁ śaṁtaṁ*ttu*lābhavanti

tvāliṁśa | dūnāchaurāśī . . . *tataṁktulā* | dvichatvāriṁ

ṁ . . ṁ . . vanti etevṛīhakāsarvattraḥ sthāp*anaṁ*asya

. . . yatrairāśikena | 5/1 ā | 2/1 . ṁ | 210/1 | pha . m 84 |

. . yasyakṛiyate

. . . . yate || 7 . | 2 . | *210 ā* | *pha* . ṁ

. . . . k . taṁ 105 [69v.]

. . || *udā* || . . . . tṛibhirdatt*ai*tṛiguṇatṛiguṇenatu | vi

. . mi*veṅsi*taduchyatām | 1/1 | 3/1 | 9/1 | dṛishya 130/1 | prakshepa

10 | 30 | 90 e katraṁ 1

. . . . . . *vān* || taṁ *śa*taṁ*tri*bhirdatyai*pa*ravaptrāpavaptṛi . . .

phalaṁ | . . | chh . . . y . | 4/1 | 6/1 | 9 | dṛi 190 |

| 60 | 90 | ekatraṁ

[70r.] (a) tola 5 . . *akbhito* (b)

35/2	etebhāgā *5*
. 17	117/70

(c) . . . . e . . . .

. . . 2 to 0 rītā *7* pala 2 tola 1

. . traṁpala 6 ‖ ‖ udā ‖ samā . i

. . . . ṇipesīkṛitā*nicha* | *dva*chatisraś . ṁ

. . . tisrasamā*dā*yatulitānitrayodaśe |

. *e*kaika*s*ya*śārdhaaḥ* ‖ 1/2 ‖ 1/3 ‖ 1/4 ‖ d .

[70v.] (b) . . . . . ri . . . . . . ri (a) prakshepayuktyāphala*ṁ*

. . guṇyaphalarāśī

(c) . . . katra*ṁ*pala 8 ‖ ‖ udā ‖ ardhatṛi . .

. . . . . dāṅśaṁpaṁchaśashṭinṛipodadau | sevakānāṁtud .

. . . . 1/2 ‖ 1/3 ‖ 1/4 ‖ dṛishya 65/1 ‖ sadṛi

. . . . *30* *20* | .

. . . . ‖ ‖ . . .

PART II

iii.—*Facsimiles of the manuscript.*

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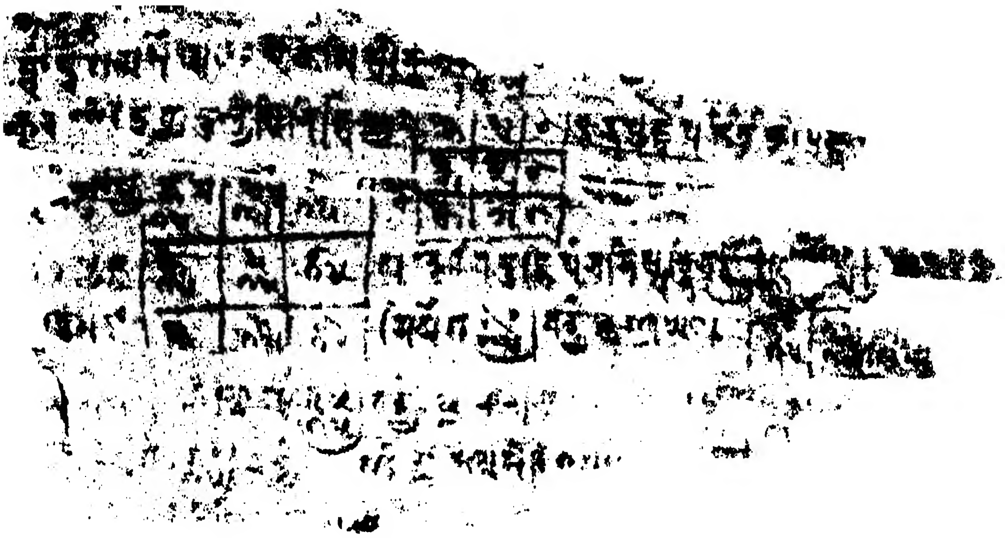

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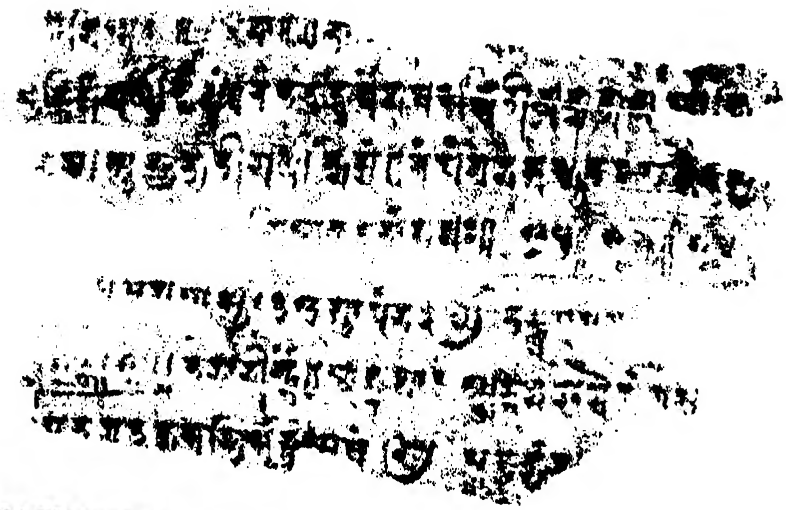

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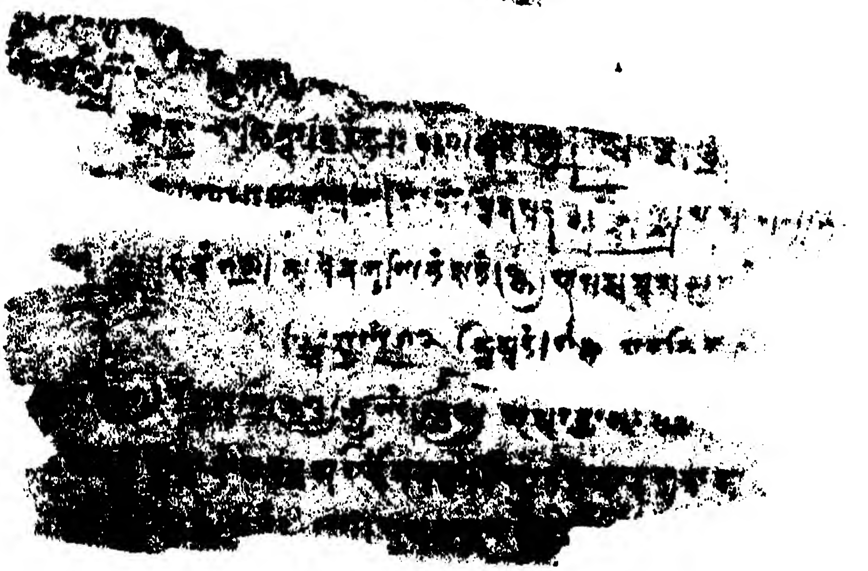

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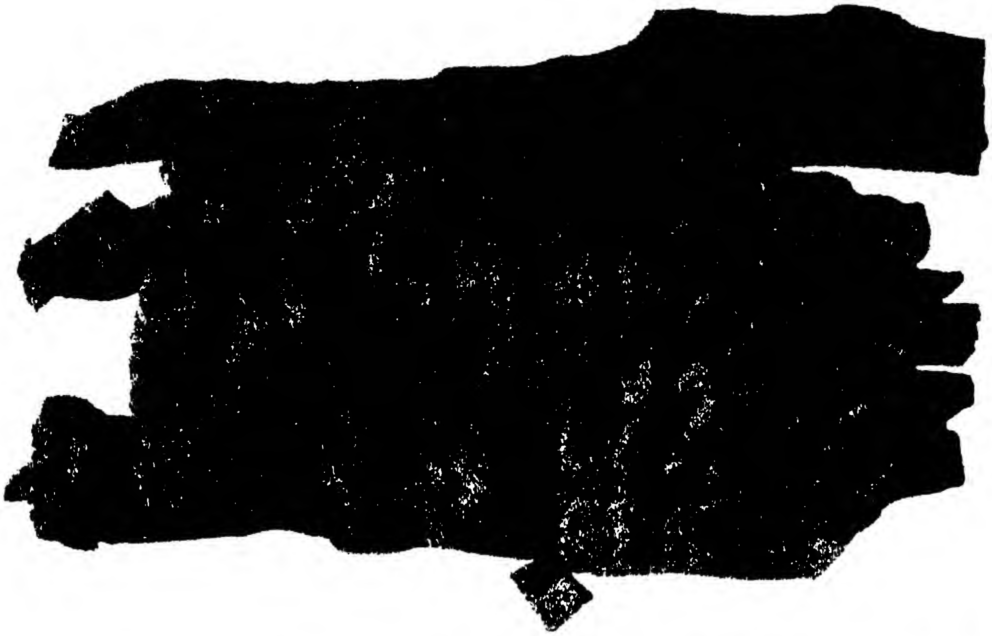

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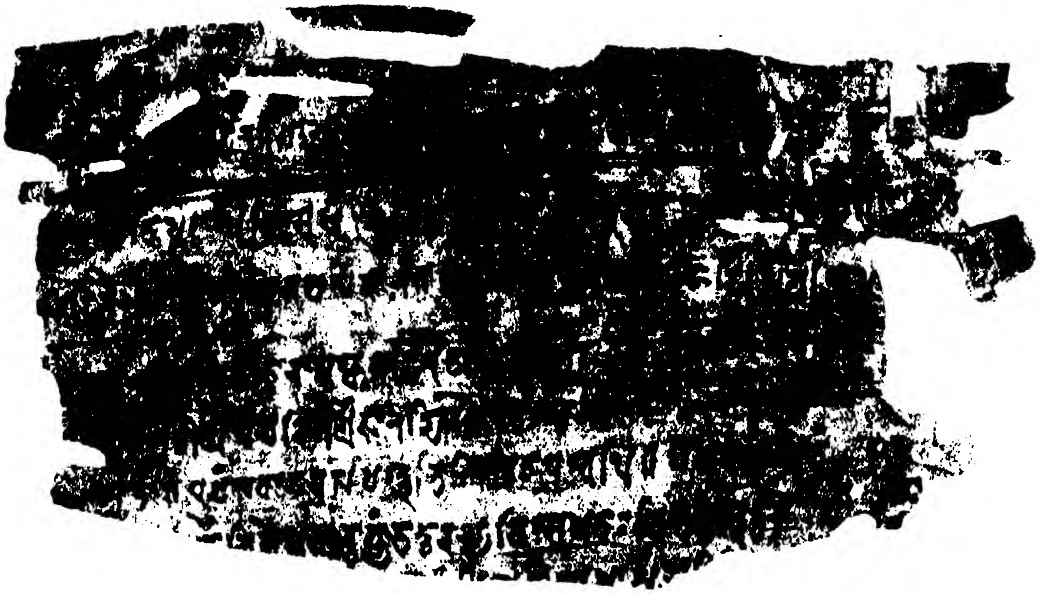

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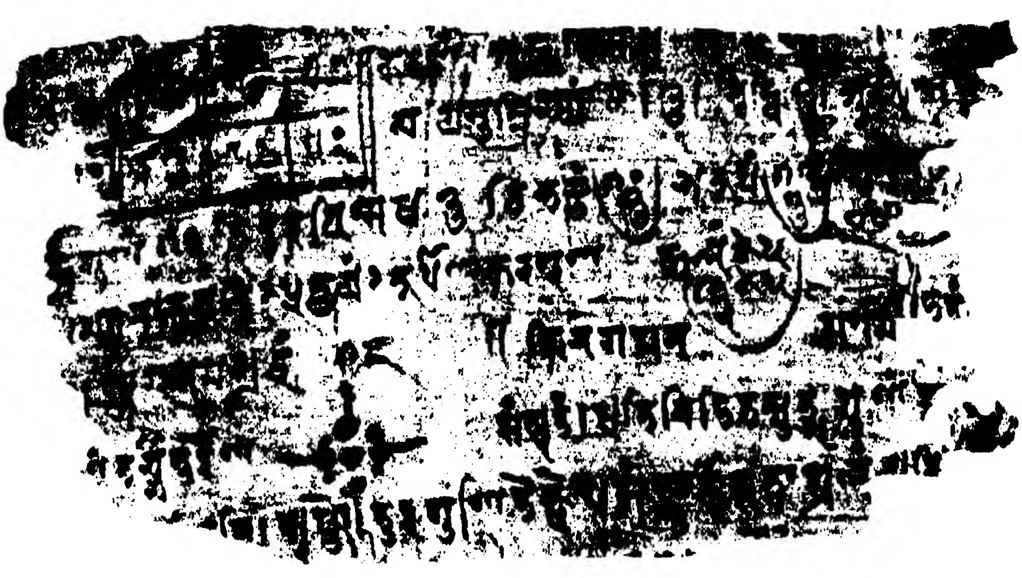

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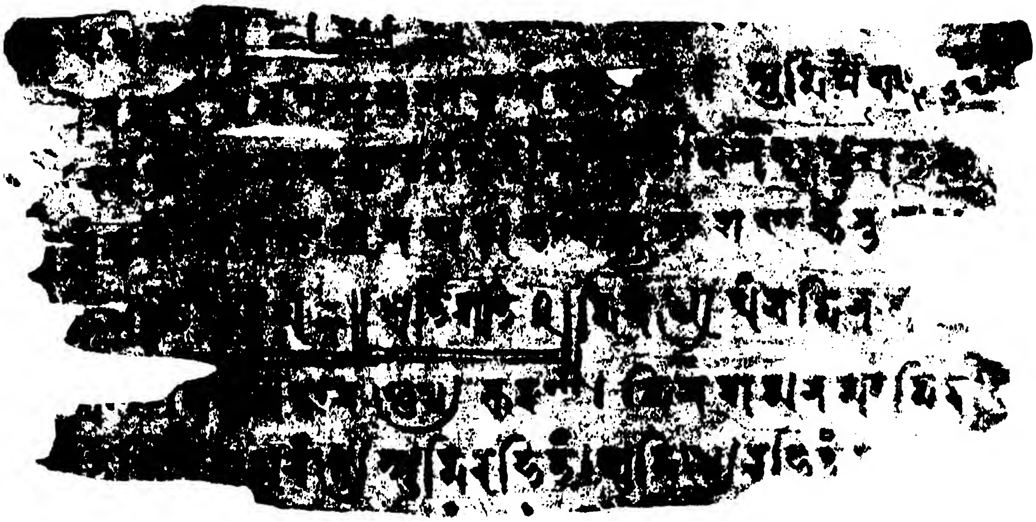

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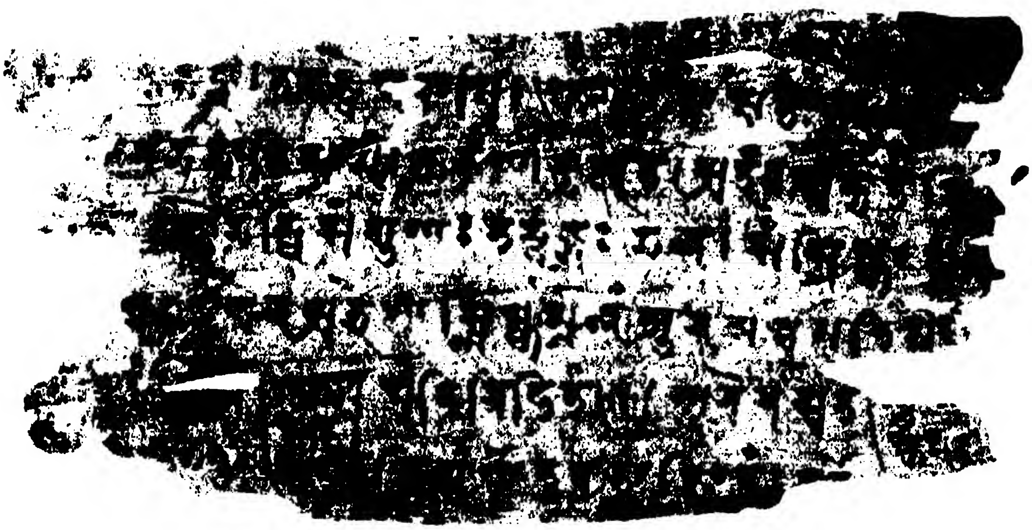

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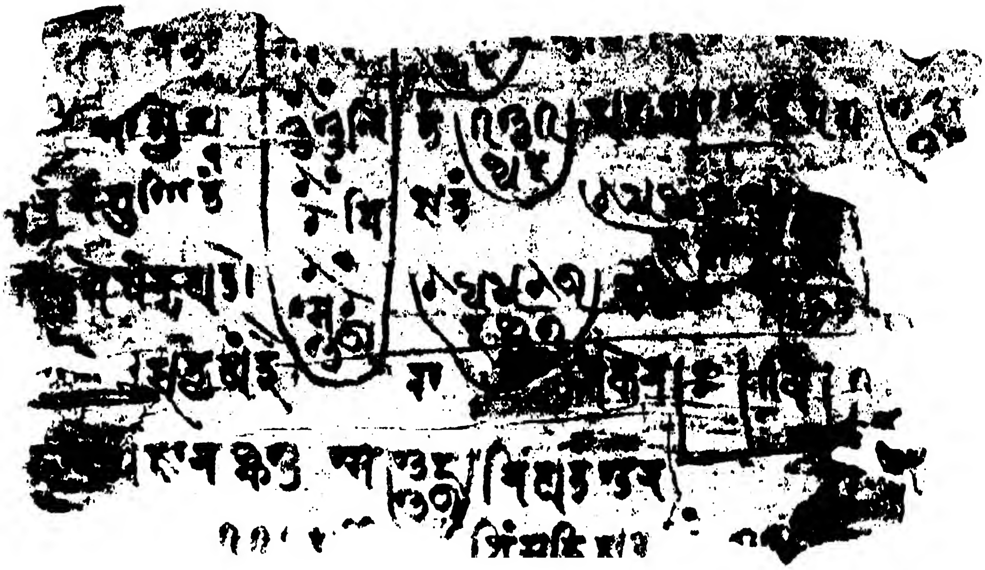

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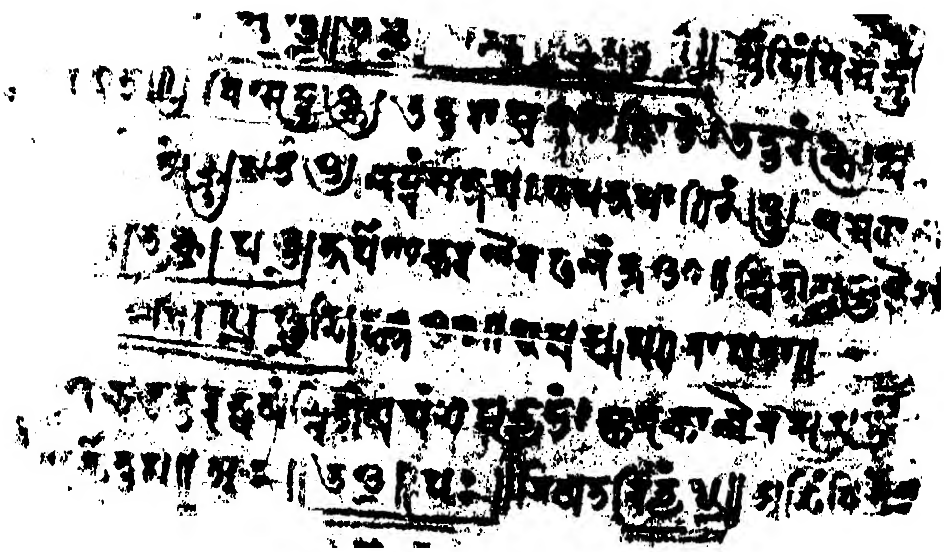

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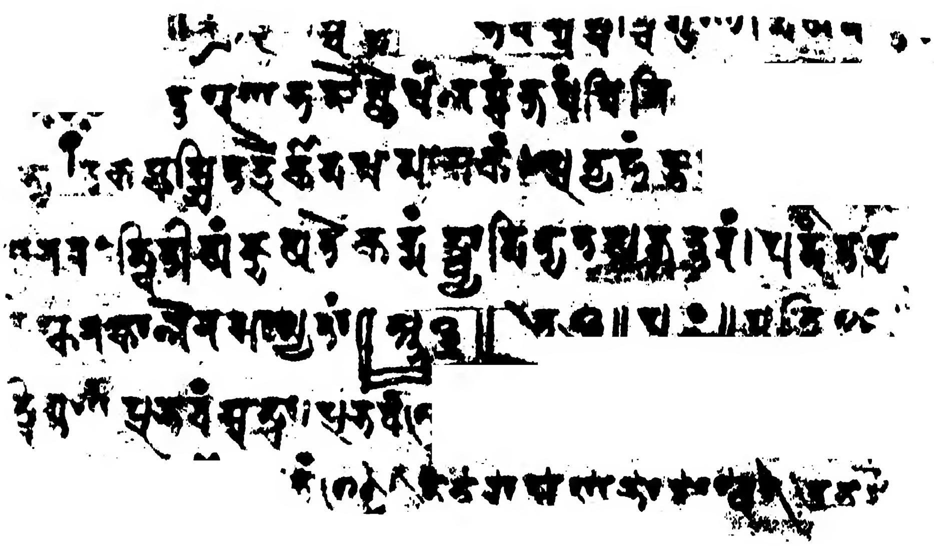

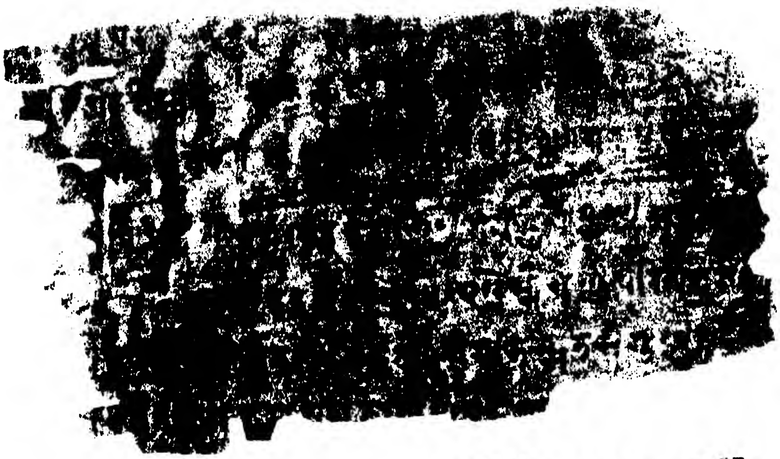

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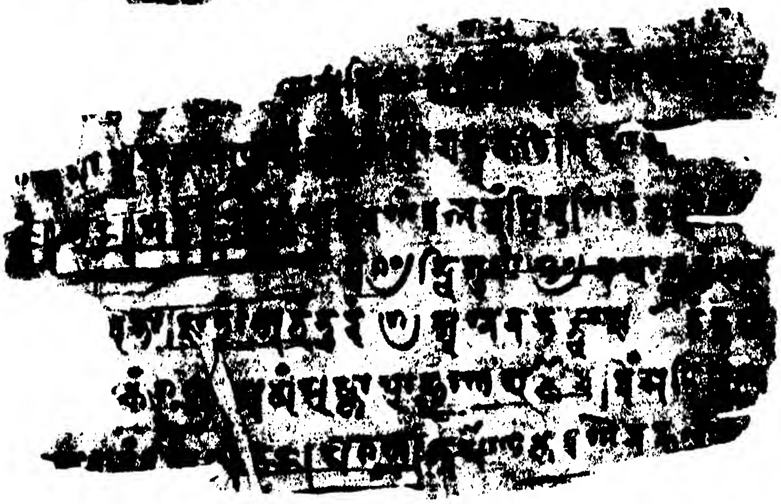

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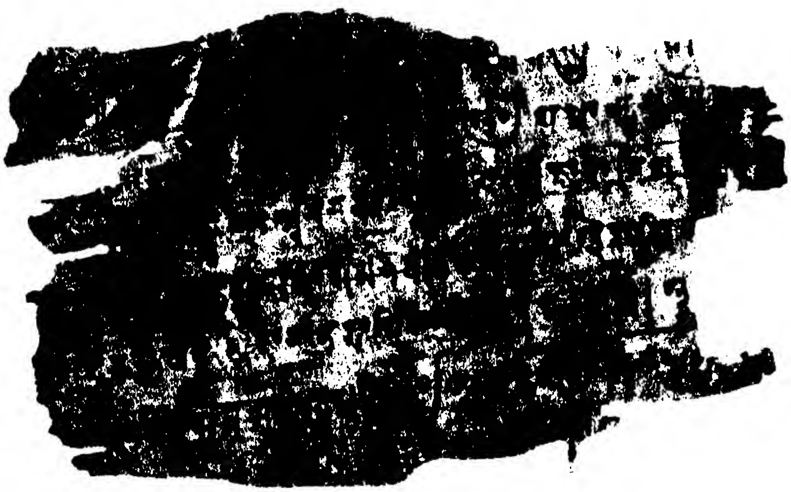

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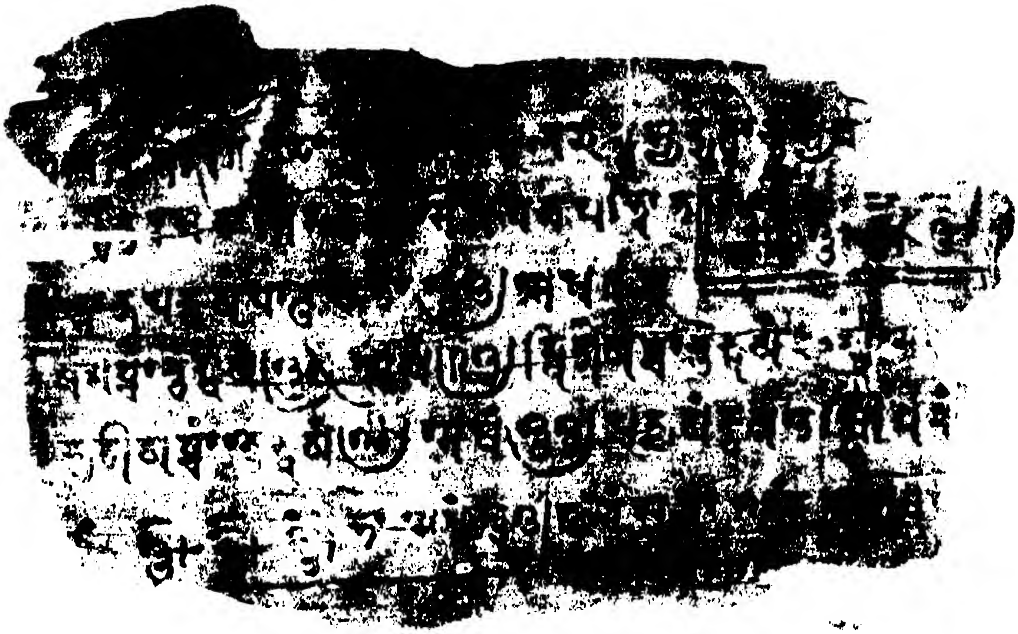

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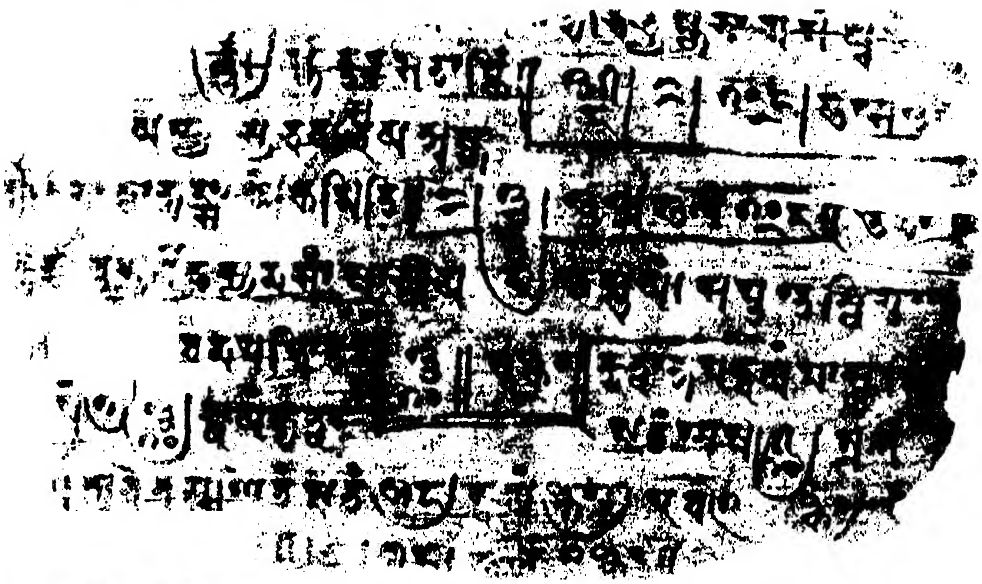

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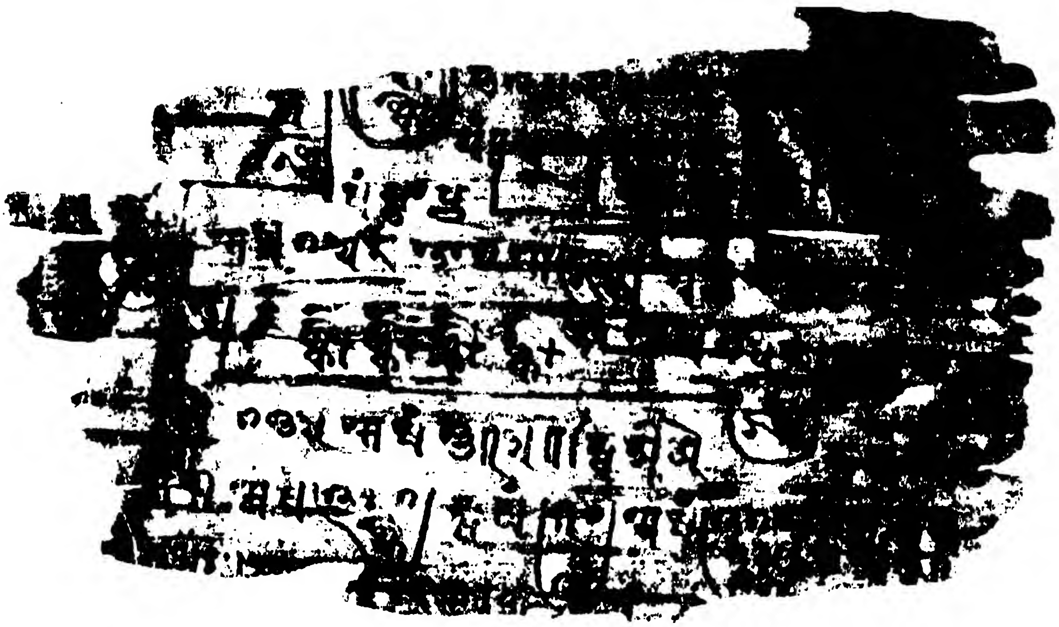

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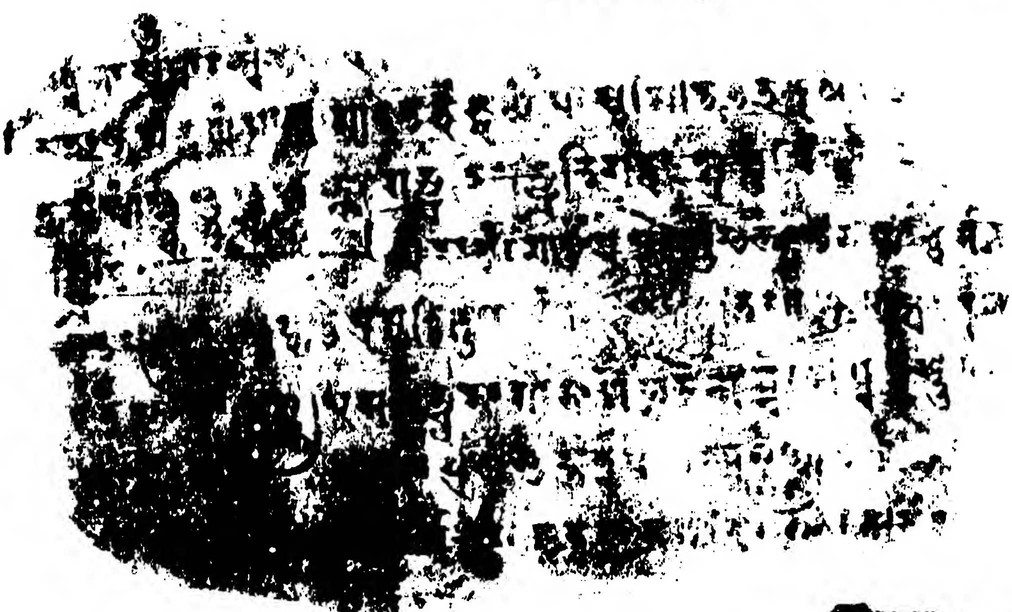

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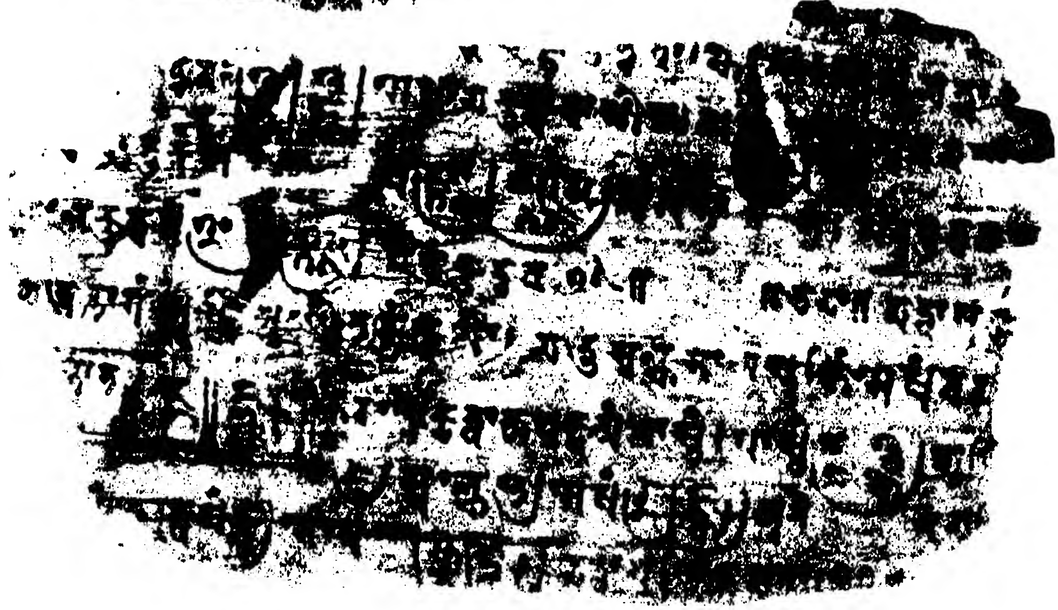

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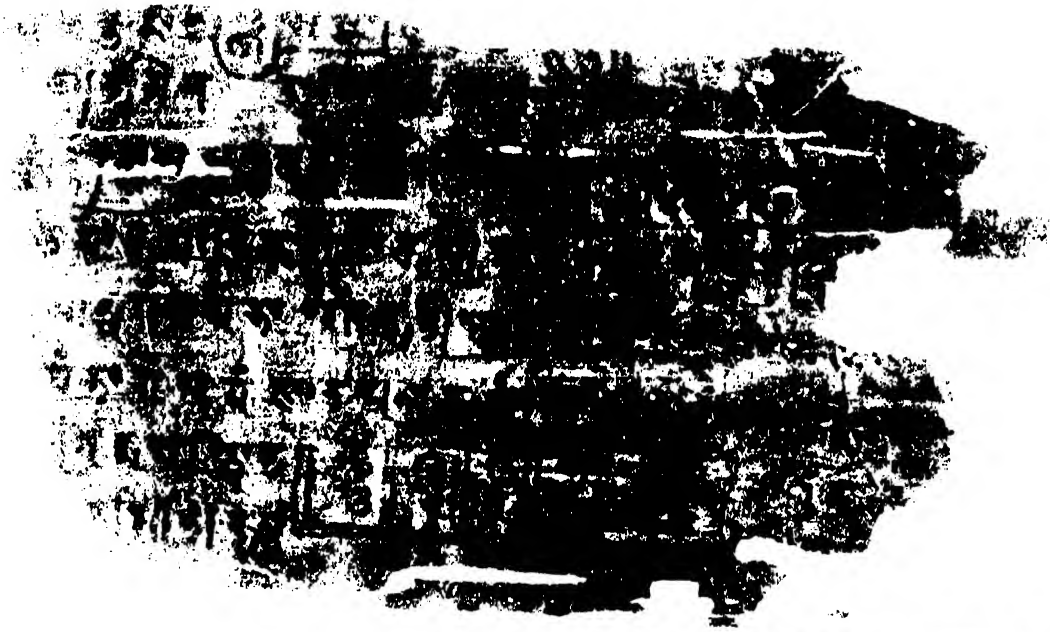

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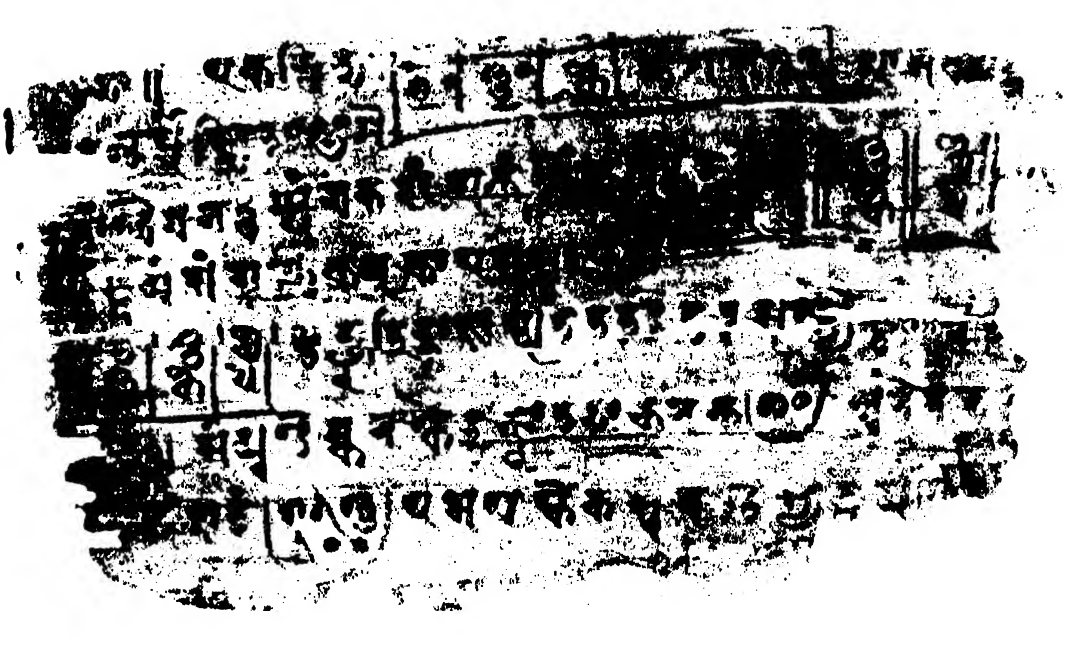

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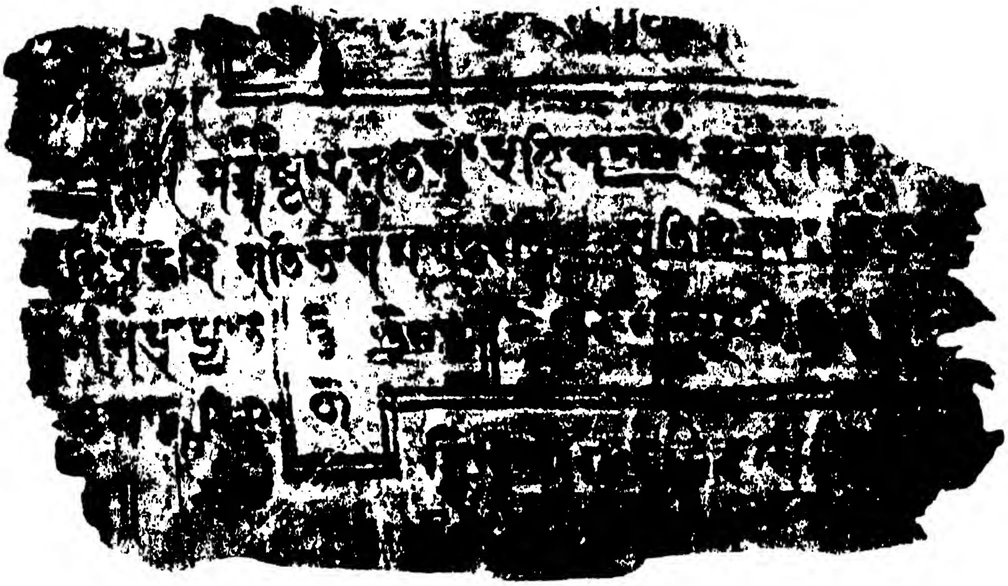

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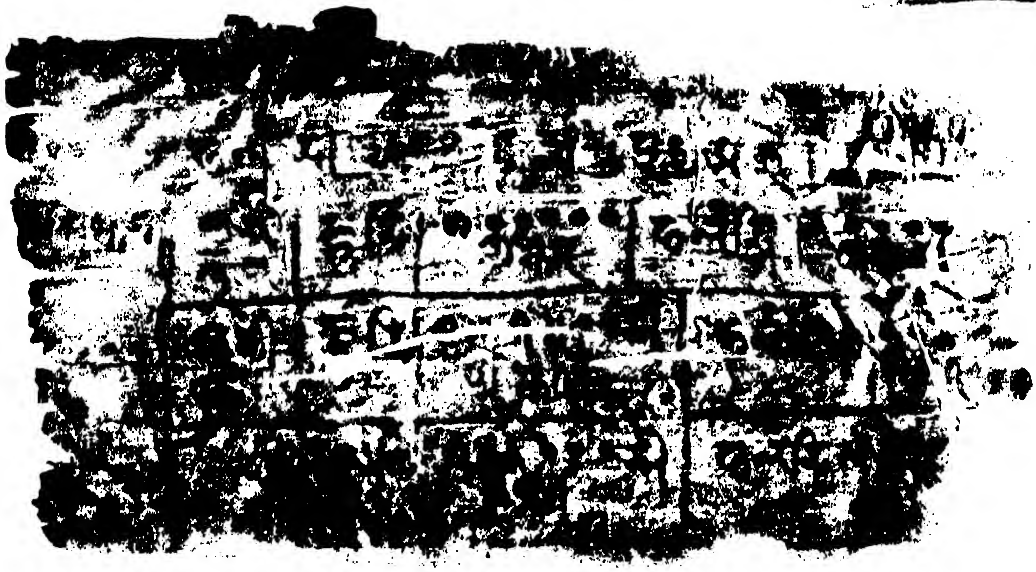

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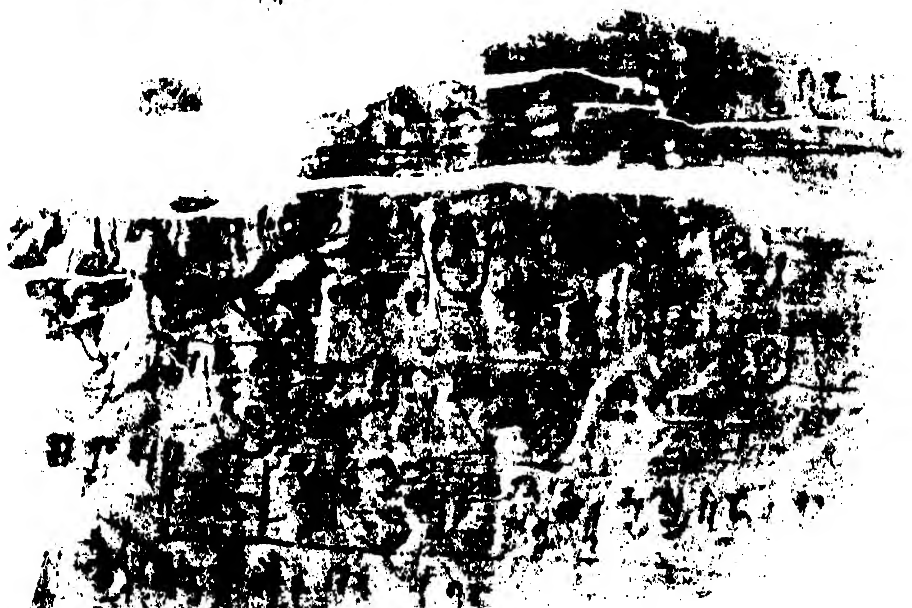

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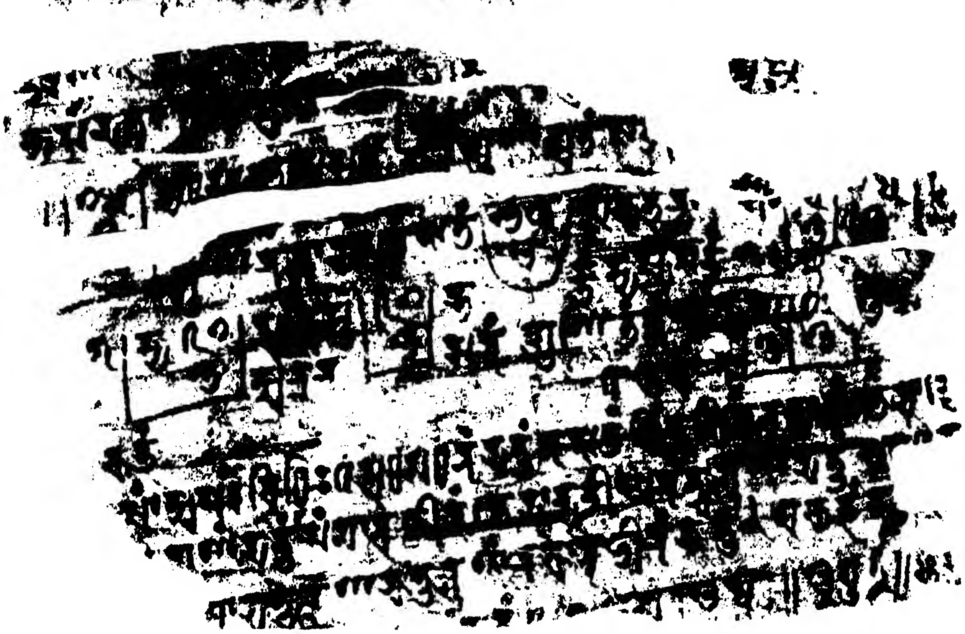

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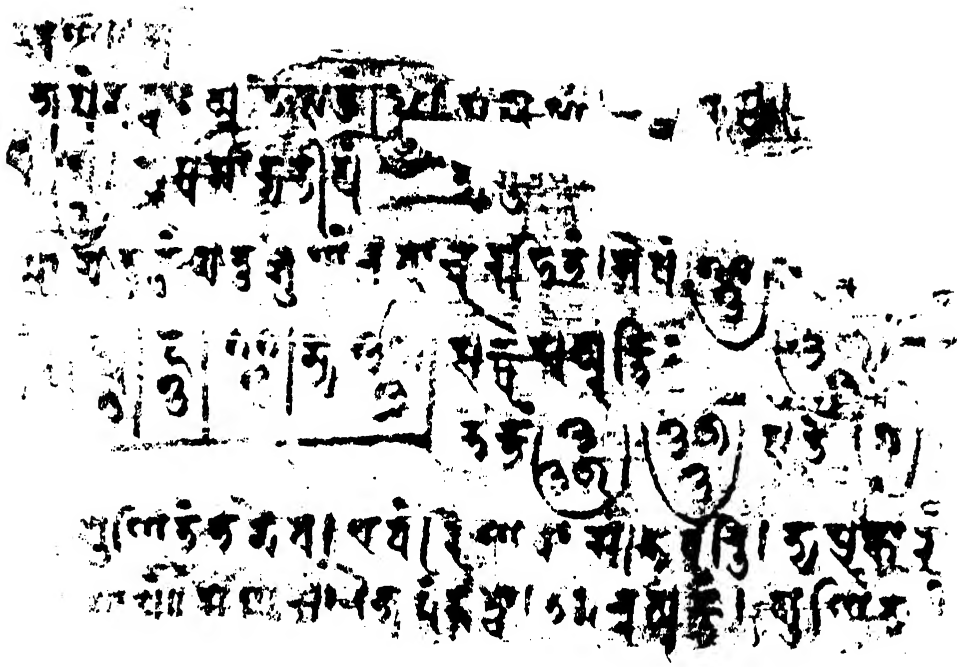

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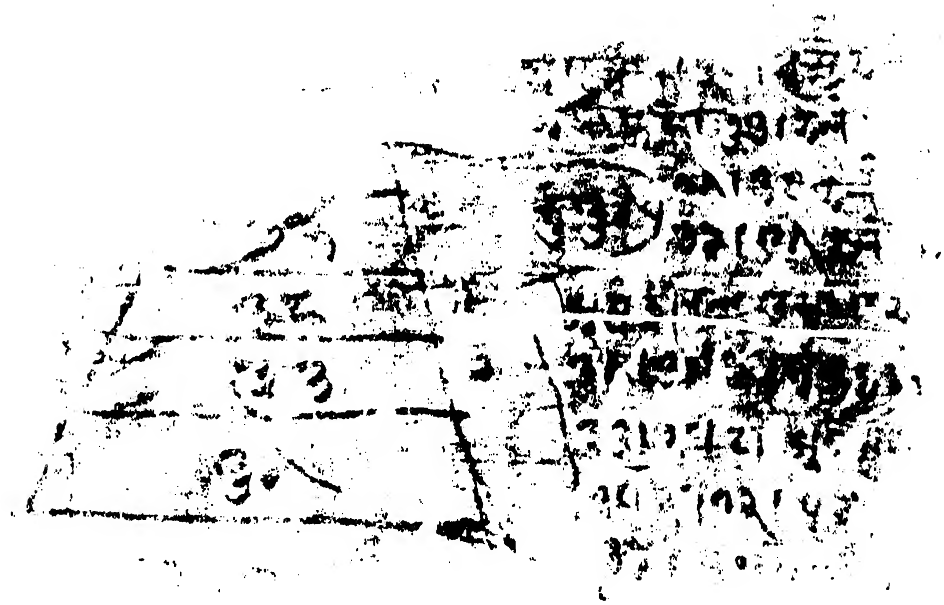

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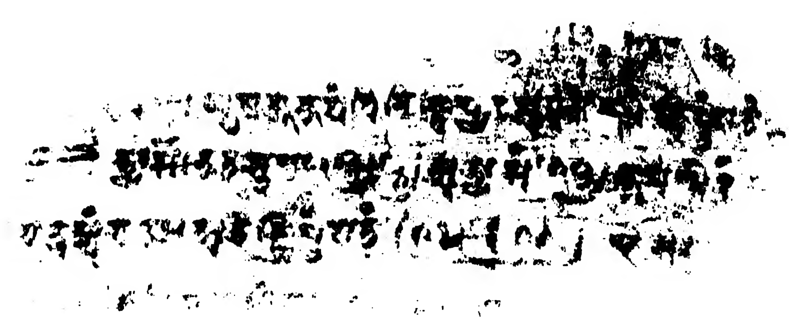

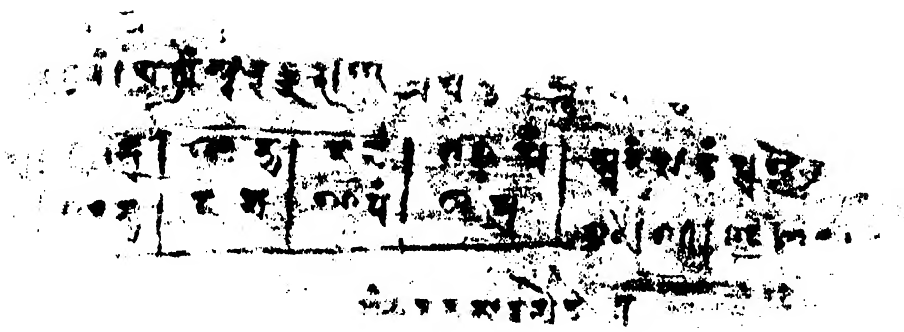

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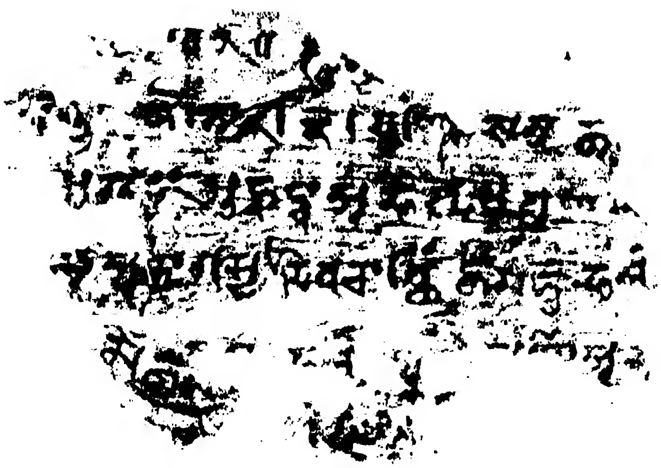

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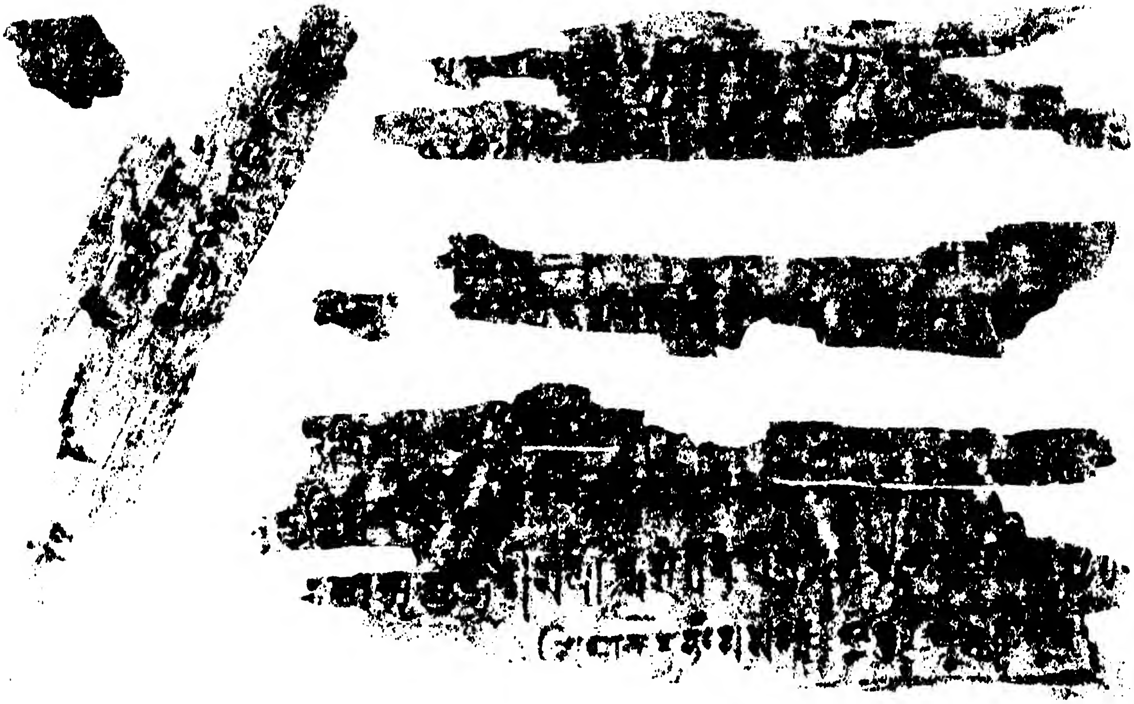

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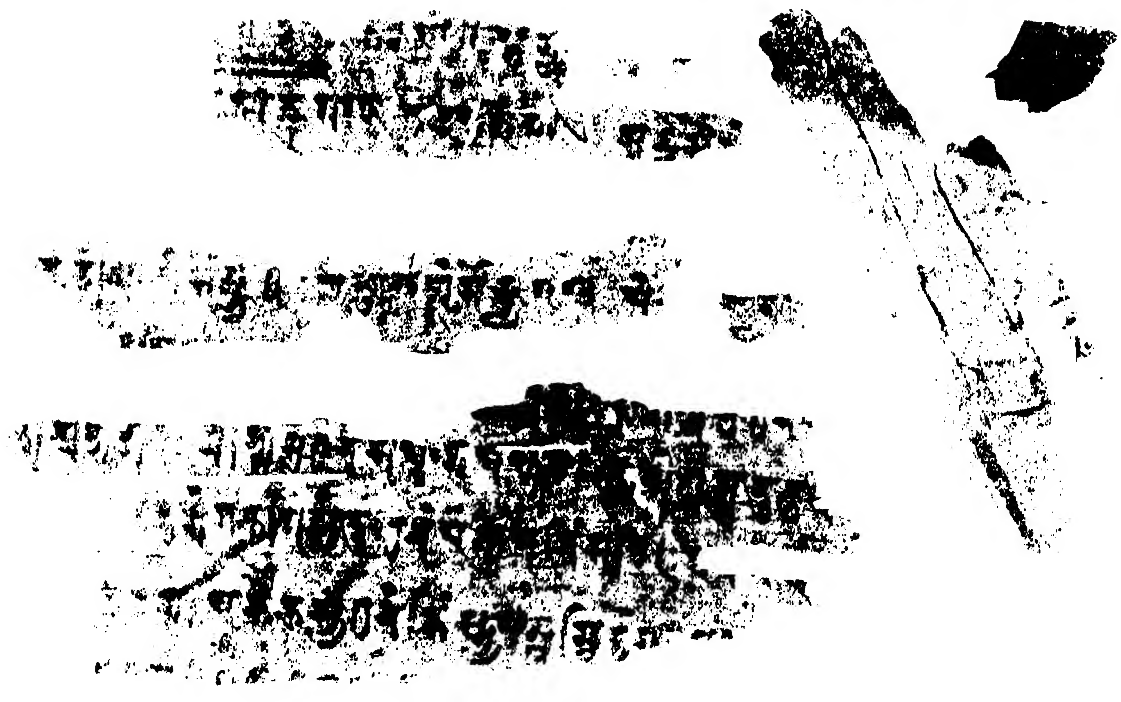

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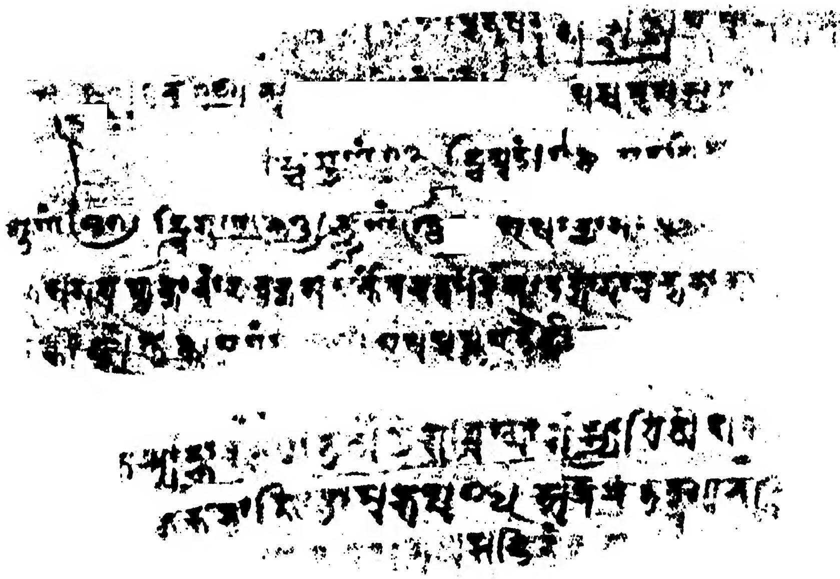

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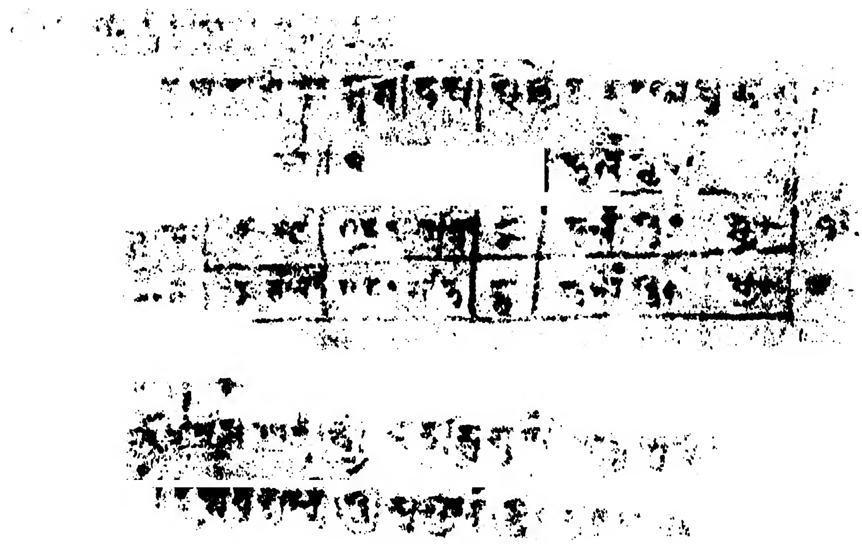

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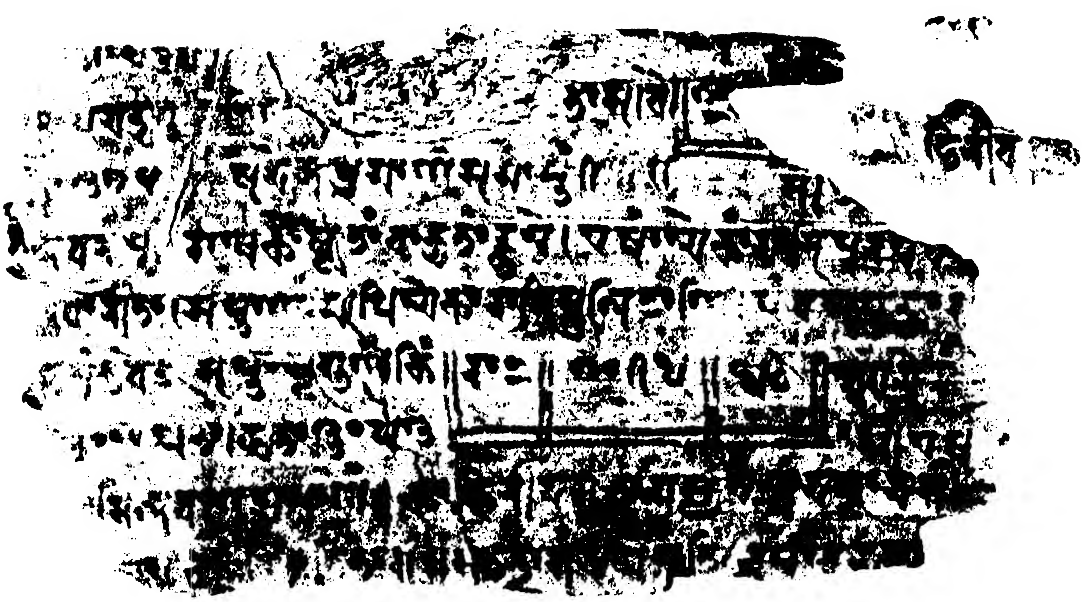

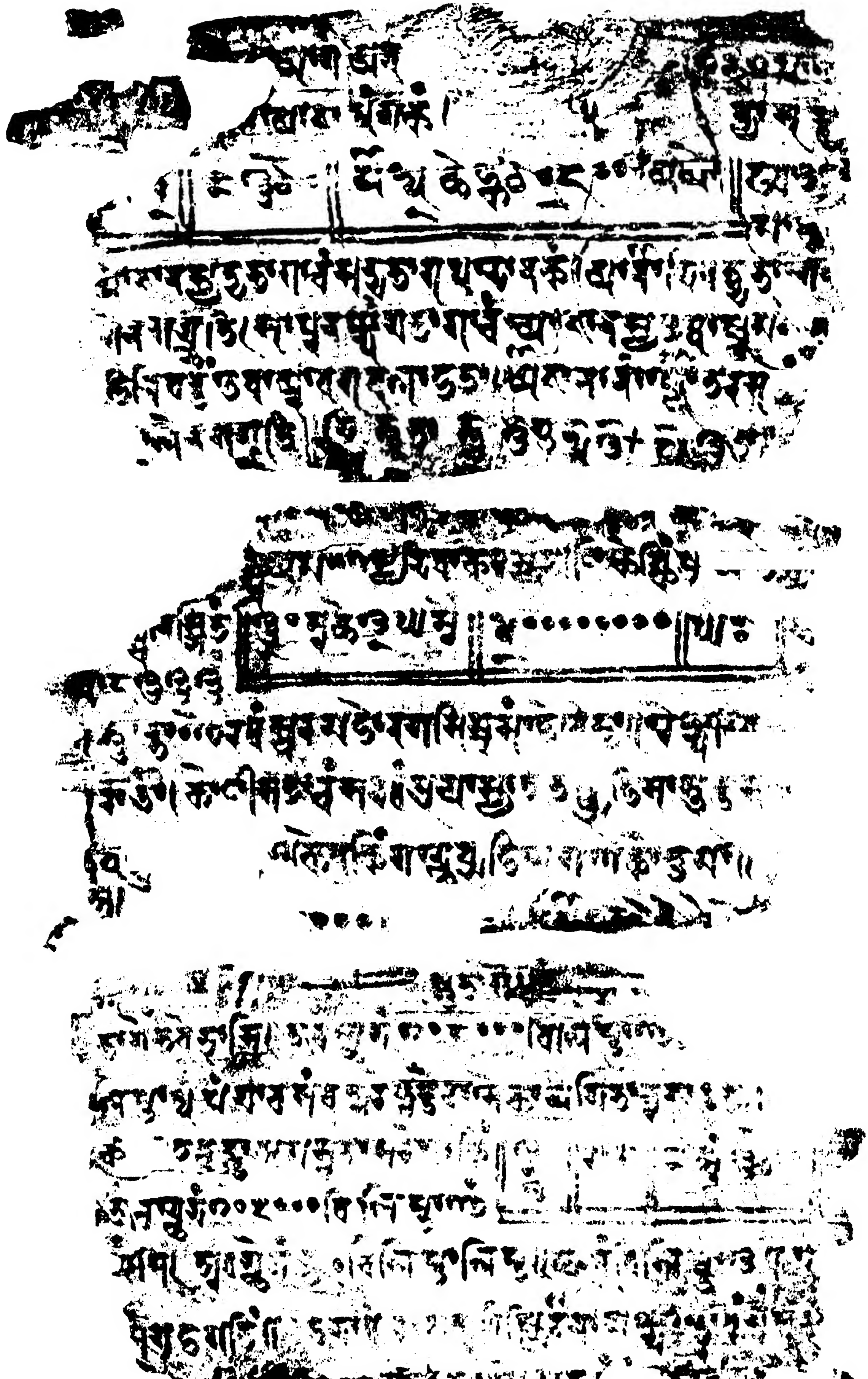

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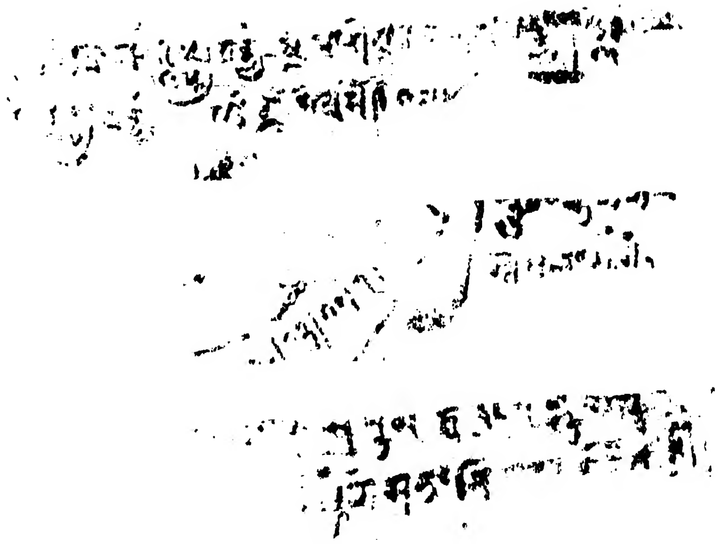

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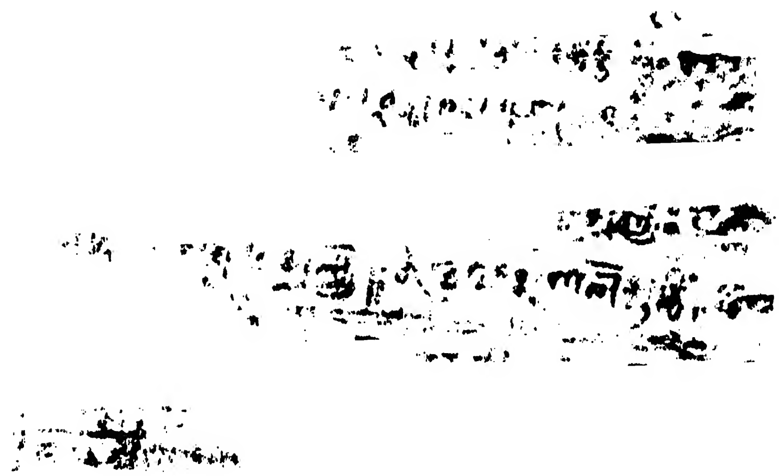

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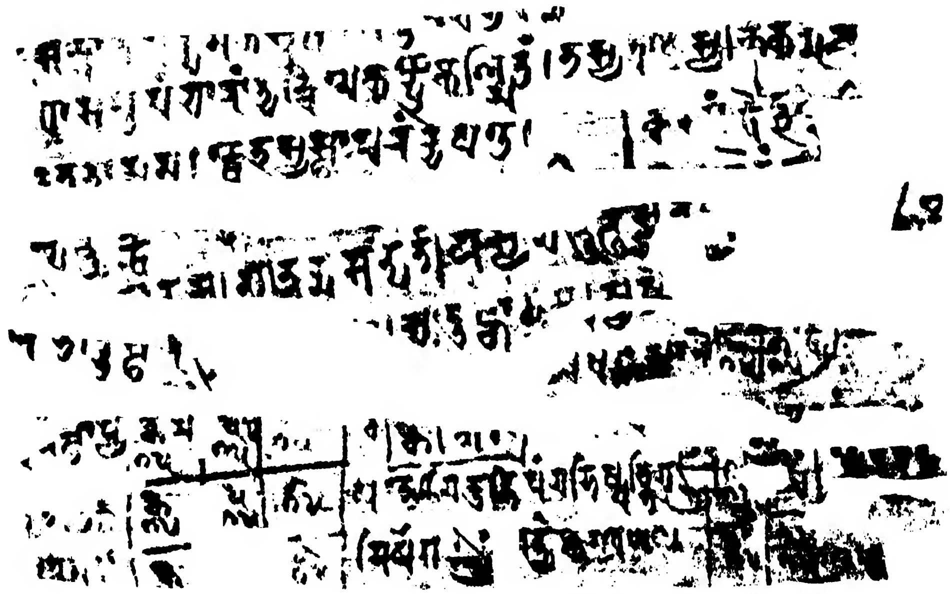

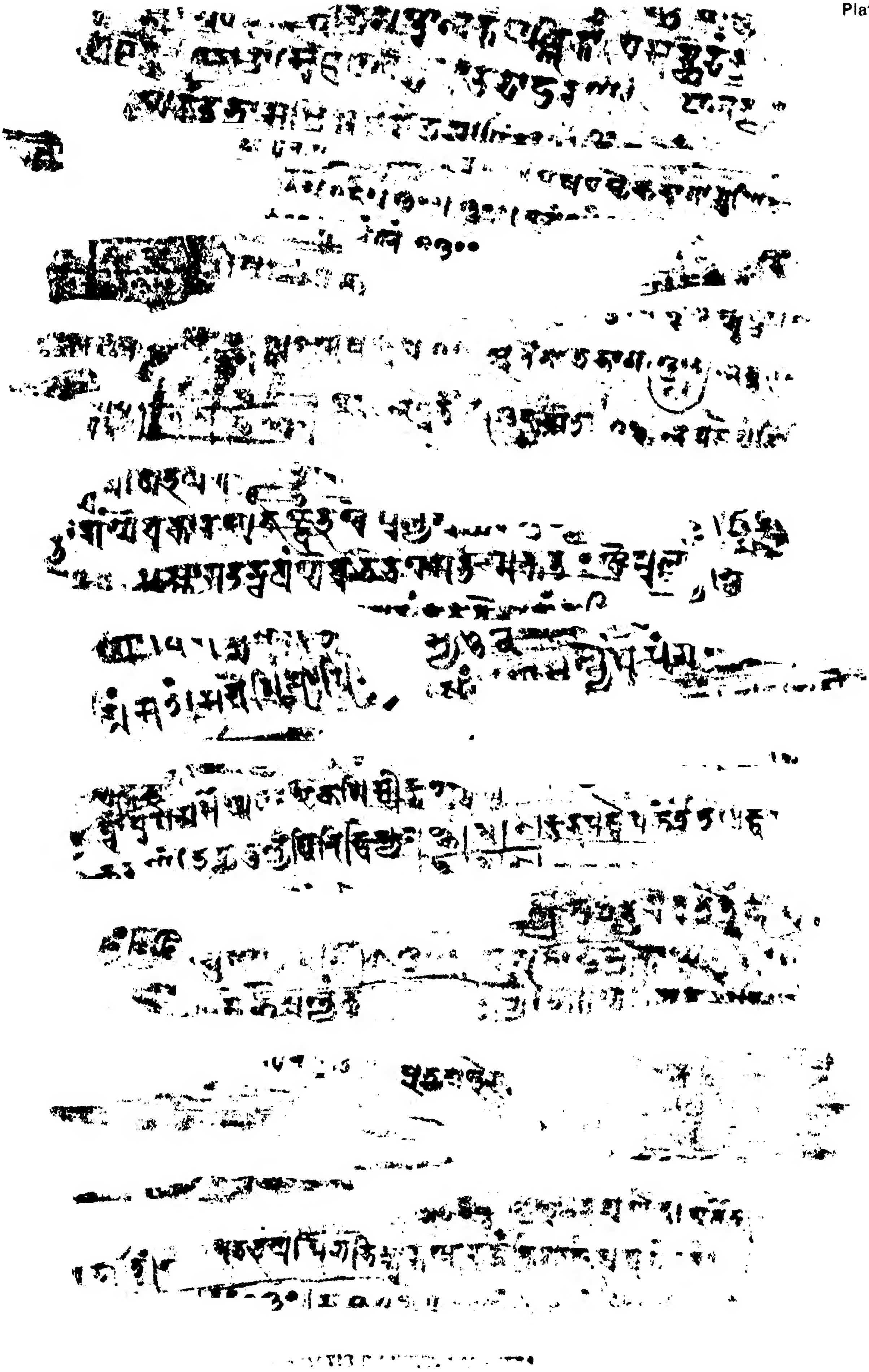

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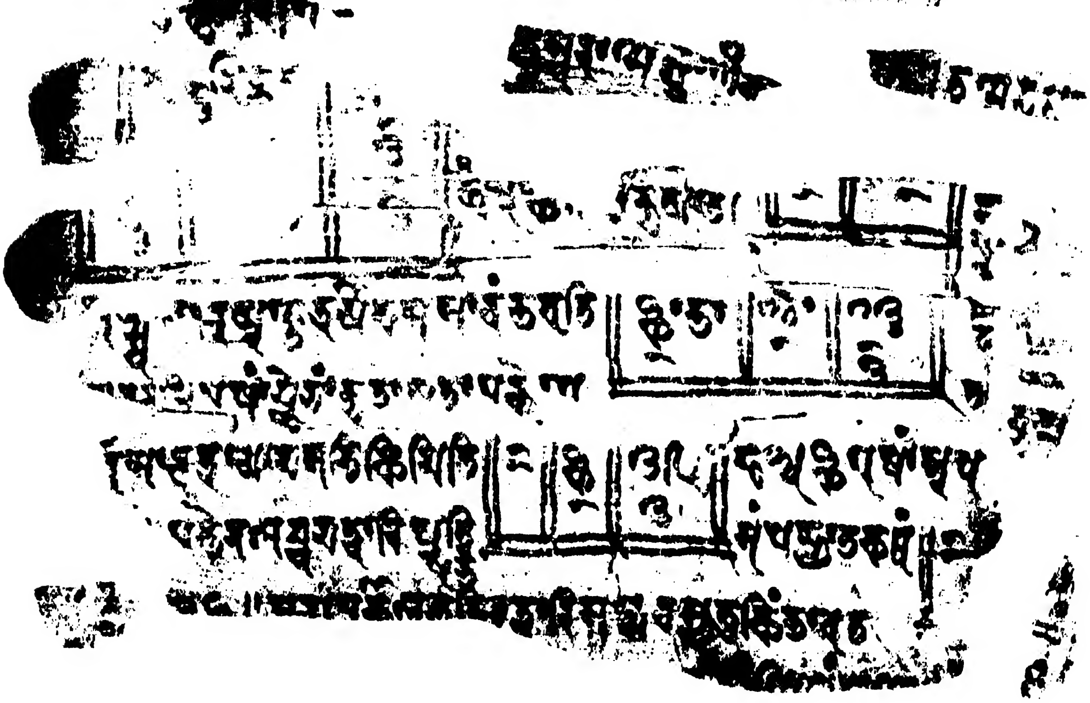

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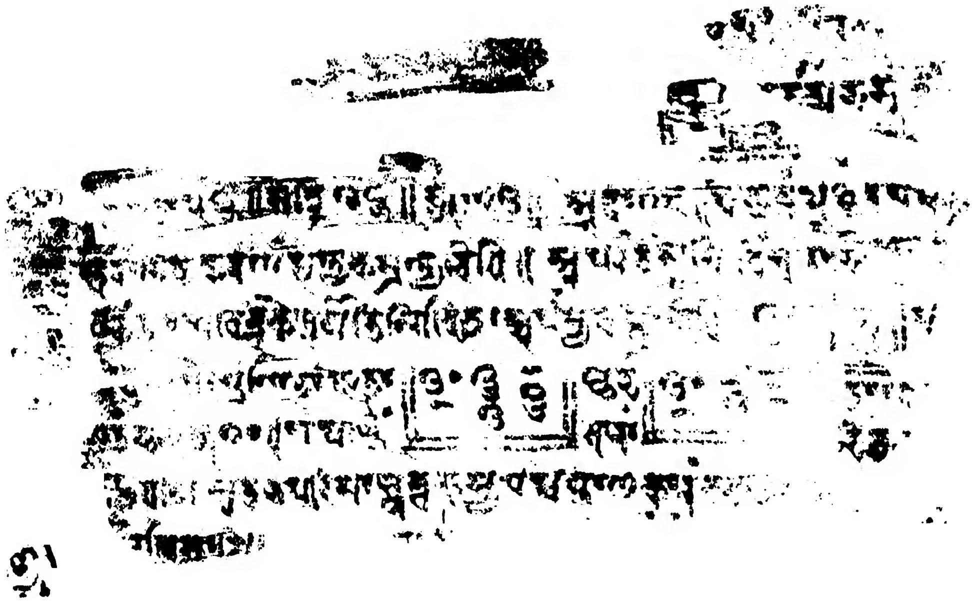

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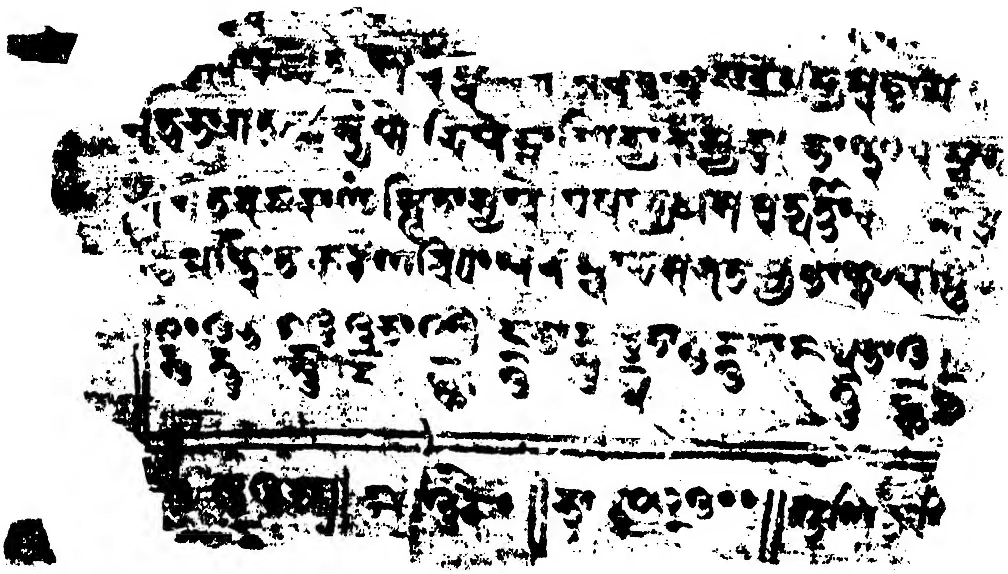

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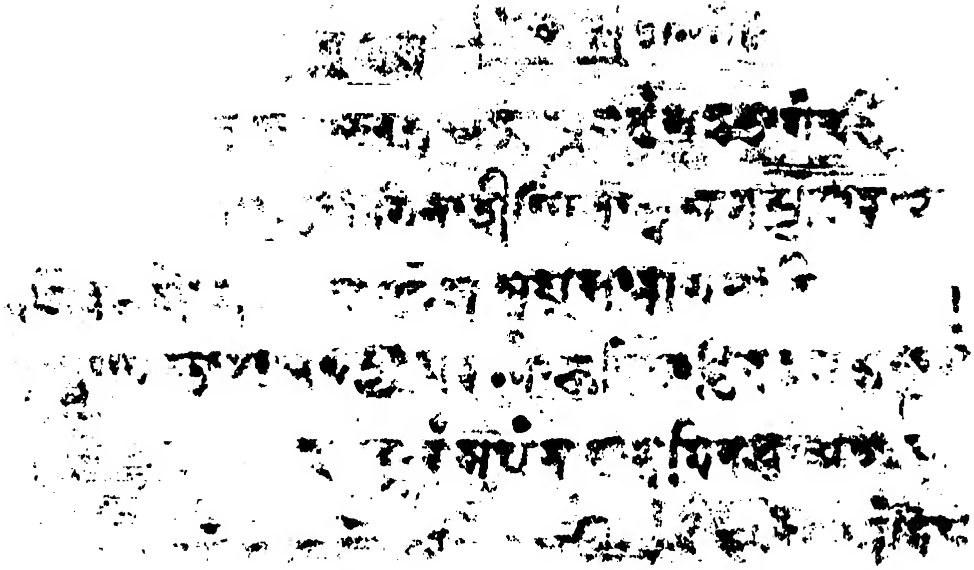

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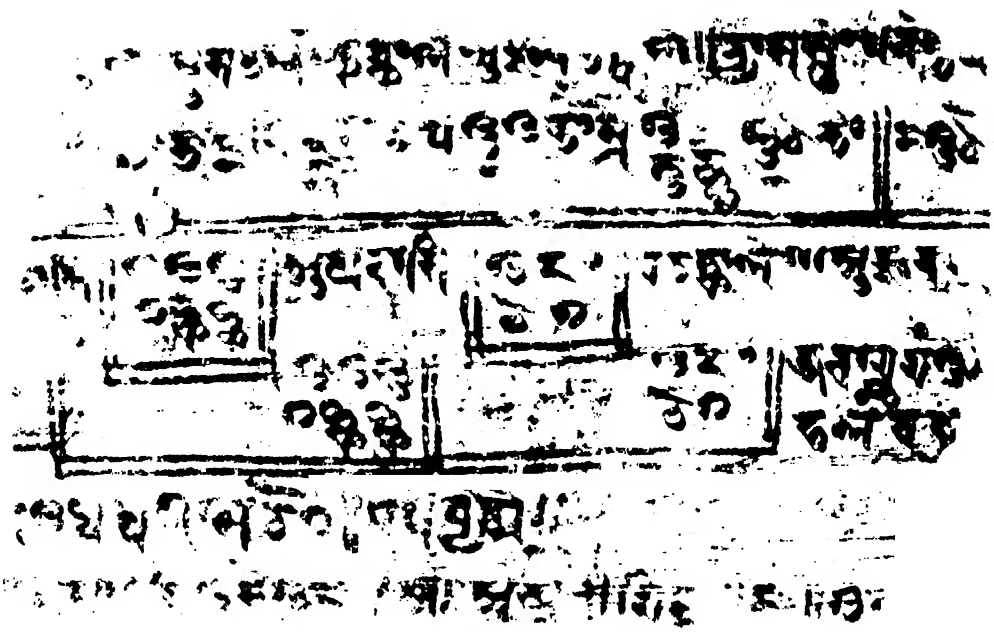

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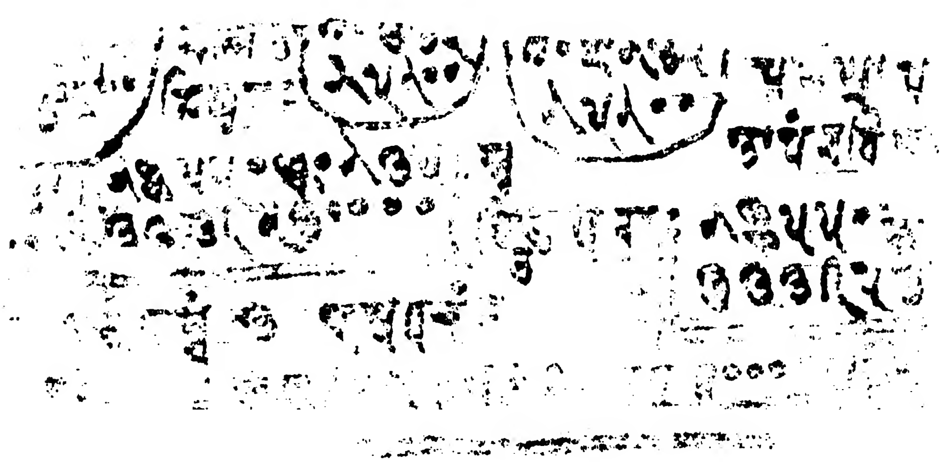

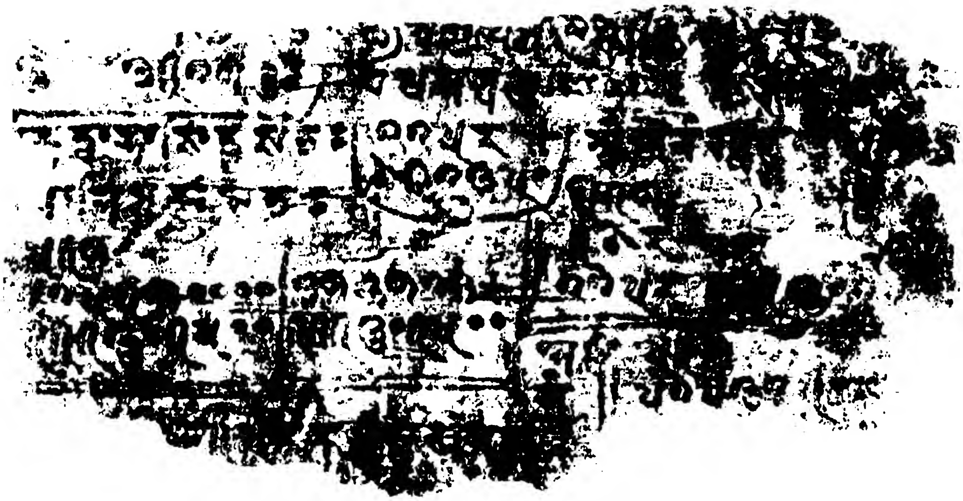

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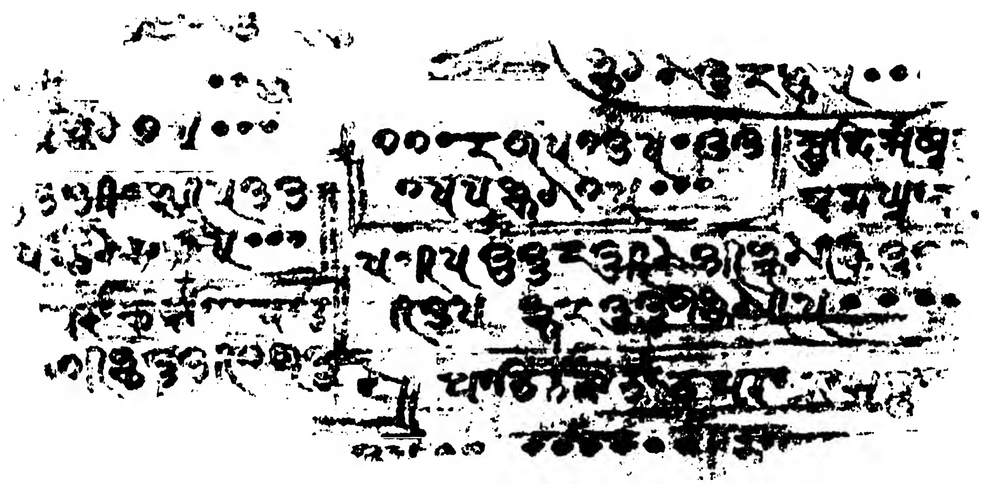

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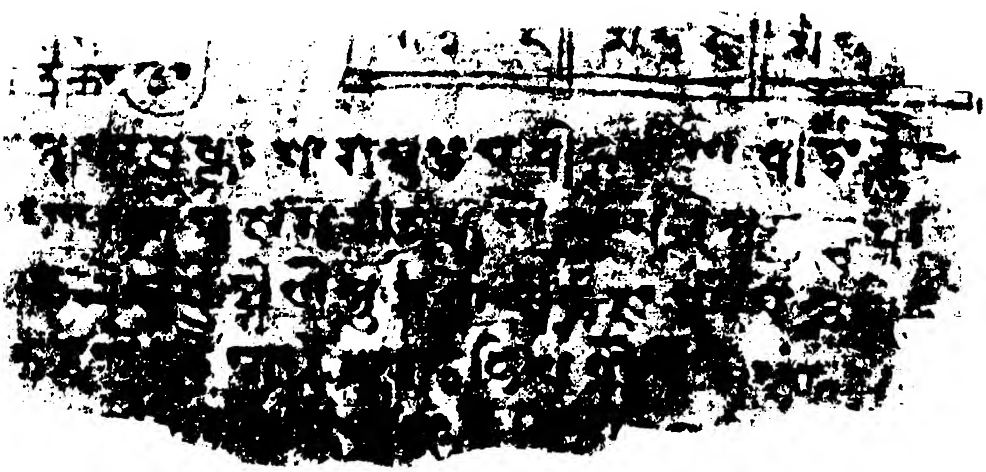

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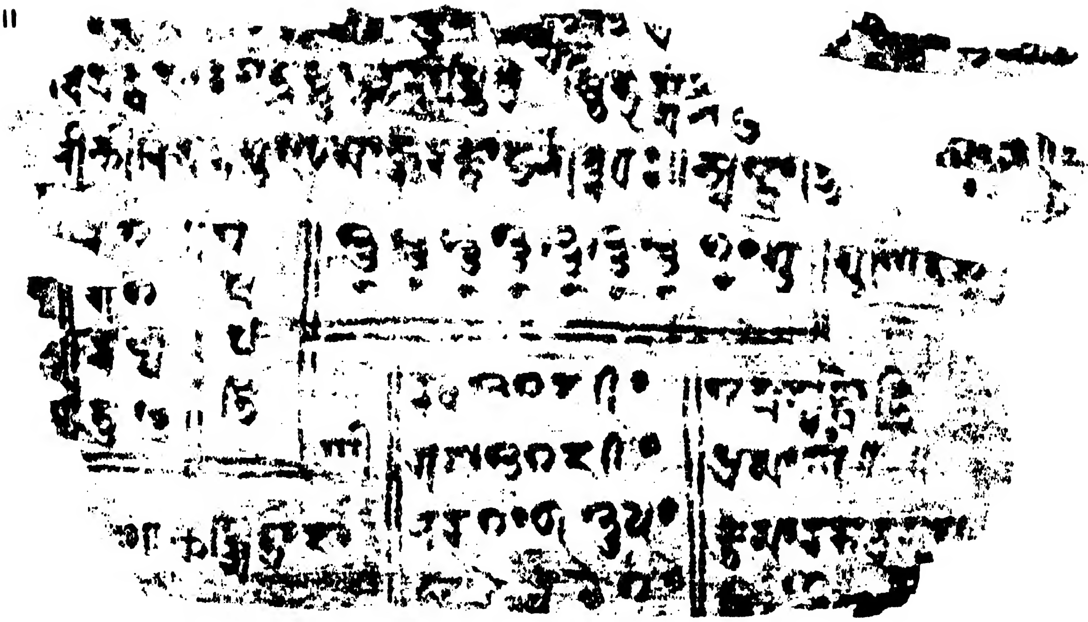

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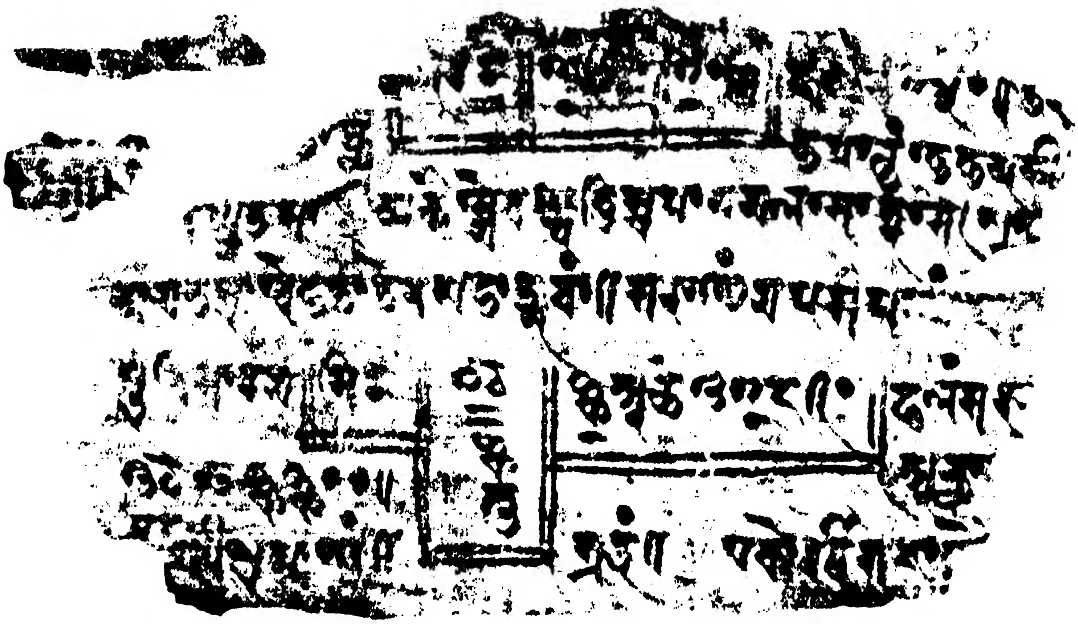

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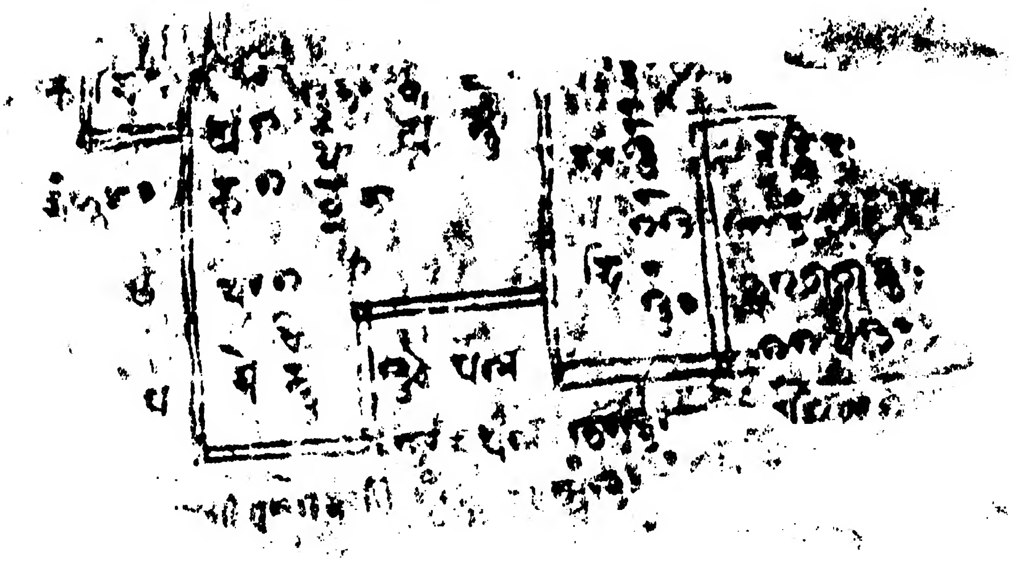

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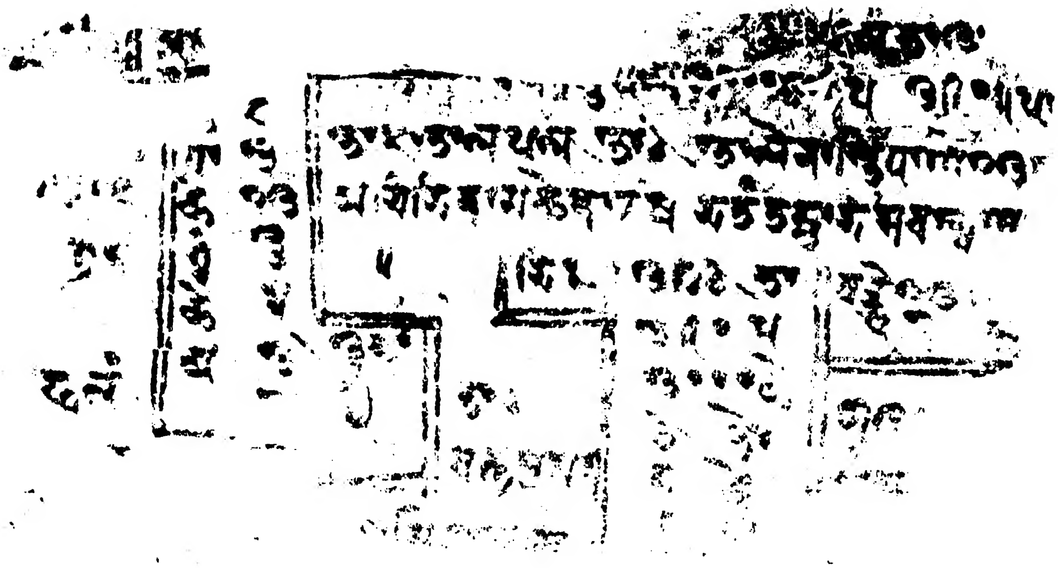

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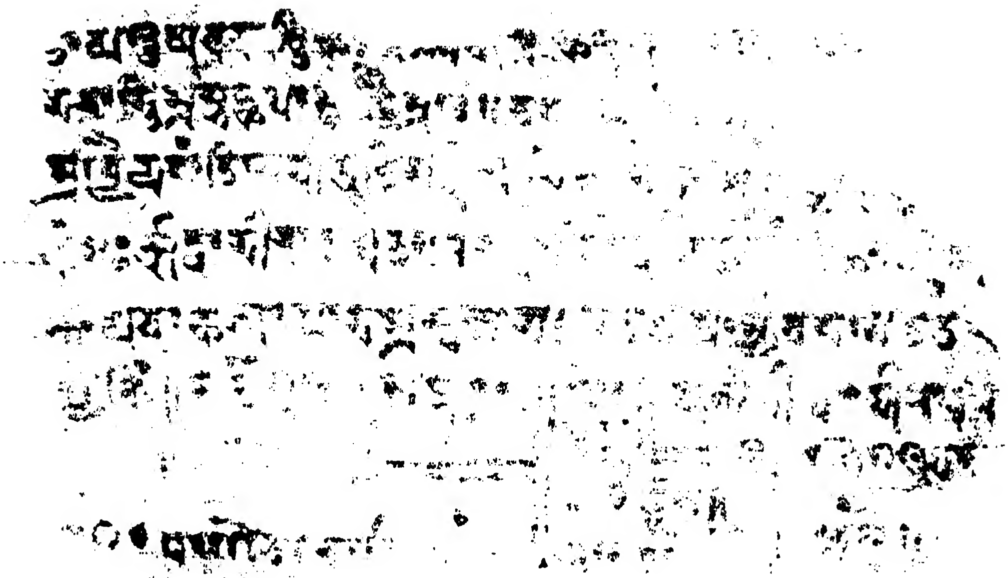

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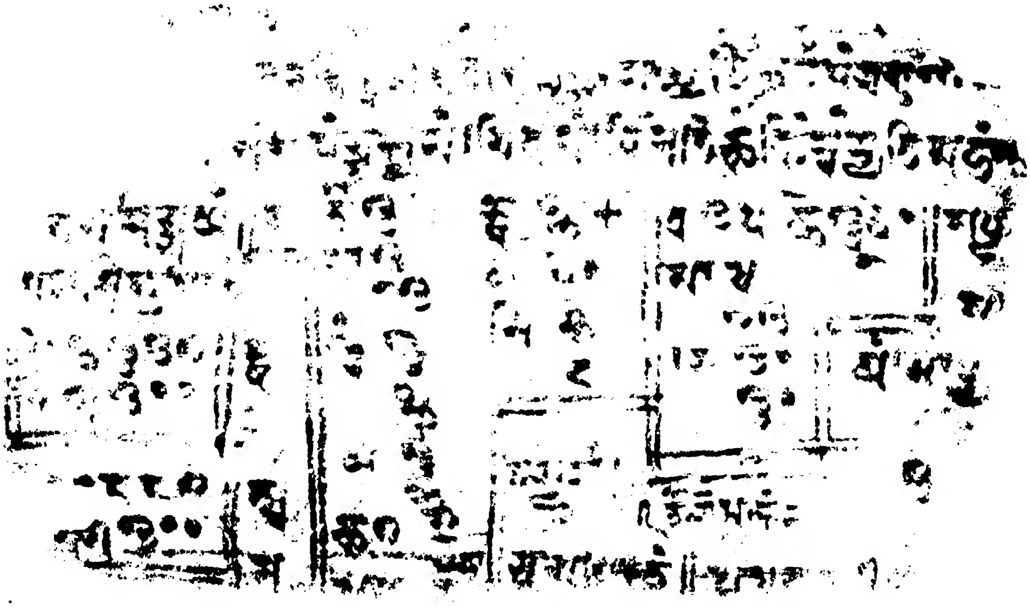

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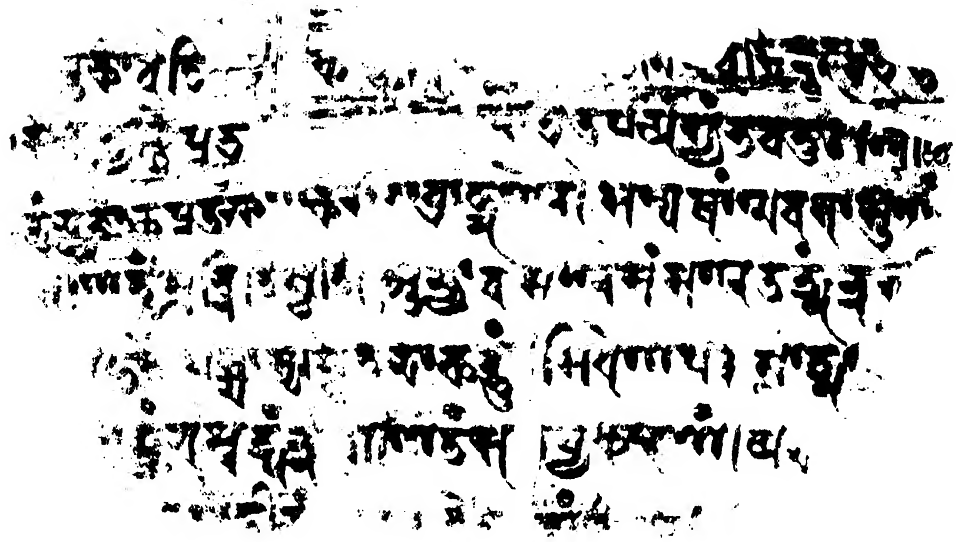

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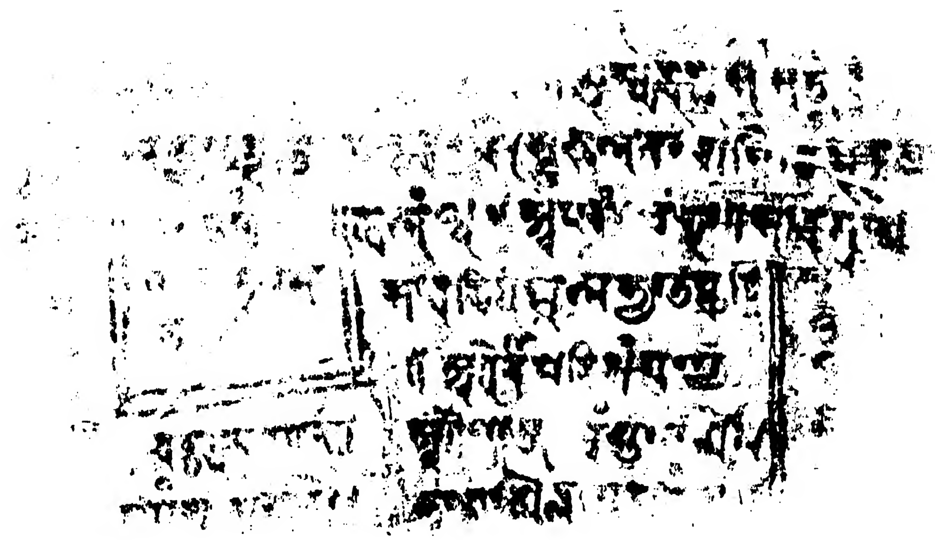
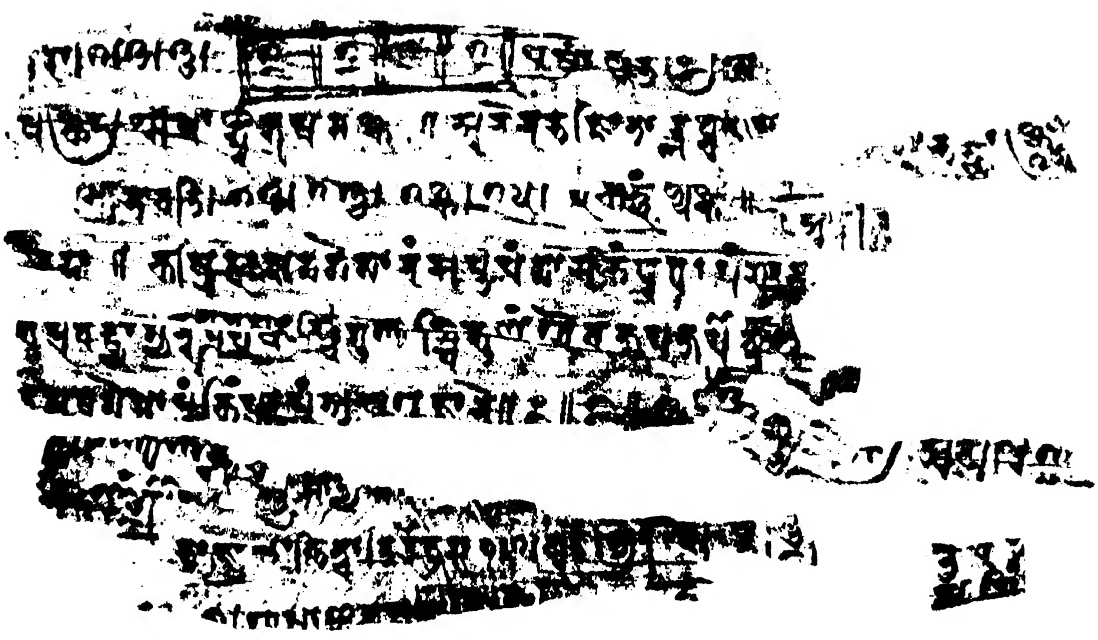
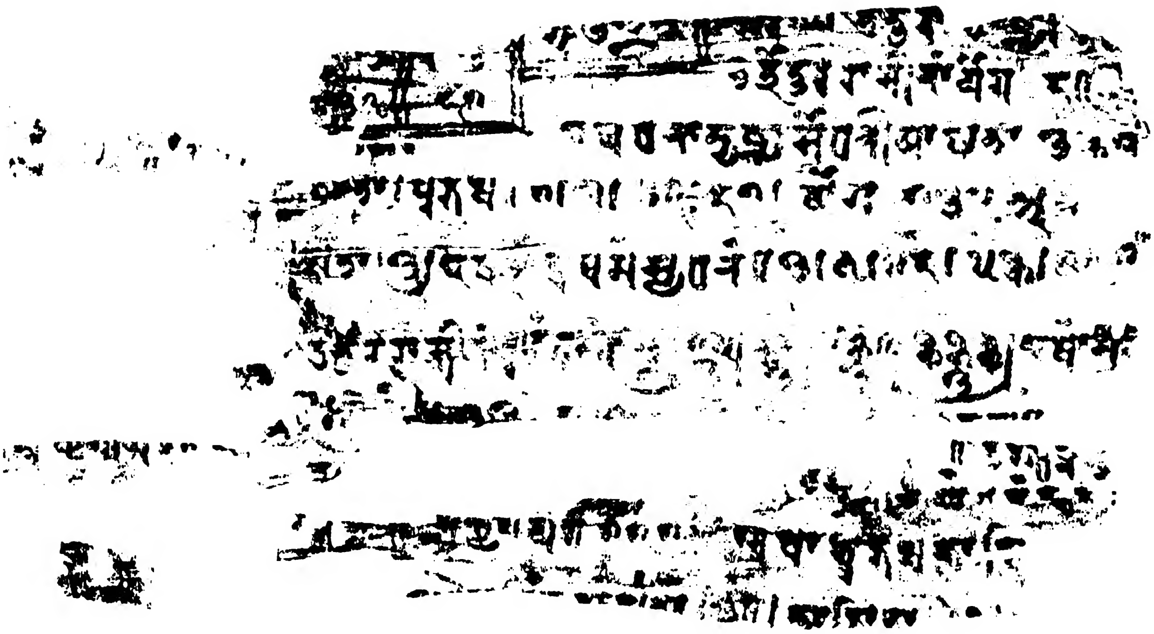


61 RECTO





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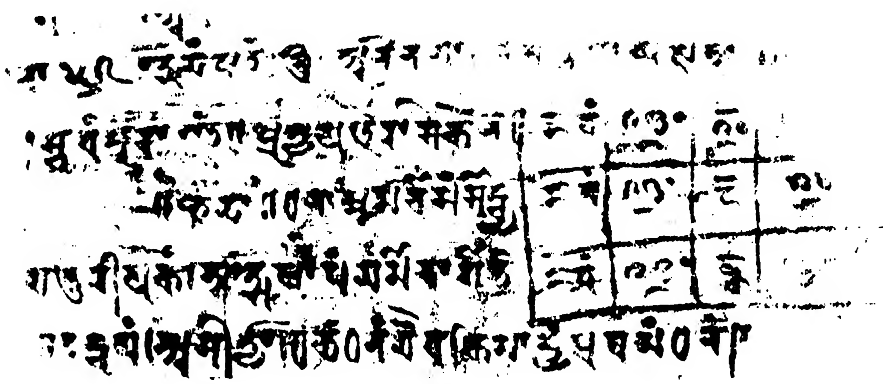
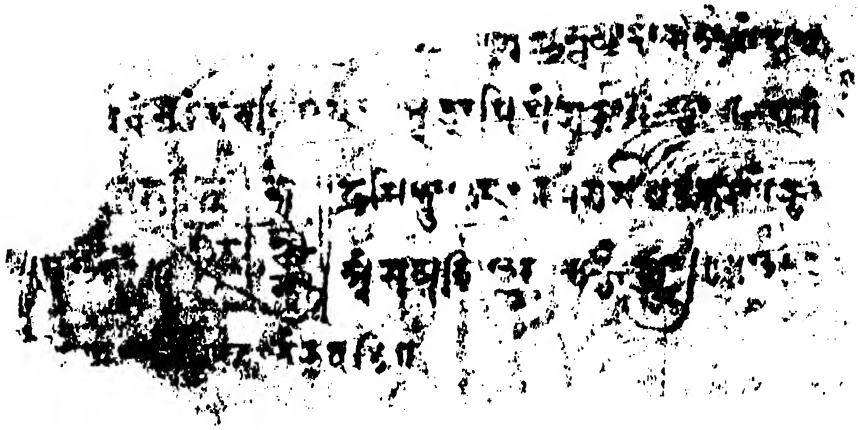


62 RECTO

Plate XXXVI

62 VERSO



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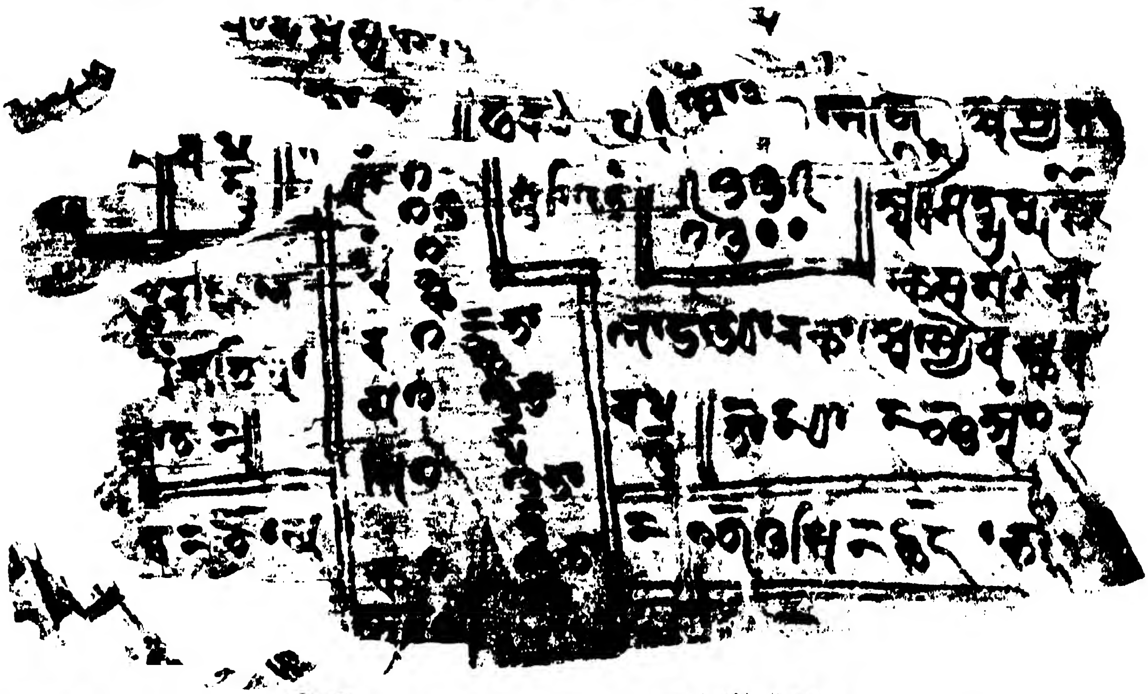

65 RECTO

66 VERSO

Plate XXXVIII

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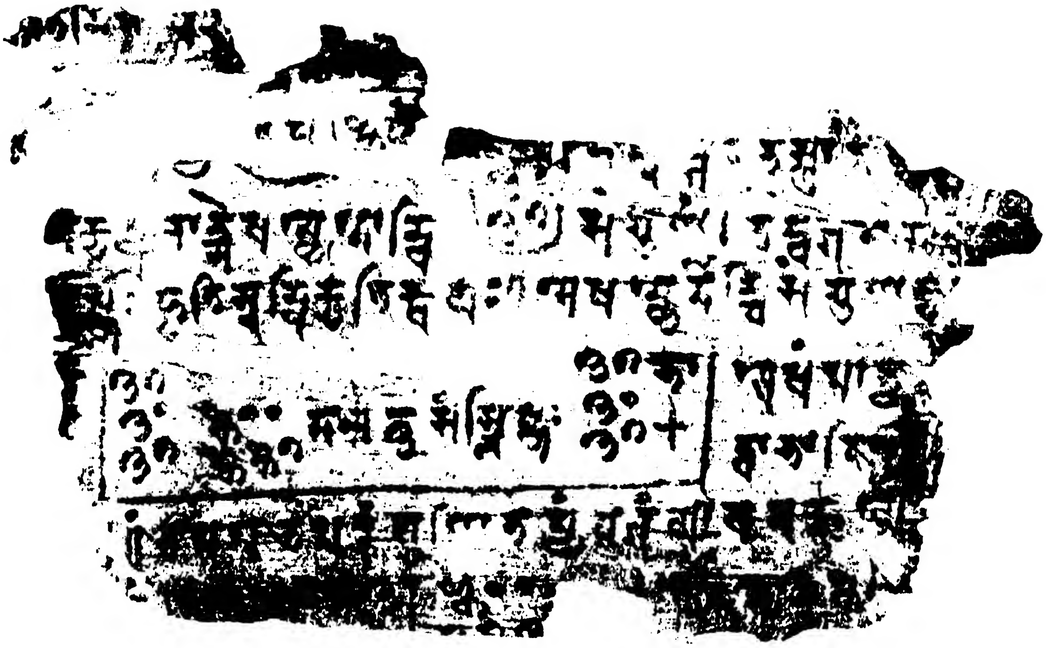

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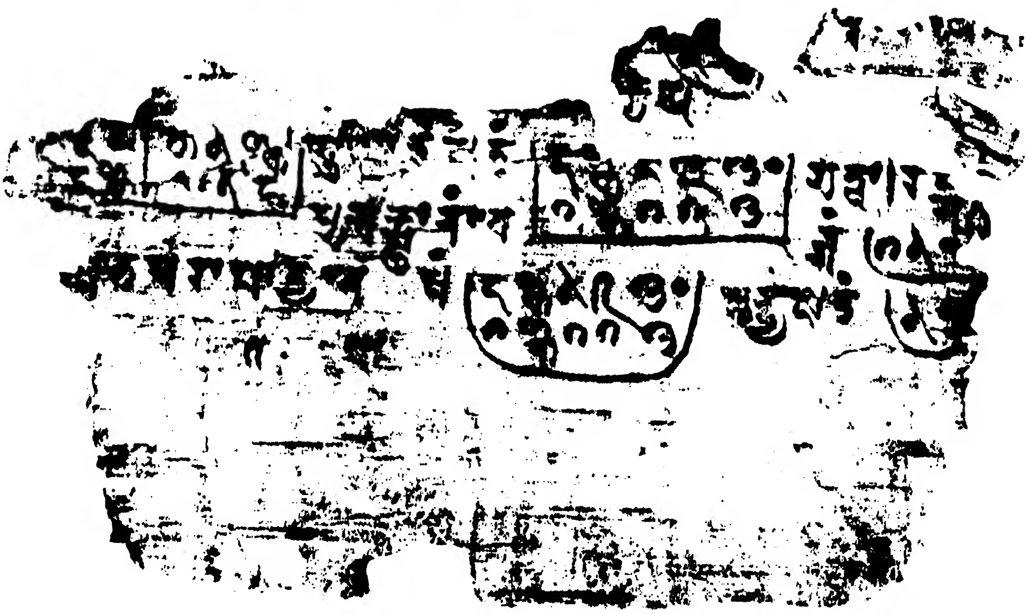

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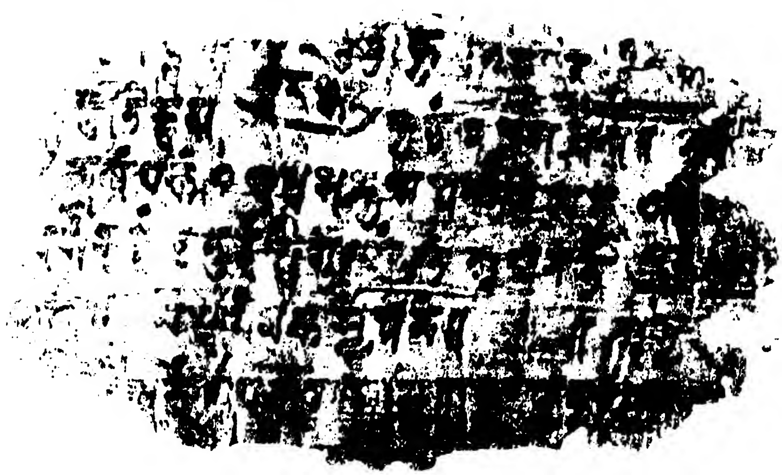

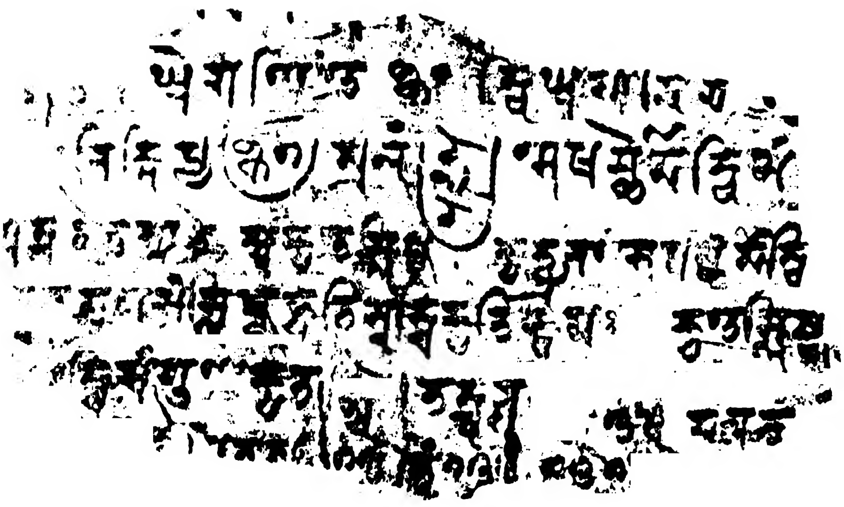

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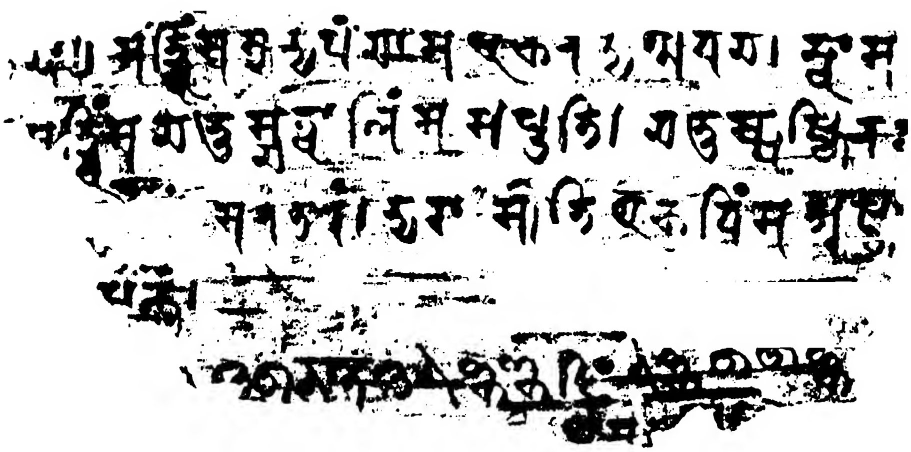
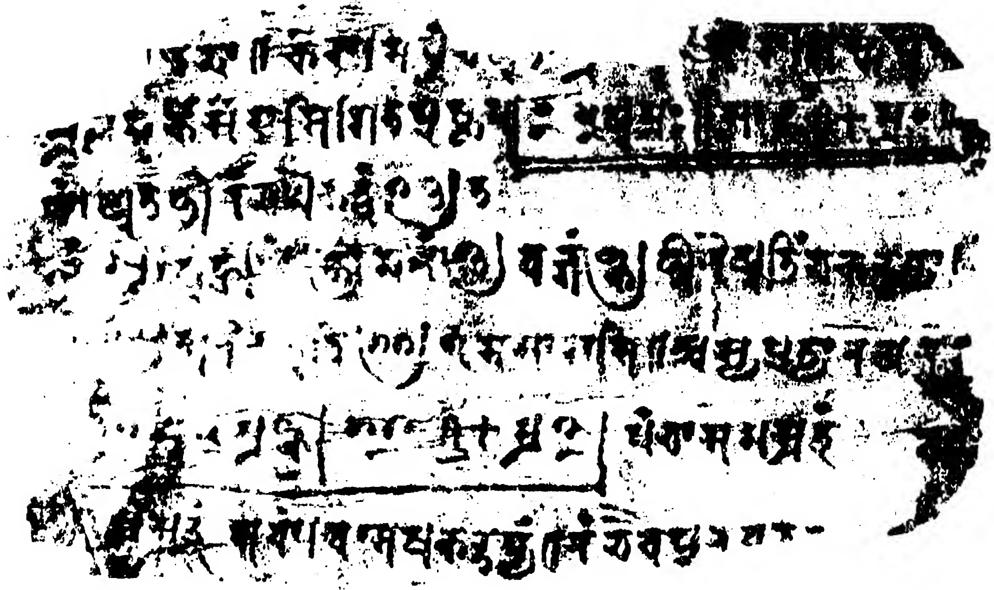


68 RECTO

68 VERSO

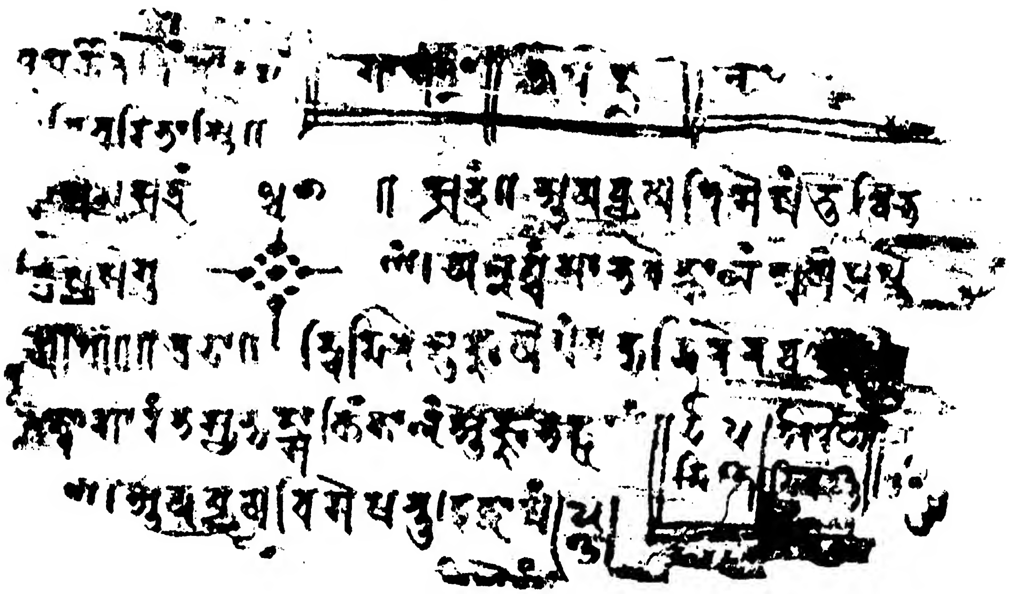
Plate XL

69 RECTO



69 Verso is blank

60 RECTO



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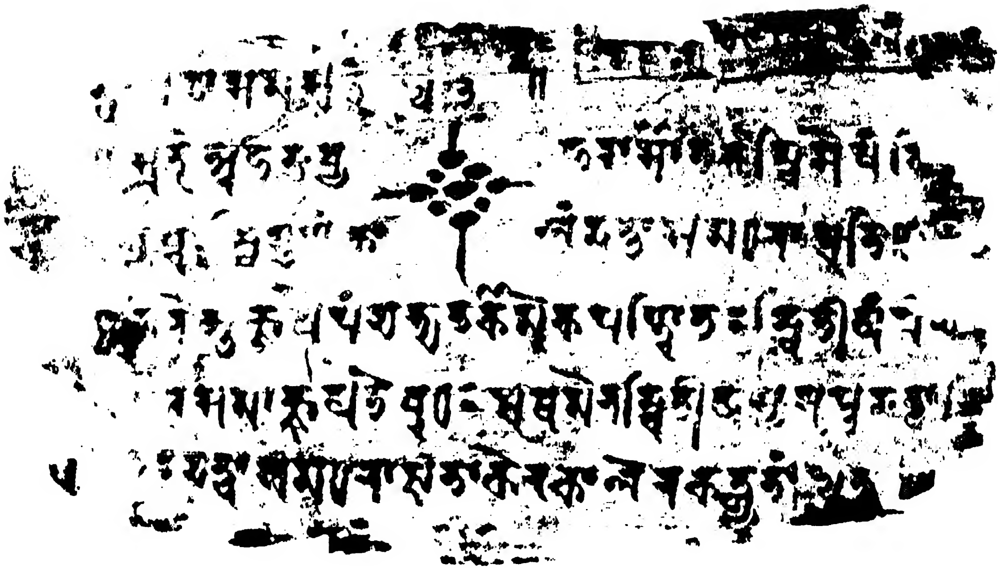

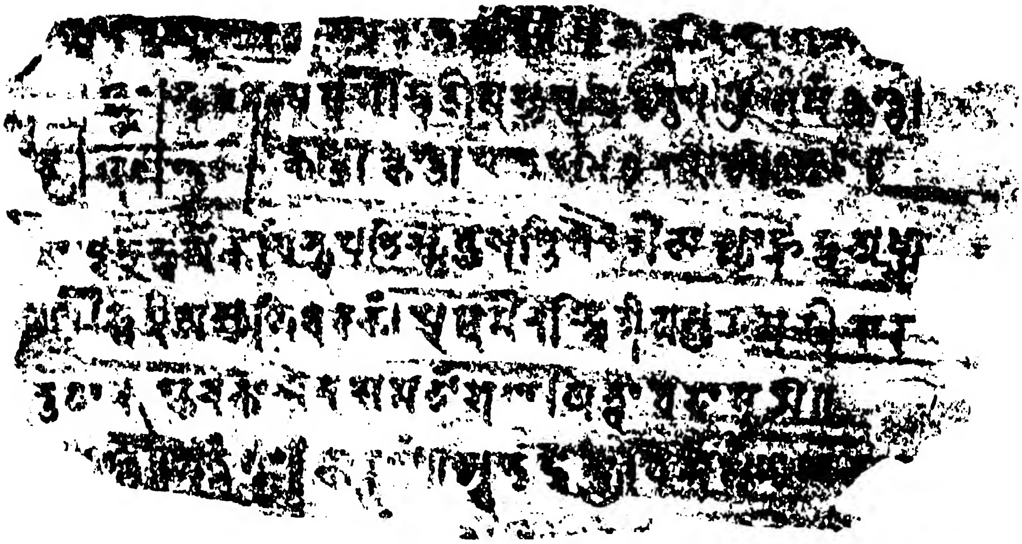

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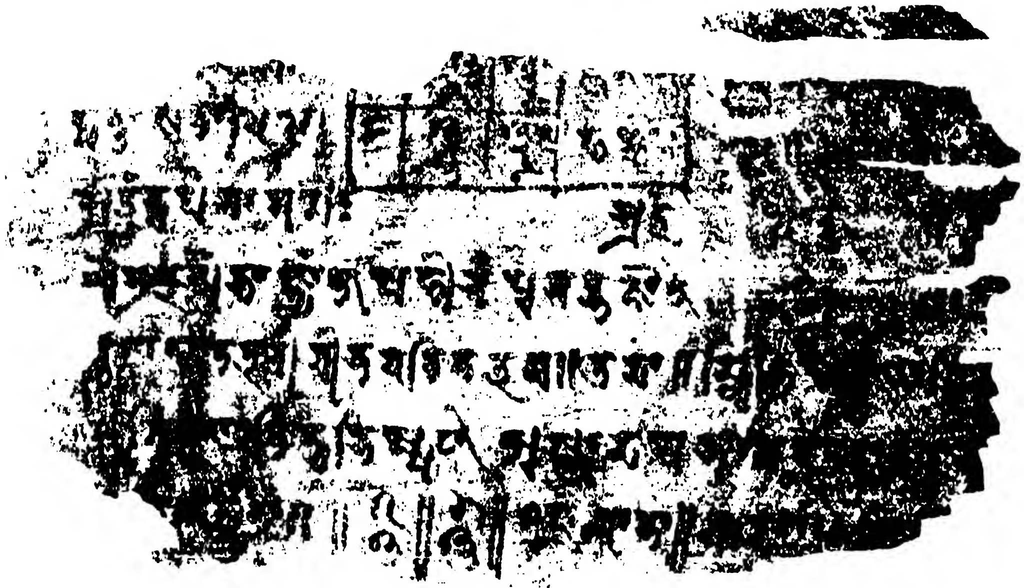

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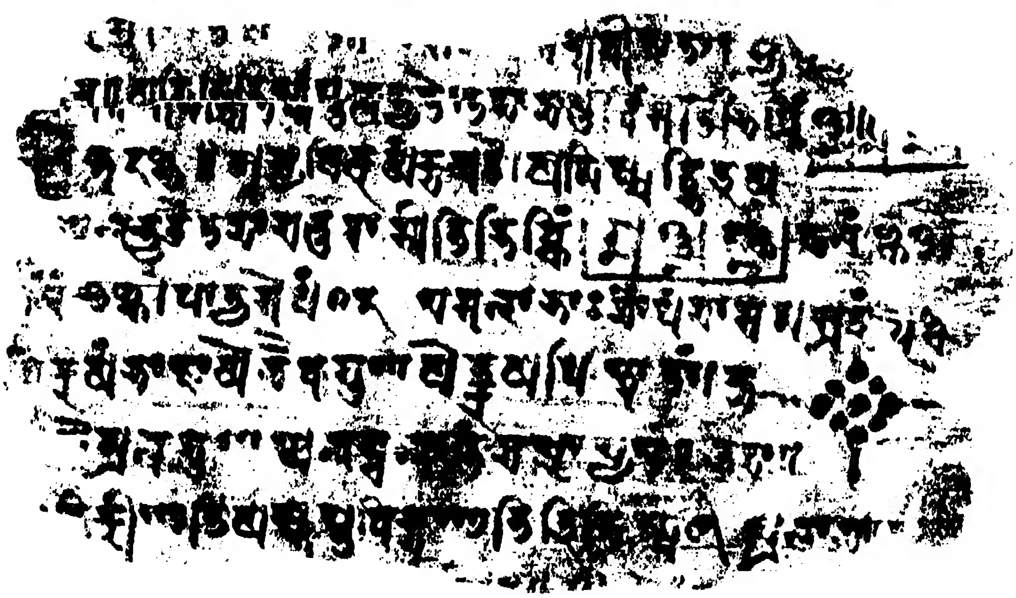
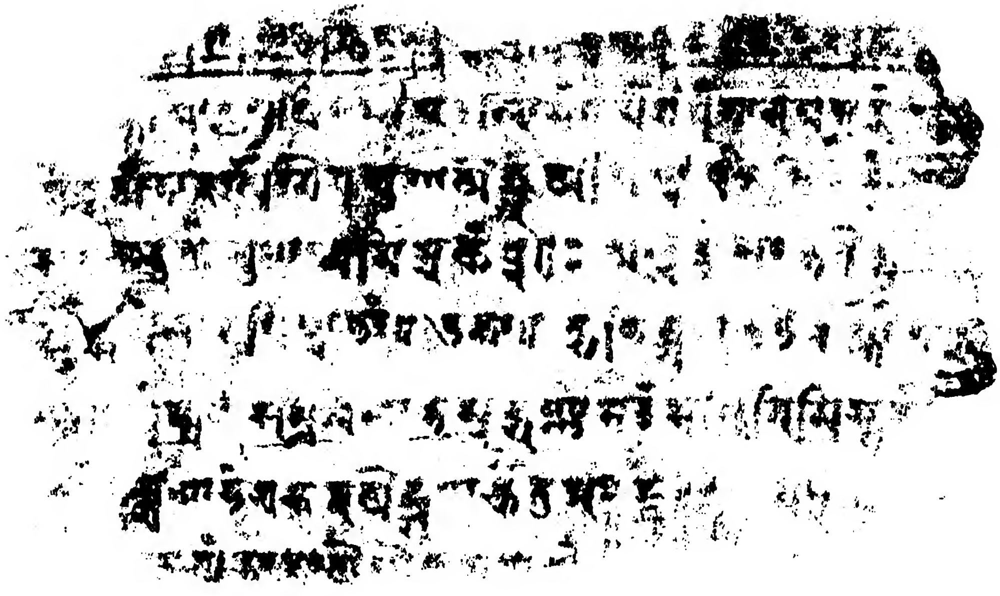


62 RECTO

Plate XLII

62 VERSO



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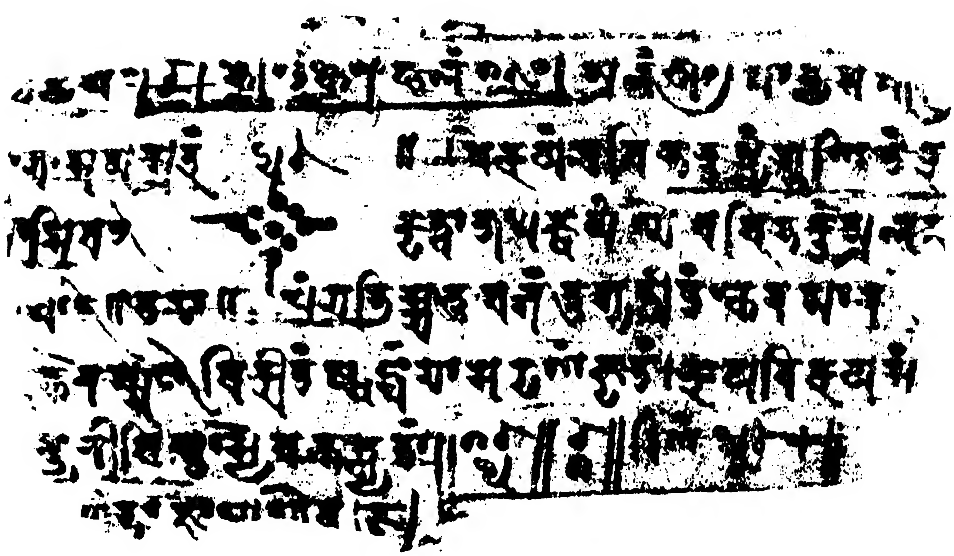

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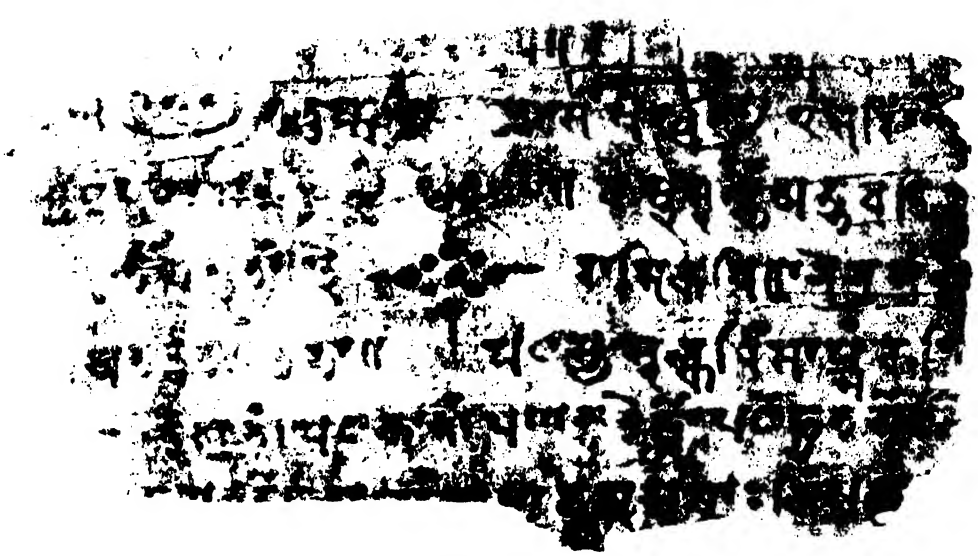

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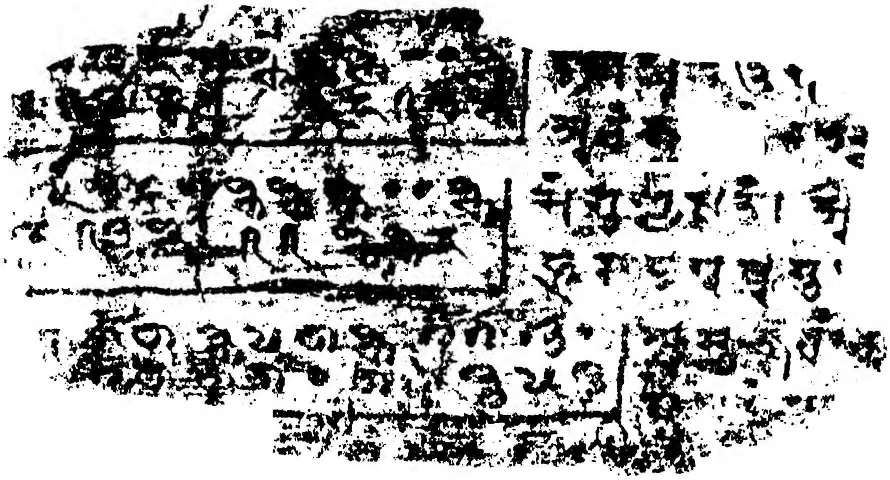

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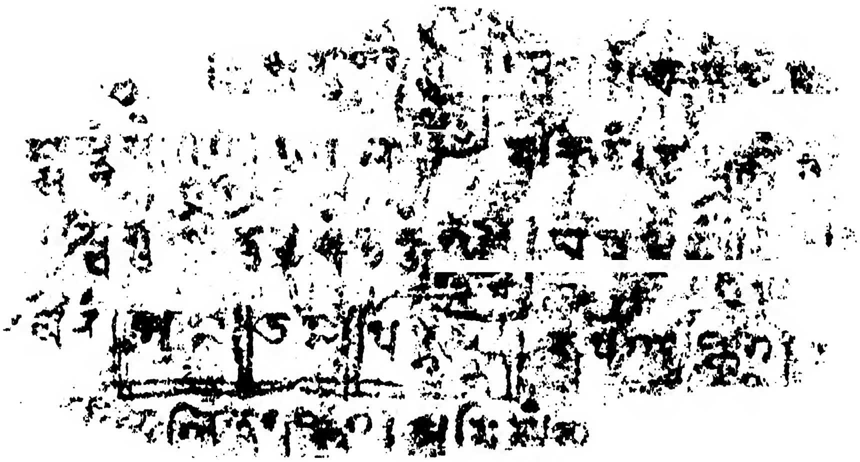

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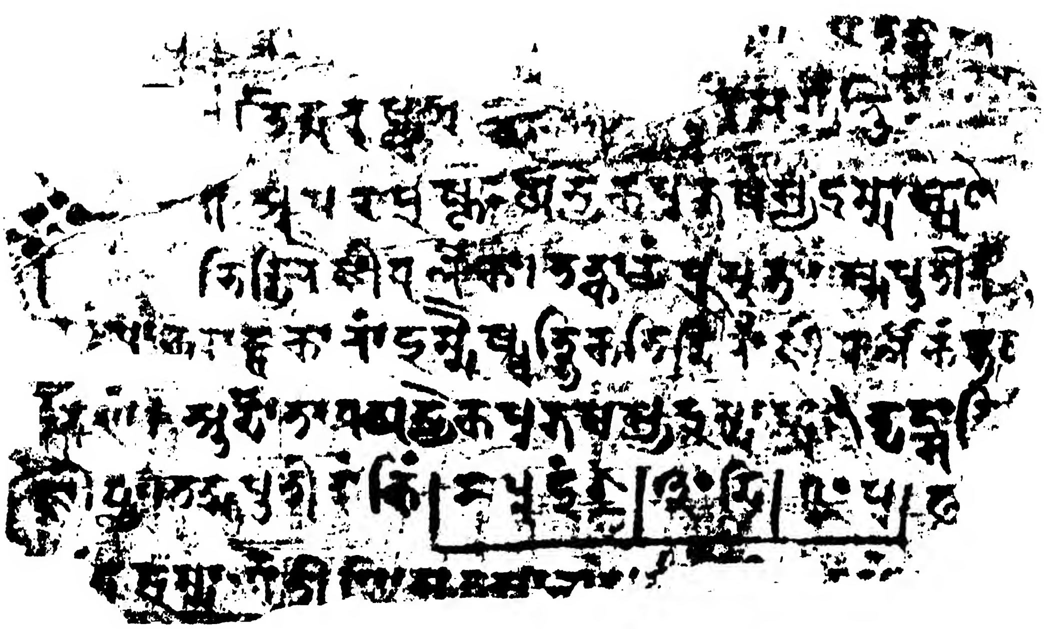

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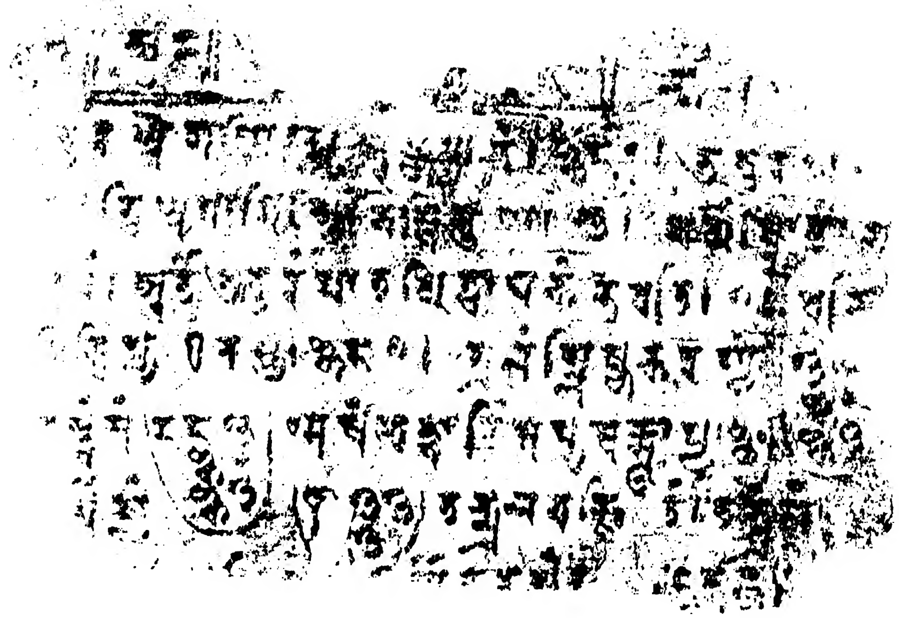

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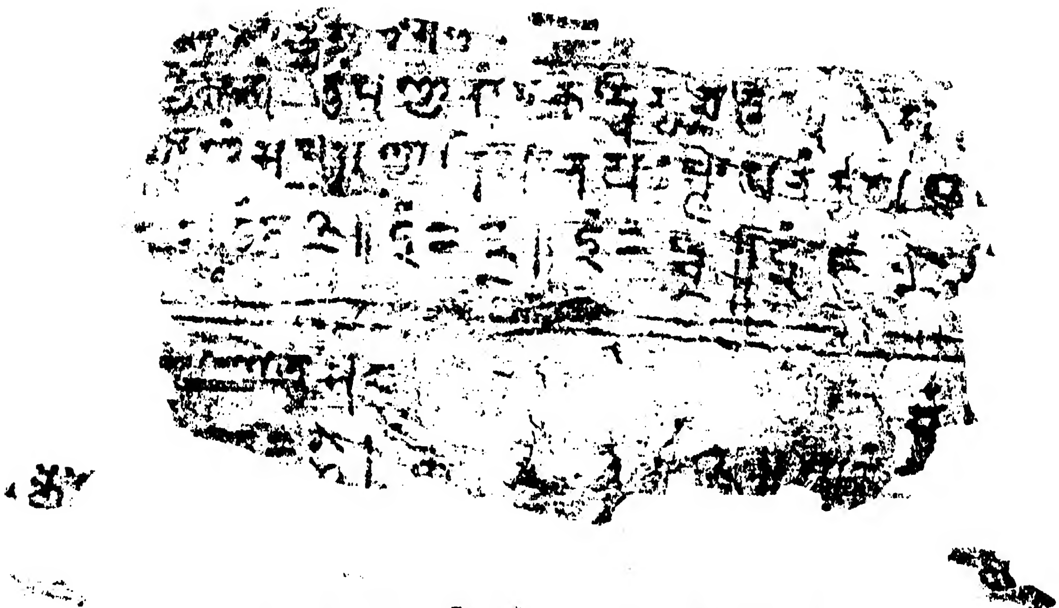

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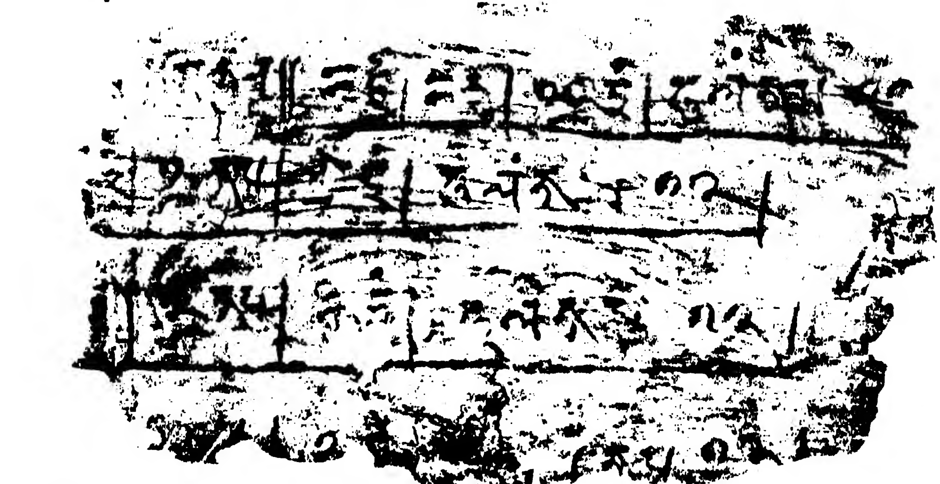

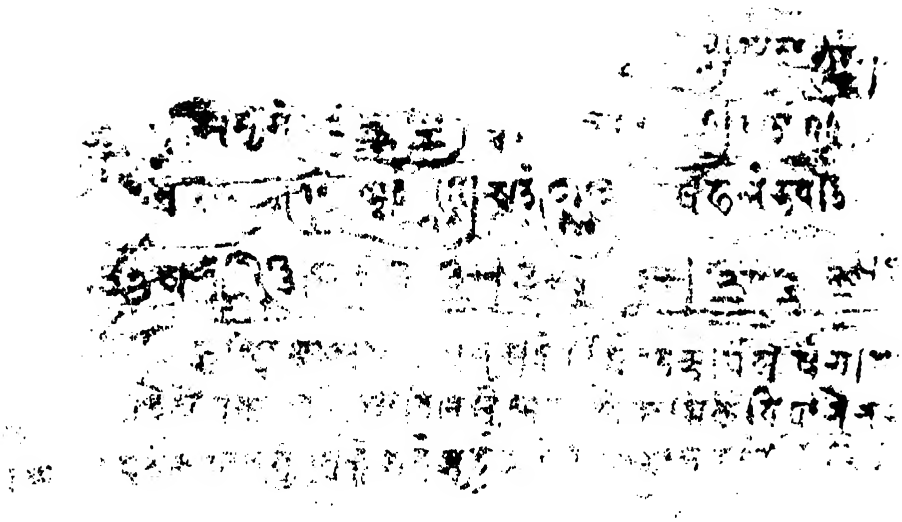

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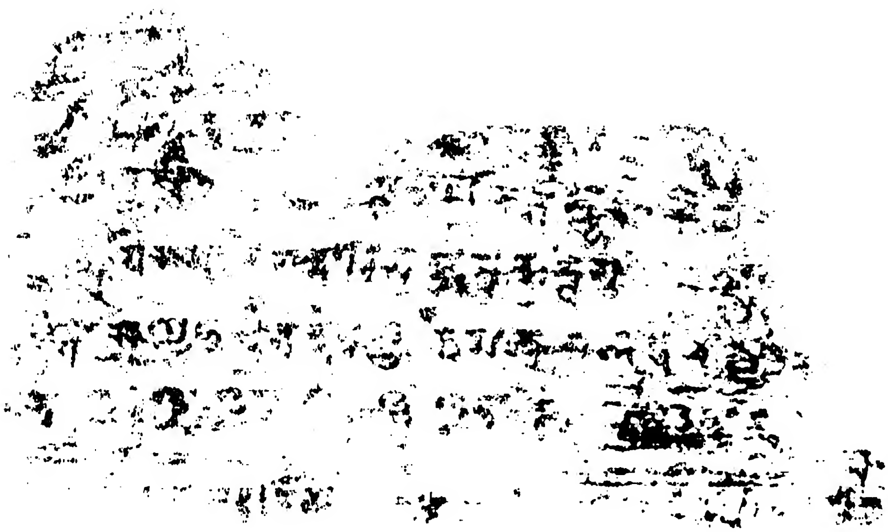


67 VERSO

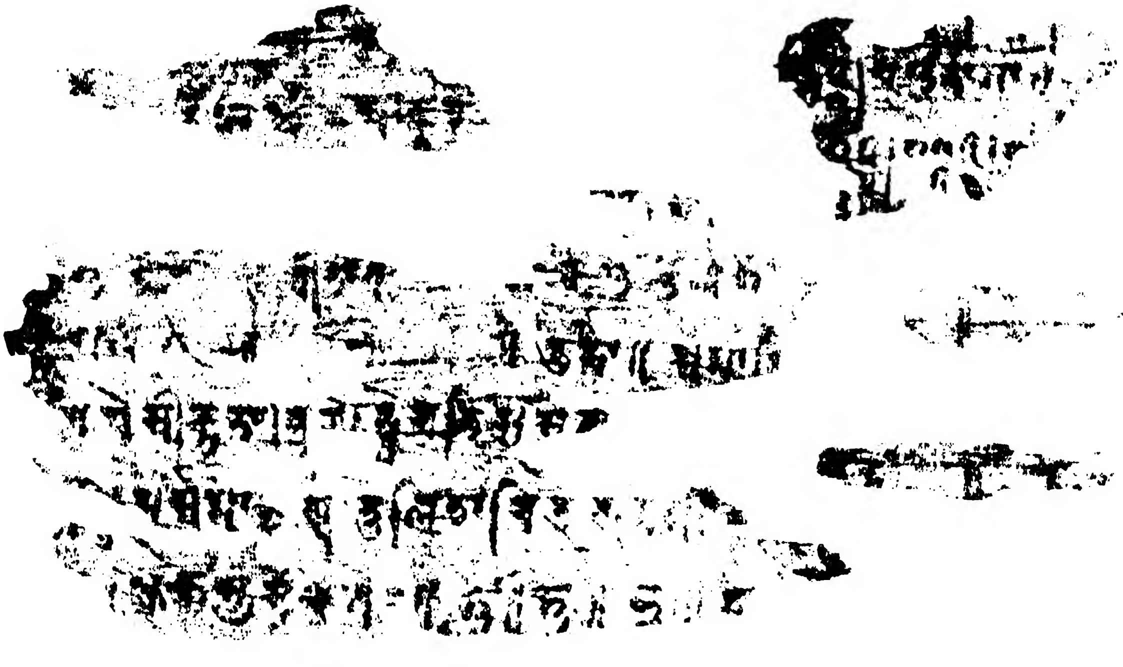
68 RECTO

Plate XLVI

RECTO

VERSO

Plate XLVII



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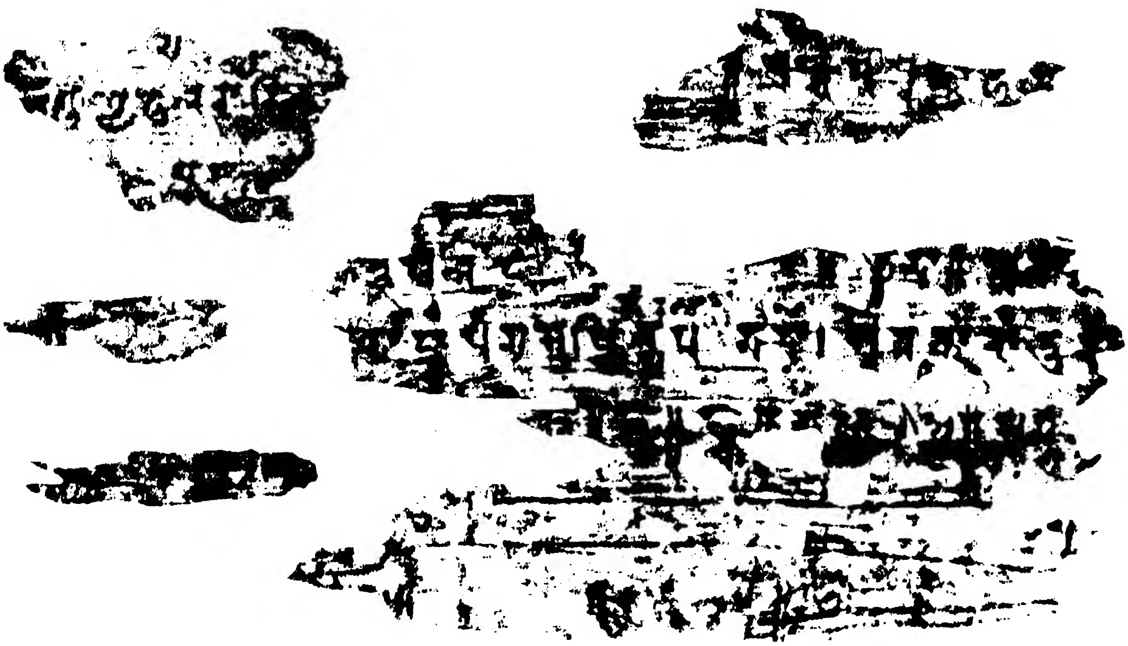

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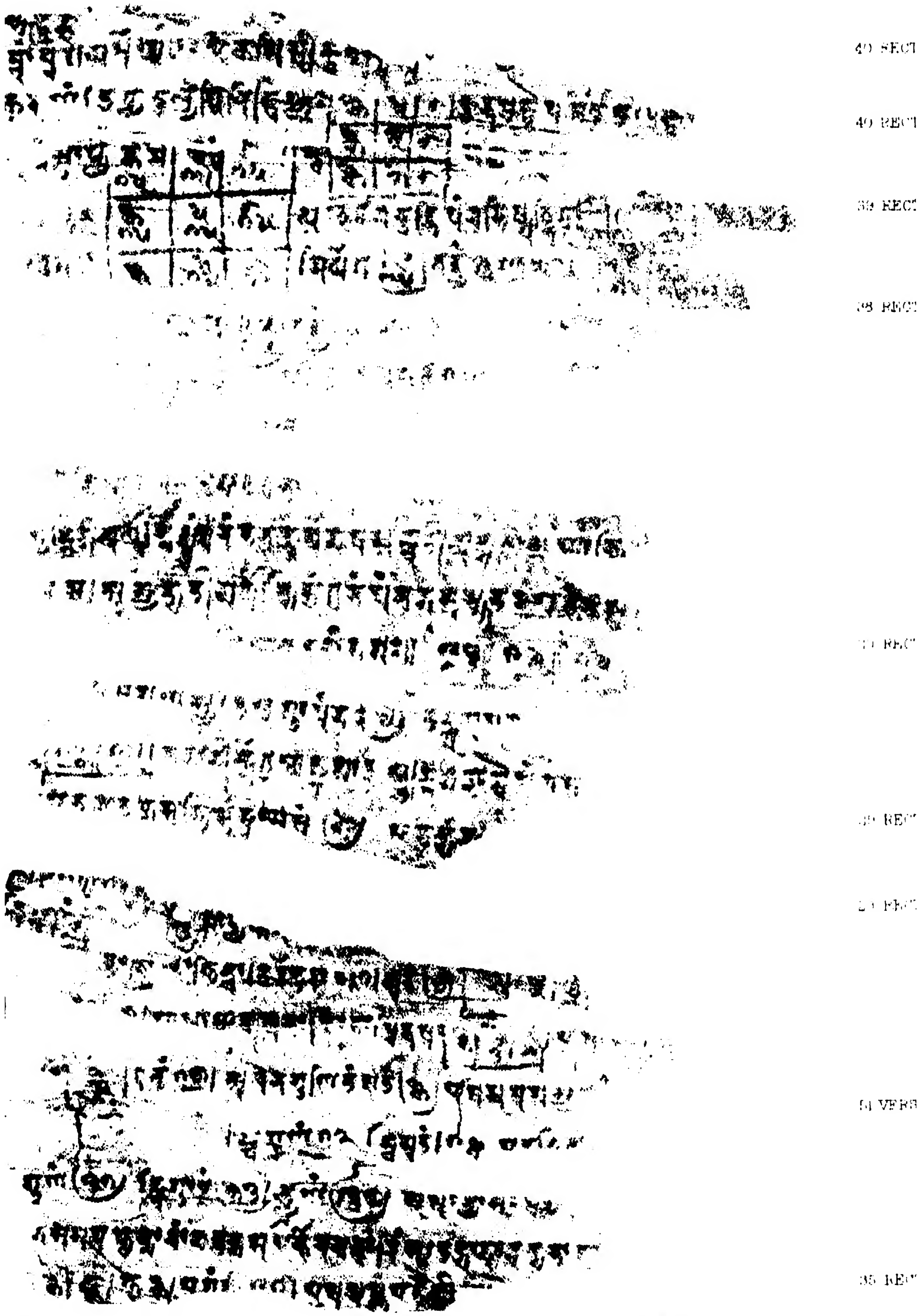

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RE-ARRANGED FRAGMENTS.

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